Exhibit H

NAVIGANT

UNITED STATES DISTRICT COURT

NORTHERN DISTRICT OF OHIO

EASTERN DIVISION

OHIO PUBLIC EMPLOYEES RETIREMENT SYSTEM, On Behalf of Itself and all Others Similarly Situated,

Plaintiff,

v.

FEDERAL HOME LOAN MORTGAGE CORPORATION, a/k/a FREDDIE MAC, RICHARD F. SYRON, PATRICIA L. COOK, ANTHONY S. PISZEL AND EUGENE M. McQUADE,

Defendants.

CIVIL ACTION NO. 4:08-CV-00160

JUDGE BENITA Y. PEARSON

MAGISTRATE JUDGE
WILLIAM H. BAUGHMAN, JR.

EXPERT REPORT OF MUKESH BAJAJ, PH.D.

September 1, 2017

Table of Contents

Case: 4:08-cv-00160-BYP Doc #: 385-2 Filed: 09/01/17 4 of 370. PageID #: 18831

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 5 of 370. PageID #: 18832

I. Qualifications

1. I am a Managing Director and the Global Head of the Securities and Finance Practice at Navigant Economics LLC, a consulting firm that applies economic and financial analysis to legal, regulatory and business matters. My curriculum vitae is attached as Appendix I.

2. In 1988, I graduated from the University of California at Berkeley earning a Ph.D. in Business Administration with a specialty in finance. I was awarded an M.B.A. from the University of Texas at Austin in 1987. I was awarded a Bachelor of Technology degree in 1981 from the Indian Institute of Technology in Delhi, India.

3. As a financial economist, I specialize in the study of capital markets, including the valuation of stocks, bonds, warrants, restricted stock and other complex contingent securities, intellectual property, intangible assets, corporate hedging practices (through derivatives and other methods), conducting event studies to determine the significance of stock price reactions to particular events, and analyzing market efficiency, materiality and loss causation issues related to securities class action claims.

4. Since 1996, I have been engaged as an expert on numerous matters involving valuation of firms and their securities, intangible assets and intellectual property. I have testified as an expert either in court or at deposition in over 60 matters, including almost 30 matters concerning liability and/or damages issues in securities fraud cases. In such securities fraud cases, I have testified on behalf of the U.S. Securities and Exchange Commission ("SEC") and the U.S. Attorney's Office in a criminal matter, as well as on behalf of both plaintiffs and defendants in civil and criminal matters.

5. In addition to my work with Navigant, I also have taught graduate-level courses in

Case: 4:08-cv-00160-BYP Doc #: 385-2 Filed: 09/01/17 6 of 370. PageID #: 18833

NAVIGANT

corporate finance, investments and financial engineering as a visiting lecturer with the Haas School of Business ("Haas") at the University of California at Berkeley. At Haas, I served as a Graduate Student Instructor while earning my Ph.D. between 1983 and 1988. From 1988 to 1995, I was an Assistant Professor of Finance and Business Economics at the University of Southern California.

6. I have authored or co-authored more than 25 publications and working papers in the field of financial economics. My research has been published in *The Journal of Finance, The Journal of Financial Economics, The Journal of Financial Research, The Journal of Applied Finance, International Economic Review, Research in Finance, The Journal of Corporation Law, The Journal of Derivatives* and *Research in Law and Economics.*

7. I am a member of the American Finance Association, the Western Finance Association and the European Finance Association, and I have lectured widely on a variety of issues in financial economics. Navigant is being compensated for my work on these matters at my regular hourly rate of $1,050 per hour. That compensation is not in any way dependent on the opinions I express on issues in these cases. I am independent of the Plaintiffs and the Defendants in this matter. I have been assisted in my work on this case by my colleagues at Navigant, for whose work Navigant is being paid at their regular hourly rates, which range from $300 an hour to $700 an hour.

8. The documents I have considered are cited throughout the report and/or listed in Appendix II. If additional information becomes available, I reserve the right to supplement or modify the opinions set forth in this report.

II. Background and Scope of Assignment

9. The Plaintiff, Ohio Public Employees Retirement System ("OPERS"), seeks to represent a class of investors who purchased the common stock of the Federal Home Loan Mortgage Corporation ("Freddie Mac" or the "Company") from August 1, 2006 through and including

November 20, 2007 (the "Proposed Class Period" or the "Class Period").[1] The Plaintiff alleges that Defendants made alleged material misrepresentations and omissions[2] "throughout the Class Period… relating to, among other things":[3]

(i) [Freddie Mac's] exposure to or risk of loss from subprime mortgage loans and other nontraditional, high risk mortgages, including "Alt-A" mortgages (a mortgage industry term to describe reduced documentation/higher credit risk loans);

(ii) its underwriting guidelines and Defendants' adherence to those guidelines,

(iii) its loan analysis software and fraud detection systems,

(iv) its risk management measures and its risk management performance, and

(v) its capital position.

10. In the TAC, the Plaintiff alleges that curative disclosures related to the alleged misrepresentations began on November 20, 2007 (the last day of the Proposed Class Period) and continued until at least September 23, 2008.[4]

11. The Plaintiff further alleges that, in seeking damages related to the alleged misrepresentations, it is entitled to the presumption of reliance established by the fraud-on-the-market doctrine and asserts that:

a. "At all times relevant to this Complaint, the market for Freddie Mac common stock was **an efficient market**," (*i.e.*, "promptly digested current information regarding Freddie Mac from

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1 Third Amended Complaint For Violations Of Federal Securities Laws, United States District Court, Northern District Of Ohio, Eastern Division (Youngstown) Civil Action No. 4:08-cv-160 (the "Complaint" or "TAC"), ¶13. Henceforth I refer to Freddie Mac's specific financial quarters in numeric terms as Q1, Q2, Q3 or Q4 denoting the first through fourth quarter, respectively.

2 To be concise, I will refer to alleged misrepresentations and alleged omissions together as "misrepresentations," except when I treat them separately, in which case I will indicate that I am doing so.

3 TAC, ¶2.

4 TAC, Section V.D.

Case: 4:08-cv-00160-BYP Doc #: 385-2 Filed: 09/01/17 8 of 370. PageID #: 18835

all publicly available sources and reflected such information in the market prices for Freddie Mac common stock **at all relevant times**");[5]

b. The Defendants "failed to disclose **material information** regarding Freddie Mac's subprime exposure, internal controls, risk management, financial condition, results and business operations";[6] and

c. The Plaintiff and other members of the Class, "as purchasers of Freddie Mac common stock during the Class Period, suffered similar injury through their purchase of Freddie Mac's common stock at **artificially inflated prices** and a presumption of reliance applies."[7]

12. I previously filed an expert report in this matter commenting on the opinions of Plaintiff's expert Dr. Greg Hallman concerning market efficiency for Freddie Mac's common stock (or "Freddie Mac stock").[8] In that report I offered two main opinions: (1) Dr. Hallman failed to establish that the market for Freddie Mac common stock was semi-strong form efficient during the proposed class period, and the results of his event study suggest that it was not; and (2) the economic evidence does not support a finding that the alleged misrepresentations and omissions inflated the price of Freddie Mac's common stock or that they were material.[9]

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5 TAC, ¶266-267 (emphasis added). Henceforth, all quoted text in bold indicates emphasis that I added and bracketed text in quotes indicates text that I added, unless indicated otherwise.

6 TAC, ¶268.

7 TAC, ¶267.

8 Expert Report of Mukesh Bajaj, *OHIO PUBLIC EMPLOYEES RETIREMENT SYSTEM, v. FEDERAL HOME LOAN MORTGAGE CORPORATION a/k/a FREDDIE MAC, et al.*, CIVIL ACTION NO. 4:08-cv-160, December 14, 2012. ("2012 Bajaj Report").

9 In a related case concerning Freddie Mac Series Z Preferred Stock class action litigation in the Southern District of New York [In re Federal Home Loan Mortgage Corp., No. 09 Civ. 832 (MGC)], I filed two expert reports and offered testimony on whether plaintiffs' expert evidence in that case sufficiently demonstrated that Freddie Mac's Series Z Preferred Stock traded in an efficient market during the Class Period. In that case Judge Cedarbaum credited my testimony and did not certify the class because Plaintiffs had "not shown by a preponderance of the credible evidence that the market for Series Z was efficient" and hence "the fraud on

Case: 4:08-cv-00160-BYP Doc #: 385-2 Filed: 09/01/17 9 of 370. PageID #: 18836

NAVIGANT

13. On December 16, 2016, Dr. Steven Feinstein filed a declaration ("Feinstein Declaration") in support of Plaintiff's motion to replace Dr. Hallman on the grounds that since Hallman's reports in 2012 "substantial developments" "beckoned the application of new empirical tests" and that "[a]ccordingly," "new empirical tests" relating to market efficiency "have been created," which "[v]ery recently" "have been gaining more widespread use and acceptance by courts."[10] In his declaration, Dr. Feinstein stated that, "if permitted by the Court to provide a report and opinion on market efficiency, I would prefer to conduct my own event study, so that I may apply the methodology in the manner I usually do."[11]

14. On June 12, 2017, Dr. Feinstein filed a "Report on Market Efficiency" ("Feinstein Report"). Dr. Feinstein's primary opinion was that "Freddie Mac common stock traded in an efficient market over the course of the Class Period."[12] Dr. Feinstein also concluded that "[e]vent study analysis demonstrates that there was a cause and effect relationship between the release of new, Company-specific information and movements in Freddie Mac's stock price during the Class Period" and that "Freddie Mac's stock price responded in a statistically significant fashion to new Company-specific information, which not only indicates market efficiency, but provides direct demonstrative evidence of market efficiency."[13]

15. I have been asked by Freddie Mac's counsel to review and comment on the opinions

the market presumption of collective reliance does not apply." [In re Federal Home Loan Mortgage Corp., No. 09 Civ. 832 (MGC), March 27, 2012.] Judge Cedarbaum, noted that "[e]ven if McCann [the Plaintiff's expert] demonstrated that news probably had some effect on price, plaintiff's burden is higher. A plaintiff must show that the market price responds to most new, material news." [In re Federal Home Loan Mortgage Corp., No. 09 Civ. 832 (MGC), March 27, 2012, at 18] Below, I will discuss how the approach presented by Dr. Feinstein in this matter utilizes similar techniques and is similarly deficient.

10 Declaration of Professor Steven P. Feinstein, OHIO PUBLIC EMPLOYEES RETIREMENT SYSTEM, v. FEDERAL HOME LOAN MORTGAGE CORPORATION a/k/a FREDDIE MAC, et al., CIVIL ACTION NO. 4:08-cv-160, December 16, 2016 ("Feinstein Declaration"), ¶21.

11 Feinstein Declaration, ¶28.

12 Report on Market Efficiency Professor Steven P. Feinstein, OHIO PUBLIC EMPLOYEES RETIREMENT SYSTEM, Individually and on Behalf of All Others Similarly Situated, Plaintiff, vs. FEDERAL HOME LOAN MORTGAGE CORPORATION a/k/a FREDDIE MAC, et al., Defendants, Case No. 4:08-cv-00160-BYP, June 7, 2017 ("Feinstein Report"), ¶18.

13 Feinstein Report, ¶19.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 10 of 370. PageID #: 18837

in the Feinstein Report. I have also been asked to review the economic evidence as it relates to the Plaintiff's allegations that alleged misrepresentations and omissions had an impact on the price of Freddie's Mac's stock.

III. Summary of Opinions

16. Based on my expertise and experience as a financial economist and my analysis described in this report, I have reached the following primary opinions:

1) Dr. Feinstein fails to prove a "cause and effect" relationship between material information and Freddie Mac's stock price and thus fails to demonstrate that the market for Freddie Mac stock was efficient over the Class Period.

2) The economic evidence supports a finding that the alleged misrepresentations and omissions had no impact on the price of Freddie Mac's common stock.

17. I explain the bases for these opinions below. In addition, attached to this report are Appendices and Exhibits that include information referenced herein and analyses that further support my opinions. These Appendices and Exhibits are as follows:

Case: 4:08-cv-00160-BYP Doc #: 385-2 Filed: 09/01/17 11 of 370. PageID #: 18838

- Appendix I: Curriculum Vitae of Mukesh Bajaj
- Appendix II: Documents Considered
- Appendix III: Application of Event Study Method
- Appendix IV: Dr. Feinstein's Z-Test Sensitivity Table
- Appendix V: 23 Misrepresentation/Omission Dates Mentioned in the Third Amended Complaint
- Appendix VI: Dr. Feinstein's Z-Test for 23 Misrepresentation/Omission Dates mentioned in the Third Amended Complaint
- Appendix VII: Dr. Feinstein's Z-Test using Results from a Regression Model that does not "Dummy" out News Dates
- Appendix VIII: VIX Index and Freddie Mac's Implied Volatility Chart
- Appendix IX: Dr. Feinstein's Regression Model Over Three Estimation Periods and Number of Statistically Significant Days by Period
- Exhibit I: Dr. Feinstein Report on Market Efficiency in Petrobras
- Exhibit II: Dr. Feinstein Report on Market Efficiency in Eletrobras
- Exhibit III: Excerpt from Statistics Textbook Cited by Dr. Feinstein [Macfie and Nufrio (2006)]
- Exhibit IV: Excerpt from Statistics Textbook Cited by Dr. Feinstein [Devore (2016)]

IV. Dr. Feinstein Fails to Demonstrate That the Market for Freddie Mac Stock Was Efficient Over the Class Period.

18. As I explain in this section, Dr. Feinstein has failed to demonstrate a cause and effect relationship between material information and Freddie Mac's stock price, and therefore has failed to demonstrate that the market for Freddie Mac's common stock was efficient over the Class Period. Dr. Feinstein's opinion that Freddie Mac's common stock traded in an efficient market throughout the Class Period rests on: (1) a mechanical review of certain factors that have been

discussed by some courts as indicators of market efficiency; and (2) two flawed studies that Dr. Feinstein conducted: – a single date event study and a "z-test".[14]

19. In particular, Dr. Feinstein cites the following eight factors drawn from the judicial decisions in *Cammer v. Bloom*, 711 F. Supp. 1264 (D.N.J. 1989) and *Krogman v. Sterritt*, 202 F.R.D. 467 (N.D. Tex. 2001) (hereafter, *Cammer* and *Krogman*):[15]

(1) the average weekly trading volume expressed as a percentage of total outstanding shares;

(2) the number of securities analysts following and reporting on the stock;

(3) the extent to which market makers and arbitrageurs trade in the stock;

(4) the company's eligibility to file SEC registration Form S-3[16] (as opposed to Form S-1 or S-2);

(5) the existence of "empirical facts showing a cause and effect relationship between unexpected corporate events or financial releases and an immediate response in the stock price";[17]

(6) the company's market capitalization;

(7) the bid-ask spread for stock sales; and

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14 VIDEOTAPED DEPOSITION OF: STEVEN P. FEINSTEIN, PH.D. in Re: OHIO PUBLIC EMPLOYEES RETIREMENT SYSTEM, on Behalf of Itself and all Others Similarly Situated, Plaintiff vs. FEDERAL HOME LOAN MORTGAGE CORPORATION, a/k/a FREDDIE MAC, RICHARD F. SYRON, PATRICIA L. COOK, ANTHONY S. PISZEL and EUGENE M. MC QUADE, Defendants, CA NO. 4:08-CV-00160, August 10, 2017 ("Feinstein Dep."), at 21:18-22:3 (testifying that he conducted two studies in connection with the fifth Cammer factor).

15 Feinstein Report, ¶¶46-48.

16 While OPERS contends that the first four Cammer factors support a finding of market efficiency, Dr. Feinstein admitted at deposition that Freddie Mac was not eligible to file Forms S-3 during the Class Period. Feinstein Dep. 98:6-99:11.

17 *Cammer* at 1287 quoted in the Feinstein Report, ¶46.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 13 of 370. PageID #: 18840

NAVIGANT

(8) float, the stock's trading volume without counting insider-owned stock.

20. As Dr. Feinstein notes, these factors are divided into two types: "structural" and "direct."[18] Factors 1-4 and 6-8 are referred to as "structural" factors that facilitate or "indicate" market efficiency. Factor 5, the cause-and-effect factor, calls for direct, empirical tests of market efficiency. Neither Dr. Feinstein's review of the structural factors nor his empirical studies, taken individually or collectively, demonstrate market efficiency.

A. Dr. Feinstein's Analysis Of The Structural *Cammer* and *Krogman* Factors Are Insufficient To Support His Opinion.

21. While the *Cammer* and *Krogman* structural factors are important in an analysis of market efficiency, they are not sufficient to establish market efficiency.[19]

22. The *Cammer* court considered the fifth factor, *i.e.*, "empirical facts showing a cause and effect relationship between unexpected corporate events or financial releases and an immediate response in its stock price" to be the most important factor in assessing market efficiency. As the *Cammer* court itself explained in referring to this cause and effect relationship: "This, after all, is the essence of an efficient market and the foundation for the fraud on the market theory."[20]

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18 Feinstein Report, ¶99; VIDEOTAPED DEPOSITION OF STEVEN PHILLIP FEINSTEIN IN RE: GROUPON, INC. SECURITIES LITIGATION, Case No.12 CV 2450, February 12, 2014 ("Feinstein Groupon Deposition"), at 111:21-113:8:,115:21-116:19.

19 In his report, Feinstein repeatedly claims that my past testimony in this case contradicts my opinions about *Cammer* / *Krogman* factor tests for market efficiency in a more recent case. Dr. Feinstein is simply wrong. I have never stated that structural factors were not indicia of market efficiency, but rather cited studies supporting the point that these factors "are not, alone, sufficient to establish market efficiency." *See*, e.g., 2012 Bajaj Report, ¶141. Dr. Feinstein repeatedly makes similarly incorrect statements about my 2012 Report regarding the other *Cammer* factors. E.g., ¶62, ¶86 and ¶90. These claims similarly mischaracterize my prior testimony.

20 *Cammer* at 1287.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 14 of 370. PageID #: 18841

23. Dr. Feinstein appears to agree that the fifth factor is the most important factor. He notes that the court in *Halliburton Co. v. Erica P. John Fund, Inc., 134 S. Ct. 2398, 2410, 189 L. Ed. 339 (2014) ("Halliburton II")* found that the "cause and effect relationship" was "at the center of market efficiency"[21] and one of "the most convincing" ways to demonstrate market efficiency.[22] He also testified in another matter that the fifth factor is "the essence of market efficiency," whereas the first four are merely "indicators of market efficiency."[23,] Further, when asked whether he would be willing to opine that a market was efficient if it merely satisfied the structural factors, Dr. Feinstein admitted that he had not, and would not, author a report concluding that a market was efficient without "some empirical demonstration" of efficiency.[24]

24. Even Dr. Feinstein concedes that the structural *Cammer* / *Krogman* factors are not a sufficient basis for him, *as an economist*, to conclude the market was efficient, and that he has never concluded a market to be efficient based solely on the structural factors in any expert report

21 Feinstein Report, ¶39.
22 Feinstein Report, ¶98-99; *Cammer* at 1291.
23 Feinstein Groupon Dep. 111:18-20, 115:21-116.:19.
Q: Would you ever perform a Cammer analysis without considering the fifth factor?
A: No. I always consider the fifth factor.
…
Q: In your view, is the fifth Cammer factor irrelevant?
A: No.
Q: Why do you say that?
A: Well, I mean it's – as I described in the report, the fifth factor is the essence of market efficiency, whereas the first four are indicators of market efficiency. So to the extent that we can run the analysis, the empirical analysis, you come up with much more certainty about the conclusion. I mean I – you know, based on the first four factors, I can make a determination is it more likely than not. Based on the fifth factor, I can – I feel more comfortable making a determination that, you know, with a high degree of professional certainty, the stock is trading either efficiently or inefficiently.
Q: In your opinion, is the fifth factor the most important of the factors?
A: It's different. It's different because it's the essence of market efficiency, rather than an indicator of market efficiency. So it's – it's very important.

24 Feinstein Dep. 384:11-19.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 15 of 370. PageID #: 18842

in a securities case.[25] However, he also claimed that as a matter of law, and after *Halliburton II*, he believed analysis of the structural factors is sufficient and that the empirical factor tests were just "icing on the cake."[26] This claim has no basis. To my knowledge, Dr. Feinstein is not a lawyer or legal expert. Crucially, Dr. Feinstein would not himself conclude that a market was efficient without an empirical demonstration of efficiency, i.e., economic evidence of a cause and effect relationship between news and stock price movement under *Cammer* factor 5. Dep. 384:11-19. As I noted above, I have reviewed *Halliburton II*, and as an economist I see no change in the definition of market efficiency for reliance purposes that would change the standard of proof required of an economist to establish efficiency. Moreover, I am not aware of any economic literature that concludes that the structural factors are sufficient to prove market efficiency. In fact, as noted below, the economic literature demonstrates precisely the opposite.

25. I agree with Dr. Feinstein that the fifth *Cammer* factor is the most important factor in assessing market efficiency. "Cause and effect" is the "*sine qua non*"[27] and necessary condition of informational market efficiency.[28] The other seven *Cammer* and *Krogman* factors, as Dr. Feinstein

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25 Feinstein Dep., 383:23-384:23.

26 Feinstein Dep., 380:9-381:20, 384:24-386:12.
This appears to echo Plaintiffs' legal brief which claims that Halliburton II established a new, "relaxed" "standard" and "degree" of proof of market efficiency for reliance purposes that does not require the fifth empirical Cammer factor. [Memorandum In Support Of Lead Plaintiff's Renewed Motion For Class Certification And To Appoint Class Representative And Class Counsel in re: *OHIO PUBLIC EMPLOYEES RETIREMENT SYSTEM, etc., Plaintiff, vs. FEDERAL HOME LOAN MORTGAGE CORPORATION*, et al., Defendants., Civil Action No. 4:08-cv-160 ("Plaintiff's Brief"), June 12, 2017, pages 8-9.]

27 *In re Federal Home Loan Mortgage Corp., No. 09 Civ. 832 (MGC), Opinion and Order, p. 23.*

28 Informationally efficient markets have security prices that reflect all material current information. In such informationally efficient markets, "prices reflect information to the point where the marginal benefits of acting on information (the profits to be made) do not exceed the marginal costs" of gathering such information. [Fama, Eugene F., "Efficient Capital Markets: II," *The Journal of Finance* 46, no. 5 (Dec. 1991), ("Fama (1991)"), pages 1575-617] Informational market efficiency can be thought of as distinct from "fundamental" market efficiency where the stock price must accurately reflect the fundamental value of the stock, rather than the incremental impact of new information.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 16 of 370. PageID #: 18843

acknowledges, only refer to "structural" market conditions. He notes that "[t]he "convincing nature of the empirical [fifth *Cammer*] factor is justified by economic principles, in that this factor provides a direct demonstration of market efficiency, whereas the other four [*Cammer*] factors are structural indicators of market efficiency."[29] These structural market conditions may promote, but do not necessarily result in, a cause and effect relationship between new information and stock price movements. Accordingly, from an economic perspective, these conditions alone, are not sufficient evidence of market efficiency.

26. In an "efficient market," stock prices at all times reflect all material, publicly available information and therefore stock prices should react promptly. to new material information regarding stock value. That is, in an efficient market, the stock price should promptly increase following unexpected positive news and promptly decrease following unexpected negative news, and such price changes should fully and correctly reflect the value of the unexpected news. If not, stock prices would not reflect all value-relevant information that was then publicly available. This is known as a "semi-strong form efficient market."[30] In this report, unless the context makes it

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The following hypothetical illustrates this distinction in the context of event study analysis, a technique economists use to test whether stock prices correctly react to new information [I discuss the event study method in more detail later in this report and in Appendix III]. Suppose a stock traded at $10 per share, whereas its true "fundamental" value was $15, i.e., its price was inconsistent with fundamental efficiency. Suppose further that material negative information is released to the public which should lead to a decline in stock price by $1 a share. An event study would test whether, upon receipt of this material negative information, the stock price declined from $10 to $9 per share. No event study test I have ever seen in thousands of academic as well as expert analyses would ever test whether the market price reaction to this type of new material information leads to the stock being correctly valued in a fundamental efficiency sense. If an event study were designed to test for fundamental efficiency, one would expect the price to rise to $14, i.e., a 40% increase rather than a 10% decline in response to this material negative news.

29 Feinstein Report, ¶99.

30 While the focus here is on semi-strong, the financial economics literature denotes three theoretical forms of efficiency: weak form efficiency, semi-strong form efficiency and strong form efficiency. In a weak-form efficient market, security prices reflect the information contained or present in "just historical prices." [Fama, Eugene F. (1970), "Efficient Capital Markets: A Review of Theory and Empirical Work," Journal of Finance

clear, when I refer to an "efficient market," I mean the semi-strong form of the efficient market hypothesis.[31]

27. As I discuss in Section B below, Dr. Feinstein has failed to demonstrate the "cause and effect relationship" between unexpected corporate announcements by Freddie Mac and contemporaneous changes in its stock price that the *Cammer* court deemed "the essence of an efficient market and the foundation for the fraud on the market theory."[32] Indeed, like Dr.

25(2), pages 383-417 ("Fama (1970)"), at page 383.] (In other words, there are no discernible trends that can be exploited to profitably trade in stock in a weak form efficient markets.)

In a semi-strong form efficient market, security prices reflect the information in historical prices as well as "other information that is obviously publicly available (e.g., announcements of annual earnings, stock splits, etc.)." Finally, in a strong form efficient market, security prices reflect "any information relevant for price formation" (i.e., either public or non-public information). [Fama (1970), page 383] Financial economists agree that the strong form of efficient markets hypothesis is simply a theoretical benchmark and it is not realistic.

Each form of market efficiency rests on the prior form. If the market is not weak-form efficient, it cannot be semi-strong form efficient.

[31] While Fama's conception of semi-strong informational market efficiency has been around since the 1970s, Dr. Feinstein implies that the Supreme Court's decision in *Halliburton II* altered the requirements for the type of efficiency required for the presumption of reliance. He claims in his Declaration and in his Report that the *Halliburton II* decision provided a new standard of proof requiring that plaintiffs only need to establish "informational" market efficiency (rather than "fundamental" efficiency) in Rule 10b-5 class certification proceedings. [Feinstein Declaration, ¶19-20; Feinstein Report, ¶32-33.] This is not true. It is my understanding that *Halliburton II* reaffirmed the Supreme Court's seminal decision in *Basic Inc. v. Levinson,* 485 U.S. 224, 246 (1988) ("*Basic"),* and the market efficiency standard to establish reliance since *Basic* was always based on informational efficiency and not fundamental efficiency. This is the standard I have applied in my academic and economic work in securities litigation for over 20 years. In fact, Dr. Hallman, in his report in 2012, also reviewed the *Cammer* and Krogman factors with the understanding that he was examining informational efficiency (and what he called operational efficiency which equated to informational efficiency). [Hallman Report, ¶10]

The concept of reliance as used in the Supreme Court's seminal decision in *Basic*, as I understand it, cannot have any other reasonable interpretation except semi-strong form "informational" market efficiency. Indeed, the District Court in *Polymedica* (a case that Dr. Feinstein cites in his Report as support for his z-test [Feinstein Report, ¶141]) noted that "by requiring that stock price in an efficient market fully reflect all publicly available information in order to establish the fraud-on-the-market presumption, we do not suggest that stock price must accurately reflect the fundamental value of the stock" and that "[t]his distinction is well-supported by the legal and economic commentary." [Polymedica (2005), p. 35.]

[32] *Cammer* at 1287.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 18 of 370. PageID #: 18845

Hallman's study several years ago, Dr. Feinstein's studies show that Freddie Mac's common stock did not trade in an efficient market throughout the Class Period, if they prove anything at all.

28. A recent study examined whether "the *Cammer and Krogman* factors are sufficient for reliance" and found that the factors had "little relation" even to weak-form efficiency, a standard lower than what I understand is required to establish a claim of presumption of reliance under the fraud-on-the-market theory, *viz.*, semi-strong form efficiency.[33] Hence, the mere presence of the *Cammer* and *Krogman* structural factors that Dr. Feinstein discusses, without actual empirical tests, could not possibly establish market efficiency.[34]

29. Indeed, a large body of literature in finance, developed over the past twenty-five years following *Basic*, indicates that a mechanical review of the structural market condition factors (*Cammer* factors 1-4 and the *Krogman* factors) is inadequate to assess market efficiency for class certification purposes, often because the activity of arbitrageurs (investors seeking to profit from informational inefficiencies) may be limited even in a market where these structural factors would indicate market efficiency. In that regard, these structural factors may be analogized to a doctor checking a patient's vital signs before specifically addressing the patient's concerns. If the doctor determines that the patient has healthy vital signs, those signs alone would not be sufficient to discharge a patient complaining of ill health without a careful further examination.

30. Not surprisingly, several academic studies have found that even the prices of well-

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33 Erenburg, Grigori, Janet Kiholm Smith and Richard L. Smith (2011), "The Paradox of 'Fraud-on-the-Market Theory': Who Relies on the Efficiency of Market Prices?" Journal of Empirical Legal Studies, Volume 8, Issue 2, June 2011, pp. 260-303. Pg. 266.

34 There are several tests of weak-form efficiency discussed in the finance literature, including Serial Correlation, Put-Call Parity, and Y-filter Tests. See, e.g., Fama, Eugene F. (1970), "EFFICIENT CAPITAL MARKETS: A REVIEW OF THEORY AND EMPIRICAL WORK", The Journal of Finance ("Fama (1970)"), pp. 390-396.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 19 of 370. PageID #: 18848

capitalized companies' stocks that traded on major exchanges, that would easily satisfy the structural factors that serve only as indicia of market efficiency,[35] may not be semi-strong form efficient over extended periods.[36] Academic studies have documented that the market for some securities can become inefficient under certain market conditions, if, for example, the activity of arbitrageurs (who are the lifeblood of an efficient market as noted above) is limited (*e.g.*, from a lack of access to capital to fund the desired trades in a period of market turmoil).[37] Hence, even for large capitalization stocks traded on major exchanges, a careful and comprehensive cause and effect analysis is necessary to properly analyze market efficiency. The economic logic of such an approach was not changed in any way by *Halliburton II*.

31. In fact, it is my understanding that the Supreme Court went further than previous case law in *Halliburton II,* by specifically allowing defendants to defeat any presumption of reliance by demonstrating that the misrepresentations at issue did not have an impact on the defendant company's stock price a lack of price impact. This is consistent with the Supreme Court recognition that, even if generalized tests of market efficiency were to indicate an informationally

35 Bajaj et al (2014), "Assessing Market Efficiency For Reliance On The Fraud-on-the-Market Doctrine After Wal-Mart and Amgen," Research in Law and Economics, Vol. 26, 161-207. ["Bajaj et al. (2014)"]. This paper shows that simply a mechanically conducted event study on a handful of earnings announcement dates may misleadingly create a false impression of market efficiency.

36 Lamont, Owen A. and Richard H. Thaler (2003), "The Law of One Price in Financial Markets," Journal of Economic Perspectives, Vol. 17(4), pp. 191-202 ("Lamont and Thaler (2003a)"); Lamont, Owen A., and Richard H. Thaler, (2003), "Can the Market Add and Subtract? Mispricing in Tech Stock Carve-Outs," The Journal of Political Economy, Vol. 111, pp. 227–268 ("Lamont and Thaler (2003b)"); Fedenia, Mark and Mark Hirschey, (2009), "The Chipotle Paradox," Journal of Applied Finance, Issues 1 & 2, pp. 1-16. See also Mitchell, Mark, Todd Pulvino, And Erik Stafford, 2002, "Limited Arbitrage in Equity Markets," The Journal of Finance 57 (2), pp. 551 – 584.

37 See, e.g., Bajaj et al (2014).

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 20 of 370. PageID #: 18847

efficient market, market efficiency is not a "binary"[38] question, i.e., a security could trade in a generally efficient market that nevertheless did not result in distorted stock prices due to disclosure defects at issue in a particular case.[39]

32. Given the above discussion of the definition of market efficiency and the relative value of the *Cammer* and *Krogman* factors, Dr. Feinstein has failed to demonstrate that the market for Freddie Mac stock was informationally efficient during the Class Period solely on the basis of the structural *Cammer* and *Krogman* factors. As I discuss below, Dr. Feinstein's analysis of *Cammer* factor five fails to demonstrate market efficiency.[40]

B. Dr. Feinstein's Two Empirical Studies Do Not Support a Finding of Market Efficiency.

33. Dr. Feinstein purports to analyze the fifth *Cammer* factor through two analyses: (1) an "event study" of a single date marking the end of the Class Period, November 20, 2007 (which he calls an "allegation-related event"); and (2) a "z-test", a statistical test comparing the proportion of statistically significant "abnormal return" days[41] (according to his market model regression) on

[38] "Basic's presumption of reliance thus does not rest on a "binary" view of market efficiency. Indeed, in making the presumption rebuttable, Basic recognized that market efficiency is a matter of degree and accordingly made it a matter of proof." *Halliburton II* at II.B.2

[39] For example, *Halliburton II*, allows defendants to present "evidence showing that the alleged misrepresentation did not actually affect the stock's market price," in order to rebut reliance. *Halliburton II* at III.B.1

[40] In addition, Dr. Feinstein also failed to perform weak-form market efficiency tests. Price reaction on a given date cannot be associated with news on that date unless the market was weak form efficient. Thus, testing for and finding the market to be at least weak form efficient needs to be a predicate to a valid test of semi-strong form of market efficiency through event studies, which Feinstein used as the basis for his empirical tests.

[41] "Abnormal" returns (also called "excess" or "residual" returns) in an event study context are the returns of the security being analyzed (in this case Freddie Mac common stock) net of the returns predicted using a statistical "market" model. The statistical "market model" based on a regression analysis is typically designed to capture market- and industry-wide factors to reasonably isolate the firm-specific component of the stock's return. See Appendix IIIa for more detailed discussion of the event study methodology. Note that

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 21 of 370. PageID #: 18848

NAVIGANT

9 purported "news" days (that he selected) as compared to 321 non-news days (as he defines them).[42] As I explain below, Dr. Feinstein's tests are fatally and fundamentally flawed both conceptually and from a statistical methods perspective, rendering them unreliable and invalid.

1. Dr. Feinstein's Single-Date Event Study Is a Flawed Study That Cannot and Does Not "Prove" Market Efficiency.

34. Dr. Feinstein first purports to prove market efficiency by an empirical "cause and effect" analysis, a purported "allegation-related event study" of a single day, November 20, 2007. He finds that on this day "there was a strongly statistically significant price reaction to Company-specific news."[43] Dr. Feinstein claims that "[t]his finding proves that Freddie Mac common stock reacted to new information and its market was efficient, and, in particular, efficient with respect to the information at issue in this case."[44]

35. Dr. Feinstein's single-date event study is flawed in many ways that I discuss below. Before turning to those flaws, it is important to understand what an event study is, how economists construct them, and how economists use them to assess market efficiency.

36. To determine if a stock trades in an efficient market, financial economists often use a statistical technique of hypothesis testing, known as an event study.[45]

37. An event study is "rooted in the basics of scientific methodology: first a hypothesis is

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if the market model is flawed, then the FDT z-test, which critically depends on the market model to generate the input observations, will also be flawed.

42 Feinstein Report, §VII.A and §VII.B and Exhibit 9.

43 Feinstein Report, ¶137.

44 Feinstein Report, ¶137.

45 I briefly explain the event study methodology here. Also see Appendix III.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 22 of 370. PageID #: 18849

proposed and then a test is conducted to see if that hypothesis can be rejected."[46] Specifically, an event study uses statistical methods to test the hypothesis that an identified news event had no effect on the stock's price. This is commonly called the "null" hypothesis.

38. To test the null hypothesis, one must calculate the stock's "abnormal return." The abnormal return is the change in stock price following the identified event, adjusted for contemporaneous market and industry effects.[47] As I explain below, the null hypothesis can only be rejected – i.e., we can only conclude that the identified news event had an effect on the stock's price – if the abnormal return is statistically significant.

39. To demonstrate a cause and effect relationship using an event study, one must first be able to reject the null hypothesis, i.e., rule out the possibility that the abnormal return was indistinguishable from zero (or attributable to chance alone rather than the identified news event) with an acceptably high degree of certitude.

40. The steps of an event study are as follows. One must first review and select the relevant news to be examined.[48] As Tabak (2010) explains, "the relevant academic literature is clear: first one identifies news and then measures price movements associated with that news."[49]

41. The next step in an event study is to establish the event window, which is the period over which the potential impact of news on the stock's price is assessed.[50] For instance, Dr.

[46] Tabak, David (2010), "Use and Misuse of Event Studies to Examine Market Efficiency," NERA Economic Consulting, April 2010 ("Tabak (2010)"), pg. 4.

[47] Tabak, David I. and Frederick C. Dunbar (2001), "Materiality and Magnitude: Event Studies in the Courtroom," in Litigation Services Handbook: The Role of the Financial Expert, Third Edition, ed. by Roman L. Weil, Michael J. Wagner and Peter B. Frank, John Wiley & Sons, Inc. USA henceforth, "Tabak and Dunbar (2001)" at 19-2.

[48] Tabak (2010) at pg. 4; also see Tabak and Dunbar (2001) at 19-4.

[49] Tabak (2010), pg. 4.

[50] Tabak and Dunbar (2001), pg. 19-4.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 23 of 370. PageID #: 18650

Feinstein uses a one-day event window.[51]

42. After identifying relevant events and establishing the appropriate event windows, a proper event study must attempt to isolate the portion of the stock's observed price change that is not attributable to market and industry effects, i.e., calculate the stock's abnormal return. As Bodie et al. notes, "a stock's price on any day could be affected by a wide range of economic news such as updated forecasts for GDP [Gross Domestic Product], inflation rates, interest rates, or corporate profitability."[52]

43. As noted in Tabak and Dunbar (2001), the calculation of abnormal returns is accomplished by estimating "a predicted stock price return, or percentage change, from the day before the news reaches the market to the day the stock price assimilates the news, using a [regression] model that takes into account market and industry effects on stock price returns … [and subtracting] the predicted return from the actual return to compute the so-called abnormal return."[53] Further, a central assumption in using regression models to predict returns is that the volatility remains unchanged over the relevant period.[54]

44. After the stock's abnormal return is calculated, one must assess its statistical significance to determine if the null hypothesis can be rejected. As Tabak and Dunbar (2001) note: "Typically, the predicted return does not exactly equal the actual return even when no event has occurred. To decide whether the difference between the actual and the predicted return [the

[51] Feinstein Report, Exhibit 7 shows Freddie Mac's abnormal return over one-day periods following his 9 selected "news" events.

[52] Bodie, Zvi, Alex Kane and Alan J. Marcus (2011), Investments, McGraw-Hill Irwin, New York, NY ("Bodie et al.") at pg. 353.

[53] Tabak (2001), page 19-2.

[54] Kennedy, Peter, *A Guide to Econometrics*, 5th ed. MIT Press, pg. 47-48.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 24 of 370. PageID #: 18651

NAVIGANT

abnormal return] … results merely from chance, one tests for statistical significance."[55]

45. An abnormal return is considered "statistically significant" when the likelihood that it occurred purely as a matter of chance is small.[56] On the other hand, if a stock's abnormal return is statistically indistinguishable from zero, it is deemed to be "statistically insignificant." In such a case, the event study's null hypothesis cannot be rejected.[57] That is, where the results of the event study are statistically insignificant, they cannot support the conclusion that the identified news caused any stock price reaction.[58]

46. In layman's terms, event study results that are not statistically significant cannot form the basis for a conclusion that the market for a security was efficient.[59]

47. To determine the statistical significance of an abnormal return, economists first calculate how large the abnormal return is relative to its historical fluctuation. This standardized measure of the abnormal return is referred to as a t-statistic.[60] To check if a stock price reacted significantly following an event, economists measure the stock's t-statistic following the event and calculate what is known as its *p*-value, which is the probability that the observed t-statistic (and the associated abnormal return) could be due to chance alone. Conventionally, if the *p*-value is less than 5% (the conventional significance level threshold), one can reject the

55 Tabak and Dunbar (2001), page 19-3; also see Bodie et al., page 354.

56 "In general, a test of significance aims to answer the question of whether an observed difference is real or simply occurred by chance." [Mitchell, Mark L. and Jeffry M. Netter (1994), "The Role of Financial Economics in Securities Fraud Cases: Applications at the Securities and Exchange Commission," The Business Lawyer, Vol. 49; henceforth "Mitchell and Netter (1994)," pg. 564.]

57 Kaye and Freedman, Judicial Reference Guide, pg. 251.

58 Tabak and Dunbar (2001), pg. 19-3.

59 Tabak and Dunbar (2001), pg. 19-3.

60 The t-statistic is defined as a ratio, which is calculated by dividing the abnormal return by its volatility ("standard error"). Consequently, if the abnormal return is zero, so is the t-statistic. [Mark P. Kritzman (1994), "What Practitioners Need to Know About Event Studies," Financial Analysts Journal, pp. 17 -20].

possibility that the abnormal return was due to chance alone and conclude that the abnormal return is statistically significant at a 95% confidence level.

48. As noted above, the null hypothesis in an event study is that there is no cause and effect relationship between material information and stock price reaction. In mathematical terms, this means that the null hypothesis is that the abnormal return following an event is zero.[61]

49. Dr. Feinstein's single-date event study does not prove market efficiency because: (1) no academic literature supports the view that market efficiency over an extended period of time may be established by testing only one date, and the existing literature supports the opposite conclusion; (2) Dr. Feinstein admitted at deposition that his study did not prove market efficiency for the duration of the Class Period; (3) Dr. Feinstein's selection of November 20, 2007 injects bias into his test, rendering the result scientifically unreliable; and (4) Dr. Feinstein's selection of a single date on the grounds that it is an "allegation-related" event is unsubstantiated.

a. No Academic Literature Supports the View that Market Efficiency Over an Extended Period of Time May Be Established by Testing Only One Date, and the Existing Literature Supports the Opposite Conclusion.

50. It is my understanding that, for a class to be certified, Plaintiff must prove by a preponderance of the evidence that Freddie Mac's stock traded in an efficient market *during the Class Period*, not merely on a single day (which also happens to be the last day of the Class Period). I am aware of no scientifically validated theory that would permit the conclusions from an analysis

[61] MacKinlay, A.C., 1997, "Event Studies in Economics and Finance," Journal of Economic Literature 35, pages 13-39 [henceforth "MacKinlay (1997)"].

of a single date on the last day of the Class Period to be imputed *to the entire Class Period*, especially one which spans more than one year. To the contrary, it is well accepted in the economic and finance literature that event study analyses designed to test market efficiency must analyze events *throughout* the class period, because the efficiency of a market for a given stock may change over time.[62]

51. The academic finance literature also makes clear that even the stock prices of well-capitalized companies' trading on major exchanges may not continue to be efficient over extended periods. Likewise, the market can become inefficient under certain market conditions, such as periods of limited liquidity.[63]

52. These principles are especially pertinent here given that Dr. Feinstein recognizes a "structural break" in Freddie Mac's stock return relationship with his market and industry-wide benchmark indices starting on August 9, 2007 which, according to Dr. Feinstein marked the start of a "systemic market liquidity crisis," "market dislocation," and heightened volatility tied to the credit crisis.[64] Because Dr. Feinstein admits that the market fundamentally changed with this structural break ushering in a systemic liquidity crisis, his attempt to impute his observations post-structural break apply equally to the market pre-structural break is thus pure conjecture.

53. In fact, the "FDT" Law Review paper that Dr. Feinstein bases his other empirical test of market efficiency on (i.e., his "z-test" which I discuss in detail later in this report), itself critiques such analysis based on one or few events, noting that "[m]erely demonstrating a single or small number of cases where there is an apparent cause and effect relationship is not enough, since this

62 Bajaj et al.(2014), pp. 190-192.
63 See Section IV.C below.
64 Feinstein Report, ¶126 and Dep., 243: 14-23. See also Dep., 244: 16 – 245: 2.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 27 of 370. PageID #: 18654

NAVIGANT

measures only one point in time during the class period, and only the stock's response to one or a handful of disclosures."[65]

54. In a 2016 working paper by one of the co-authors of FDT, Dr. Tabak[66] (a paper that Dr. Feinstein cited in his January 2017 Declaration), Dr. Tabak explains that a "Proof-by-example" approach analyzing only a "handful" of days is fatally flawed. The reason is because an event study with too few "events" is incapable of distinguishing whether stock price movements are the probable result of news events or simply random variation. As Dr. Tabak notes, "[a] serious, and in fact, fatal, problem with this approach is that one would expect to see such results if stock-price movements were completely random and had no average correlation with news events."[67]

b. Dr. Feinstein Admitted at Deposition That His Study Did Not Prove Market Efficiency for the Duration of the Class Period.

55. At deposition, Dr. Feinstein further clarified that his single-day event study analysis, according to him, only "proves" efficiency on November 20, 2007 and not throughout the Class Period as his report implies.[68] This admission contradicted his claim in his report that, based on this single-event event study, he could conclude that the market for Freddie Mac's stock was

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[65] Paul A. Ferrillo, Frederick C. Dunbar, and David Tabak, "The 'Less Than' Efficient Capital Markets Hypothesis: Requiring More Proof from Plaintiffs in Fraud-on-the-Market Cases," 78 St. John's Law Review 81, 119-22 (2004) ("FDT"), page 128.

[66] David Tabak, "What Should We Expect When Testing for Price Response to News in Securities Litigation?", NERA Economic Consulting, Working Paper, 2016 ("Tabak (2016)").

[67] Dr. Feinstein, in his deposition, attempted to rehabilitate this flaw in his analysis by claiming that Tabak (2016) was criticizing the practice of selecting statistically significant dates first and then looking for news to explain the price reaction [See Feinstein Dep. 368:20-371:2]. Feinstein's assertion is unsupported by Tabak (2016). But even if it were true, the assertion is irrelevant, as Tabak (2016)'s point holds true irrespective of the specific context he was criticizing.

[68] Feinstein Dep. 177:19-179:7 and Feinstein Report ¶137.

NAVIGANT

efficient. As he stated in his report:

> *The event study shows that for the allegation-related event, there was a strongly statistically significant price reaction to Company-specific news. This finding* ***proves*** *that Freddie Mac common stock reacted to new information and its market was efficient, and, in particular, efficient with respect to the information at issue in this case.*[69]

56. Further, at deposition, Dr. Feinstein admitted that it was possible for a company's stock price to respond appropriately to material news on one day, even if that company's stock generally traded in a market that was not efficient.[70]

57. I agree with Dr. Feinstein that a single-date event study for a single event cannot prove market efficiency for any extended period of time, let alone throughout a 330-day period.

c. Dr. Feinstein's Selection of November 20, 2007 Injects Bias into the Test, Rendering the Result Scientifically Unreliable.

58. Dr. Feinstein's analysis of November 20, 2007 is also invalid because the selection of the event was biased in several respects. Dr. Feinstein's test of November 20, 2007, injects a bias that is unique to securities class actions. It also injects a bias into his study that is unique to this case.

59. With respect to securities class actions, securities class action plaintiffs almost universally select the final day of the Class Period to coincide with a large stock price drop. An event study that only tests the date of the alleged "corrective disclosure" – the date the Plaintiffs allege the "truth was revealed" followed by a drop in stock price – effectively presents a foregone conclusion with respect to determining whether the stock price reacted to purported material news on this date.

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69 Feinstein Report, ¶137 (emphasis added).
70 Feinstein Deposition, pp. 96:11-97: 4; 183:1-6.

60. For largely this reason, and as FDT note in discussing their z-test, "the [z-test] examination would exclude those days in which a corrective disclosure was made because plaintiffs would normally choose a class period where corrective disclosures coincide with large negative price movements." This is essentially the notion of "peek-ahead" bias, which, as Dr. Feinstein testified in his deposition, is an improper way to conduct an event study.[71]

61. Tabak (2016), also cited by Dr. Feinstein, echoes this logic in discussing the FDT z-test, noting that including the disclosure day at the end of the class period as a "news event" in an event study would raise a "selection-bias issue" "as its inclusion would have been driven, at least in part, by the fact that there was an associated stock-price movement."[72] The same logic applies to Dr. Feinstein's single day "event study" test of market efficiency. Selecting a single day that was already selected by a securities plaintiff because on that day there was a disclosure and a large price drop, and then concluding that market efficiency is proven because the stock price's movement was statistically significant, is entirely circular and proves nothing.

62. Dr. Feinstein's selection also injects bias into his analysis given that he reviewed Dr. Hallman's Report dated August 16, 2012 and the 2012 Bajaj Report prior to selecting his single date for his event study.[73] Dr. Hallman had tested six earnings dates for Freddie Mac, and he concluded that two of those dates yielded statistically significant results. One of those two dates was November 20, 2007. In my report, I explained various econometric errors committed by Dr. Hallman. Had he run his tests properly, he would have concluded that only November 20, 2007 yielded statistically significant results. Thus, Dr. Feinstein knew that Dr. Hallman and I had both tested earnings dates, and one point on which we agreed was that the stock price of Freddie Mac's stock moved in a statistically significant way on November 20, 2007 (whether or not it was in response to a corrective disclosure or materialization of the allegedly concealed risks is another

71 Feinstein Dep., 220-221.
72 Tabak (2016), endnote 5.
73 Feinstein Dep. 91:19-92:17.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 30 of 370. PageID #: 18657

matter, as I discuss below).

63. I have reviewed Dr. Feinstein's reports on market efficiency in other securities cases, and I observed that, like Dr. Hallman, Dr. Feinstein typically tests earnings dates in his event studies. Indeed, I observed that he routinely tests multiple earnings dates, and I did not observe a single case in which he tested only a single date.[74] At deposition, Dr. Feinstein testified that he did not recall ever testing only a single date in any other case.[75] The fact that Dr. Feinstein chose only November 20, 2007 to test, rather than testing other dates, such as earnings dates that he typically tests, and the fact that he already knew the results of a test for November 20, 2007, injects into his analysis the sort of bias that scientists, including economists, attempt to avoid.[76]

d. Dr. Feinstein's Reason for Selecting November 20, 2007, As an "Allegation-Related" Event Is Unsubstantiated.

64. Finally, even if an event study that analyzes only the last day of the Class Period could be deemed reliable and probative for an analysis of market efficiency over a 330-day Class Period, Dr. Feinstein's underlying premise for his selection of November 20, 2007 as the sole event required to determine market efficiency, is unsubstantiated. In particular, Dr. Feinstein asserts that November 20, 2007 is an appropriate date to serve as his single event because it is an "allegation related event." That is, Dr. Feinstein asserts that "[b]y focusing an event study on the disclosure of information (or risk materialization date) related to the allegations in the Complaint, one is able to ascertain whether the market for Freddie Mac stock is efficient, not only generally, but also with respect to the particular information at issue in this case."[77]

65. However, while Dr. Feinstein discusses what was announced by Freddie Mac on November 20, 2007, he provides no analysis linking this information to the allegations in the

74 The reports I reviewed are listed in Appendix II.
75 Feinstein Dep. 75:19-25.
76 See also discussion of "data snooping" bias below at ¶118.
77 Feinstein Report, ¶112.

Case: 4:08-cv-00160-BYP Doc #: 385-2 Filed: 09/01/17 31 of 370. PageID #: 18658

Complaint. Rather, he appears to simply assume that the Company disclosed "allegation-related" information. I conducted an analysis of the economic evidence surrounding the November 20, 2007 disclosures, and see no evidence to suggest that the Company's disclosures were corrective disclosures linked to the alleged misrepresentations and omissions in the Complaint.

66. The TAC asserts that, in the November 20, 2007 press release, "[f]or the first time, **Defendants admitted that Freddie Mac's investments in subprime and nontraditional mortgage products had subjected Freddie Mac to significant risk and caused it to sustain substantial losses**."[78] Similarly, the TAC states that "**the November 20, 2007 press release admitted that the Company had been investing in subprime, Alt-A and nontraditional mortgage products**."[79] The TAC also states that "[t]he revelation of November 20, 2007, **was the first indication that the Defendants had misrepresented the truth with regard to Freddie Mac's capital.**"[80]

67. I searched in the press release for economic evidence to support these allegations, and I have found none. As Freddie Mac's November 20, 2007 press release and the transcript of the Company's investor conference call that day make plain, the Company made no corrections to its purported disclosure defects regarding its alleged "subprime" or operational risk exposure. The Company's press release that day, which the TAC cites as a purported curative disclosure, does not even mention the terms "subprime," "nontraditional loan products" or "fraud", which according to the Plaintiff are the central issues about which the Defendants had misled investors during the Proposed Class Period.

78 TAC, ¶190.
79 TAC, ¶227.
80 TAC, ¶212

68. Although Freddie Mac acknowledged that its Q3 2007 results had been "impacted by the deterioration" in 2007 in "**the country's housing and credit markets**"[81] [which was well-known publicly], such a statement does not constitute an admission for the "first time" that "Freddie Mac's investments in subprime and nontraditional mortgage products had subjected Freddie Mac to significant risk and caused it to sustain substantial losses" as the Plaintiff asserts.[82] To the contrary, in its conference call the same day, Freddie Mac discussed its previously disclosed subprime exposure in its Retained portfolio (as the TAC acknowledges[83]) and noted that:[84]

> *The credit profile of our [R]etained portfolio* ***remains of the highest credit quality*** *with 57% in agency mortgages and 33% in non-agency securities, of which 97% is triple A rated and does not include any CDOs. … Despite the continued deterioration of the housing market and increases in non-prime delinquencies, we remain comfortable with our risk position on these assets.* ***For the subprime securities****, while we have experienced some downgrades, …* ***no losses are projected on these securities****.*

69. Notably, on November 20, 2007, Freddie Mac <u>did not disclose</u> the specific amounts of loans that were scored as "C1," "C2" or "EA" by automated underwriting systems that were contained within its single-family guarantee portfolio: rather, it provided numerous other credit characteristics regarding the composition of its guarantee portfolio, including the percentages of loans in its guarantee portfolio with certain credit scores and LTV ratios, as it also had done throughout the Proposed Class Period. The public was not informed of the specific amounts of EA, C1 and C2 loans that were contained within its single-family guarantee portfolio until Freddie Mac entered into a Non-Prosecution Agreement with the SEC on December 16, 2011, more than four years after the Proposed Class Period had ended.

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81 Quoted in TAC at ¶190.
82 TAC, ¶190.
83 As the TAC mentions, at the end of 2006, "Freddie had … about $124 billion of AAA rated subprime exposure in [its] retained portfolio." [TAC, ¶84].
84 "FRE - Q3 2007 Freddie Mac Earnings Conference Call," Thomson StreetEvents, November 20, 2007, pg. 4.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 33 of 370. PageID #: 18660

NAVIGANT

70. Similarly, the earnings release on November 20, 2007, stated that Freddie Mac would undertake certain actions to manage its capital position "as a result of GAAP losses and in order to manage to the 30 percent mandatory target capital surplus and respond to regulatory concerns."

71. In other words, I did not find any "allegation-related" curative disclosure on November 20, 2007 and accordingly, the premise underlying Dr. Feinstein's selection of that date for his single-event event study, is unsupported.

2. Dr. Feinstein's "Z-test" Cannot and Does Not "Prove" Market Efficiency.

72. Based solely upon a single 2004 law review article (and not a peer-reviewed article in an economics journal) by Ferrillo, Dunbar, and Tabak ("FDT", as defined above),[85] Dr. Feinstein also analyzes the fifth *Cammer* factor, attempting to establish a cause-and-effect relationship between information and stock price, by using a "z-test" ("FDT z-test").

73. Dr. Feinstein's z-test is an application of what is known in statistics as a "z-test of two proportions."[86] Generally speaking, the z-test of two proportions is used when one wants to test whether two populations or groups differ significantly on some single, categorical characteristic. A simple example is a survey of 100 men and 100 women (randomly sampled), asking whether they like to watch baseball, and then comparing the proportions of men and women who said "yes," to test whether the proportions are different. The idea behind Dr. Feinstein's z-test in this case is to compare two categories of trading days during the Class Period, "news days" and "non-news days" (both as defined by Dr. Feinstein), ostensibly, to test whether Freddie Mac's stock price experienced a greater incidence of "abnormal returns" on the news days, which he contends would "prove" market efficiency.

85 Dr. Feinstein also cites *In re PolyMedica Corp. Sec. Litig.*, 453 F. Supp. 2d 266 (D. Mass. 2005) [Feinstein Report, ¶141].

86 See, for example, Bhattacharyya and Johnson, Statistical Concepts and Methods, Wiley, 1977 ("Bhattacharyya and Johnson"), pp. 308-312.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 34 of 370. PageID #: 18861

74. Dr. Feinstein's implementation of the FDT z-test proceeds in three stages. First, he had to, in effect, create the two populations being tested – i.e. "news days" versus "non-news days." Notably, unlike using a z-test to assess whether there is a statistically significant difference in the proportion of men compared to women who like to watch baseball games, which involves objectively observable criteria (male or female) – whether a particular trading day in the Class Period is a "news day" or a "non-news day" is subjective. In other words, in a typical application of the z-test to determine if two proportions are statistically significantly different, the proportions for each group are exogenously identified. They are objectively determined, not subjectively determined by the researcher performing the test. In contrast, in this case, before Dr. Feinstein could compare stock price fluctuations on news days to non-news days, he had to develop a definition for each category (using his own subjective assumptions) and then apply that definition to the trading days in the Class Period to, in effect, create the two population categories. That resulted in Dr. Feinstein selecting only nine "news days" out of the 330 trading days in the Class Period. The remaining 321 trading days of the Class Period were determined to be "non-news days."[87] I discuss below the method that Dr. Feinstein used to select "news" dates and the problems associated with that method.

75. Next, in the second stage of his z-test, Dr. Feinstein used a market model regression analysis to determine which of the 330 trading days had statistically significant abnormal returns. That is, Dr. Feinstein devised a way to determine what constitutes an "abnormal return" – a stock price fluctuation that exceeds what would be expected based solely on random day-to-day movements in the stock and the market, exclusive of company-specific news and to determine whether that abnormal return was unusually large (i.e., statistically significant). Unlike the example above of measuring women and men's affinity (or lack thereof) for watching baseball,

[87] Feinstein Report, ¶¶139-144. Notably, as I discuss below, Dr. Feinstein did not select the date that the articles were published on, but rather the date of the event identified in the article, even if the event occurred before the article's publication. Feinstein Report, Exhibit 5.

where readily observable and objective criteria would be determinative – i.e., a yes or no answer – Dr. Feinstein was required in this instance to use a model, again based on his assumptions, and subject to some statistical rate of error, to determine the number of statistically significant abnormal returns on news versus non-news days. In this second stage, Dr. Feinstein used his event study market model to find 23 days during the entire Class Period that had statistically significant, abnormal returns. Of his 9 "news days," Dr. Feinstein finds that 4 were statistically significant. Of the remaining 321 "non-news" days (as per Dr. Feinstein's classification), 19 exhibited statistically significant abnormal returns.

76. These findings then form the basis for implementation of Dr. Feinstein's z-test in the third stage of his analysis that compares the proportion (or frequency) of the statistically significant abnormal return days for the "news" and "non-news" groups of days. The matrix below presents his findings:

Dr. Feinstein's Z-test Significant Day Frequency Matrix				
	Significant	Not Significant	Total	Significant Frequency
News Days	4	5	9	4/9 = 44.44%
Non-News Days	19	302	321	19/321 = 5.92%
Totals	**23**	**307**	**330**	**6.97%**

77. After each of these analyses, Dr. Feinstein then performed his z-test (the third stage) by inputting the data resulting from his selection of news and non-news days and event studies to detect statistically significant abnormal returns into his z-test formula, which results in a "z-

NAVIGANT

statistic" – a number that purports to measure the extent to which the proportion of statistically significant abnormal returns on news days exceeded the proportion on non-news days using a statistical benchmark. Using the foregoing data resulting from the first and second stages, Dr. Feinstein then inputted the data into his interpretation of the FDT z-test formula,[88] calculating a "z-statistic" number of 4.48:

n_1= number of days in news sample = 9

n_2= number of days in no-news sample = 321

p_1= frequency of events in news sample = 4/9 = 0.4444

p_2= frequency of events in no-news sample = 19/321 = 0.05919

$\hat{p}$= pooled frequency of events = 23/330 = 0.069697

and calculates the value of his "z statistic" using the following formula:

$$Z = \frac{p_1 - p_2}{\sqrt{\hat{p}(1-\hat{p})(\frac{1}{n_1}+\frac{1}{n_2})}} = \frac{0.4444 - \ 0.05919}{\sqrt{0.069697(1-0.069697)(\frac{1}{9}+\frac{1}{321})}} = \ 4.48$$

78. The "z-statistic" is the ratio of (1) the difference between the two samples (or the sample "error"), and (2) the "standard error." In this case, Dr. Feinstein finds that the ratio is 4.48 (= 0.3853 / 0.08606) and hence that the difference between the two proportions (0.3853) is more than 4 times larger than the standard error.

79. He then compares his z-statistic of 4.48 to what is known in statistics as the "critical" z-statistic threshold to evaluate whether the "error," the difference in the proportion on news days

88 Dr. Feinstein deviates from the approach used by FDT in at least two important ways. I discuss these deviations in discussions of Flaws 4 and 5 below.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 37 of 370. PageID #: 18864

versus non-news days, is likely to have occurred by random chance alone. He chooses a critical z-statistic level of 1.65, which is the threshold for statistical significance at the 95% level in a "one tailed" z-test.[89]

80. Since Dr. Feinstein finds a z-statistic of 4.48, and this is greater than his chosen critical value of 1.65, he concludes that, with a greater than 95% level of confidence, the frequency of statistically significant movements in Freddie Mac's stock price on his selected "news days" was higher than the frequency of such movements on "non-news days."

81. As I explain below, a z-test conducted in this manner cannot prove market efficiency. In addition, even if Dr. Feinstein's z-test could in the abstract be considered sufficient to indicate, though not "prove," market efficiency, Dr. Feinstein's design and implementation of the test is unreliable, as it is premised on numerous statistical errors and unsupported assumptions that bias his test in favor of OPERS.

a. Dr. Feinstein's Z-Test Is Not a Method Accepted by Economists to Establish Market Efficiency.

82. Dr. Feinstein's z-test is not a method that is generally accepted among economists to establish market efficiency. To my knowledge, while the z-test has been used for decades as a test for whether two proportions are statistically different, Dr. Feinstein's z-test has never been used in the context of a test for market efficiency in a peer-reviewed Finance or Economics journal article.

83. Dr. Feinstein's sole support for the use of a z-test to establish market efficiency is the FDT article.[90] That article was published in the *St. John's Law Review* which is not a peer-reviewed

89 Feinstein Report, ¶143.

90 Dep., 41:9-42:4. Dr. Feinstein appears to be referring to the paper titled "The Curious Incident of the Dog That Didn't Bark and Establishing Cause-and-Effect in Class Action Securities Litigation," by Michael L. Hartzmark, and H. Nejat Seyhun published in the Virginia Law and Business Review (2011)]. I have reviewed that article, and it does not do so. The article does not even mention a z-test.

finance or economics journal. For this reason alone, Dr. Feinstein's z-test lacks a proper scientific basis.

84. Moreover, even looking past the lack of scientific basis, Dr. Feinstein ignores that the FDT authors themselves caution that the FDT z-test is insufficient to prove market efficiency. As they state, the test is a "threshold step, not a sufficient condition, to show that a stock traded in an efficient market."[91] This is in direct opposition to Dr. Feinstein's claim that his z-test alone can "prove" efficiency.[92]

85. Indeed, David Tabak (one of the authors of the FDT article) in a 2010 working paper noted that "the FDT methodology may not be able to fully distinguish an efficient market from an inefficient one."[93] Tabak's 2010 paper notes that there are "conceptual questions relating to whether a market can exhibit some form of inefficiency but still pass the FDT test" and that there are "clear examples" where that could be the case. One example is where the market is inefficient because only some, but not all, material news is absorbed into the price of a security.[94]

86. Dr. Feinstein does not address these limitations. More fundamentally, he concludes that his FDT z-test alone "proves" market efficiency, despite the fact that the FDT authors specifically acknowledge that it is not sufficient to do so.

87. The mere fact that a z-test has been in existence for years does not render it an appropriate or reliable method for assessing market efficiency. It is not.

b. By Its Very Design, the Z-Test Does Not Assess Market Efficiency.

88. It should come as no surprise that the z-test has not obtained acceptance among economists as a test to establish market efficiency, because that is not what it does. Dr. Feinstein's

91 FDT (2004), p. 122.
92 Feinstein Report (2017), ¶143
93 Tabak (2010), p. 7.
94 Tabak (2010), p. 7.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 39 of 370. PageID #: 18866

NAVIGANT

z-test is incapable of demonstrating a "cause and effect" relationship between new, material news and Freddie Mac's stock price, and thus cannot "prove" market efficiency.

89. *Basic v. Levinson*, echoing Fama's notion of informational market efficiency discussed above, states that in an efficient market "*the market price of shares traded on well developed markets reflects all publicly available information.*"[95] As noted above, markets that are informationally efficient have security prices that promptly reflect *all* material news. In such informationally efficient markets, "prices reflect information to the point where the marginal benefits of acting on information (the profits to be made) do not exceed the marginal costs"[96] of gathering such information.

90. Thus, both the definition of informationally efficient markets, and the logic of *Basic*, require that price impact be observed in response to all material news, all the time.

91. Dr. Feinstein agrees, as he must, that an efficient market must consistently incorporate available information. As Dr. Feinstein testified during his deposition, "[f]or a market to be efficient, it needs to be incorporating available information, all the time."[97] Further, he conceded that a market that only "sometimes," or "rarely," incorporates available information is not efficient.[98]

92. In contrast, however, Dr. Feinstein's z-test simply tests whether, proportionally, more

[95] *485 U.S. 224, BASIC INC. ET AL. v. LEVINSON ET AL. (1988) , No. 86-279. Supreme Court of United States, 246 (emphasis added). Elsewhere, the Court notes in Basic that "[a]n investor who buys or sells stock at the price set by the market does so in reliance on the integrity of that price. Because most publicly available information is reflected in market price, an investor's reliance on any public material misrepresentations, therefore, may be presumed for purposes of a Rule 10b-5 action. 485 U.S. 224, BASIC INC. ET AL. v. LEVINSON ET AL. (1988) , No. 86-279. Supreme Court of United States,, 247 (1988) (emphasis added).*

[96] Fama, Eugene F., "Efficient Capital Markets: II," The Journal of Finance 46, no. 5 (Dec. 1991), ("Fama (1991)"), pg. 1575.

[97] Feinstein Dep., 97:5-8.

[98] Feinstein Dep., 96:11-97:4, 183:2-11.

abnormal returns are observed on news days than non-news days. As Dr. Feinstein explains, he uses his z-test to assess "whether the stock has a greater frequency of statistically significant price movements on days with greater information flow ('news dates') than on more typical days ('non-news dates')."[99] In other words, Dr. Feinstein's z-test is merely concerned with whether proportionally, there are more abnormal return days on the 9 "news days" he selected than on the 321 non-news days, not whether news is consistently absorbed into to the stock price. Accordingly, Dr. Feinstein's z-test could yield what he deems to be "proof" of market efficiency even if it only showed abnormal returns *some of the time*.

93. It is well established that when price reactions occur only "some of the time" when there is material news, that is not enough to establish market efficiency. Indeed, as Judge Cedarbaum concluded in the Freddie Mac Series Z Preferred Stock class action litigation, a "plaintiff's burden is higher" than simply proving that "news probably had some effect on price" but rather "[a] plaintiff must show that the market price responds to most new, material news."[100] Citing to my testimony in the matter, Judge Cedarbaum noted that "an economist may conclude that a market is efficient if it reacts to news 80 to 90% of the time, depending on the number of news dates at issue."[101,102]

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99 Feinstein Report, ¶139.

100 Opinion, In re Federal Home Loan Mortgage Corp. (Freddie Mac) Securities Litigation, United States District Court, Southern District of New York, No. 09 Civ. 832 (MGC), March 27, 2012, pg. 18.

101 Opinion, In re Federal Home Loan Mortgage Corp. (Freddie Mac) Securities Litigation, United States District Court, Southern District of New York, No. 09 Civ. 832 (MGC), March 27, 2012, pg. 19.

102 Judge Cedarbaum's holding was consistent with my testimony that, given the number of relevant news dates at issue, if the market reacted to such news 80% to 90% of the time, it would not be distinguishable statistically from the market reacting every time to material news. In other words, if one is testing whether a stock price reacts to all material news, statistically, the price reaction should be observed at least 80% to 90% of the time, depending on the sample size. For example, if we assume 9 news dates, and we observe a statistically significant reaction on 8 of them (or about 89%), the result would be statistically indistinguishable from observing a significant price reaction on 99% of the days [Note: This is based on a

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 41 of 370. PageID #: 18868

NAVIGANT

94. Similarly, in *Polymedica*, the District Court agreed with the Defendants' expert, Dr. Dunbar, and cited to his testimony criticizing the plaintiffs' expert for showing cause and effect on only a handful of days. The District Court agreed that the plaintiffs' expert should have "picked news days as a sample, all news days, not just the ones [he] self-selected," and not "the few news days that would prove [his] point."[103] Notably, Dr. Dunbar was one of the authors of the 2004 FDT paper, where he clearly states (with his coauthors) that "plaintiffs ought to be required to make some detailed showing that the stock in question traded in an efficient manner. Merely demonstrating a single or small number of cases where there is an apparent cause and effect relationship is not enough, since this measures only one point in time during the class period."[104]

95. In contrast to such a comprehensive study of the class period, Dr. Feinstein's z-test in this case is designed to ask only whether, on a handful of his purported news dates, the stock price moved by a statistically significant amount more than 5% of the time. In other words, depending on sample size, Dr. Feinstein's z-test implies that if the stock price were to move statistically significantly on less than 6% of news dates compared to moving significantly on only 5% of non-news dates, the market still should be deemed to be informationally efficient.[105]

96. To illustrate this fatal flaw in the z-test approach, suppose there were 100 material news

binomial test – see discussion in Footnote 178 below for a description of this test. This test is also used by Dr. Feinstein as one of his "robustness" checks on his z-test (Feinstein Report, ¶). I use 99% and 95% in this example as a binomial test cannot be computed using 100%]. Observing 7 statistically significant news dates out of 9 (or about 78%) would be statistically indistinguishable from observing a significant price reaction on 95% of the days. As the sample size increases, a higher percentage of days are required to be statistically indistinguishable from the 99% or 95% level. For example, if the sample size were 20 news days, then 19 days, or 95% would be required to be significant in order to be statistically indistinguishable from 99%. Similarly, if the sample size were 20 news days, then 17 days, or 85% would be required to be significant in order to be statistically indistinguishable from 95%.

103 Evid. Hr'g, Cited in Opinion, *In Re Polymedica Corporation Securities Litigation*, 453 F. Supp. 2d 260 (D. Mass. 2006).

104 FDT (2004), pg. 128.

105 For example, if we assume 1,000 news dates and 1,000 non-news dates and that 5%, or 50, of the non-news dates are statistically significant, then Dr. Feinstein's z-test would only require 68 of the news days (or 6.8% to be statistically significant, to conclude there was a statistically significant difference and therefore that the market was efficient according to his z-test.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 42 of 370. PageID #: 18869

days for publicly traded "Company A," over a class period of 500 days in total. The logic of *Basic* and informationally efficient markets dictates that all or most of those 100 news days should result in statistically significant abnormal returns for Company A. That is not what Dr. Feinstein's z-test measures. Rather, Dr. Feinstein's z-test measures whether the proportion of statistically significant abnormal returns on those 100 news days was larger (to a degree of statistical significance, i.e. beyond the statistical threshold of random chance according to the his z-test of the two proportions) than the proportion of statistically significant abnormal returns on the 400 non-news days. The z-test is thus incapable of demonstrating that Freddie Mac's stock price reacted to most material news, as the test simply does not test that hypothesis.

97. To further illustrate the inapplicability of Dr. Feinstein's z-test to proving market efficiency, it is helpful to understand how the test plays out in light of the assumptions he uses regarding the extent to which random chance, as opposed to material news, will cause Freddie Mac's stock price to experience abnormal returns. Dr. Feinstein's z-test assumes that abnormal returns resulting from random chance (as opposed to material news) will occur about 5% of the time.[106] Continuing the illustration with the example of Company A discussed above, assume that on non-news days, abnormal returns were observed (as a matter of random chance) on only 5% of the non-news days, or 20 out of the 400 non-news days in the example above. Dr. Feinstein's z-test simply asks whether the proportion of abnormal returns among the 100 news days was significantly larger than 5% such that, when compared to the 5% random chance of non-news days,

[106] Due to "noise," i.e., random changes, in stock prices, when applying an event study market model some proportion of dates will show a statistically significant abnormal price movement in the absence of material news. In other words, if the test of statistical significance is calibrated at a 5% level of significance, it would mean that about 5% of days would be expected to show a statistically significant price change due to no specific cause, just random noise. Dr. Feinstein's z-test comparison group, i.e., the proportion of statistically significant non-news days, is 19/321 or about 6%.

Case: 4:08-cv-00160-BYP Doc #: 385-2 Filed: 09/01/17 43 of 370. PageID #: 18670

NAVIGANT

the difference is statistically significant. Applying Dr. Feinstein's z-test in these circumstances would require only 10 of the 100 news days, or 10%, to have statistically significant abnormal returns to conclude that this proportion was significantly greater than 5%, and hence conclude that the market was efficient. However one may define an informationally efficient market, when a stock price reacts to new, material information only 10% of the time, it can hardly be considered to be trading in an efficient market as *Basic* contemplates.[107]

98. Unsurprisingly, the FDT law review article itself notes that, "[m]erely demonstrating a single or small number of cases where there is an apparent cause and effect relationship is not enough, since this measures only one point in time during the class period, and only the stock's response to one or a handful of disclosures."[108] That is, the authors of the sole article that Dr. Feinstein cites as authority for application of the z-test to market efficiency analyses expressly cautioned that small sample sizes impair whatever conclusions one might draw from the test.

99. The z-test standard proposed by Dr. Feinstein is not capable of "proving" market efficiency as he claims. I am not aware of any economic authority that supports Dr. Feinstein's z-test approach in this case, and, as noted, even FDT explicitly disavow such a use of the z-test.[109]

c. Dr. Feinstein's Z-Test Ignores the Directionality of Price Reactions, in Contradiction of the Economic Definition of Informational Efficiency and His Own Past Testimony

100. There is yet another fundamental reason that Dr. Feinstein's z-test, as a general proposition and as applied by Dr. Feinstein, is insufficient to demonstrate market efficiency. Dr.

[107] Dr. Feinstein's z-test results are demonstrative. After excluding the alleged corrective disclosure date of November 20, 2007, Dr. Feinstein's z-test results in 8 news dates. To generate a "z-statistic" of greater than 1.65 and conclude the market was efficient, he would only need 2 statistically significant days, or 25%. A conclusion that a stock reacts to news only 25% of the time is a far cry from concluding that the stock "always" reacts to value-relevant information, or even that it does so most of the time. It is therefore insufficient to conclude market efficiency. Appendix IV illustrates the number of exact statistically significant news days needed to get a z-statistic of 1.65 per Dr. Feinstein's z-test by varying news days.

[108] FDT (2004), pg. 128

[109] FDT (2004), pg. 122

Case: 4:08-cv-00160-BYP Doc #: 385-2 Filed: 09/01/17 44 of 370. PageID #: 18871

NAVIGANT

Feinstein's z-test does not account for the directionality of the price reaction, another key requisite of the cause and effect demonstration that is the *sine qua non* of market efficiency. If a market is to be considered efficient in incorporating new value relevant information, price reactions must correspond to the direction indicated by the nature of the information – i.e., all other things being equal, negative news would result in a lower stock price, while positive news would result in a higher stock price. As Dr. Feinstein testified in his AIG deposition:

> *Well, it depends on your purpose for the event study. I'm not done. If it's to determine market efficiency, what you're looking for from the event study is, did the security move in the direction that is appropriate given the nature of the information. So you need to be able to form a hypothesis about what the appropriate direction is. Otherwise you wouldn't know from the result whether the stock price moved appropriately or not.* [110]

101. Indeed, in his report in this matter, Dr. Feinstein quotes Professor Eugene Fama, noting that, in an efficient market where prices respond to new material information "prices generally move reasonably promptly *in the predicted direction* in response to unexpected material public information (favorable or unfavorable)."[111]

102. Using Dr. Feinstein's z-test, which does not account for directionality, to "prove" market efficiency is like testing a medicine for efficacy and ignoring whether it kills people or cures them, and testing instead only whether the proportion of total extreme outcomes in the group taking the medicine is different than the proportion of extreme outcomes in the placebo group, even if the difference was being driven by every patient dying, rather than recovering. In Dr. Feinstein's analysis in this case (as opposed to AIG, quoted above), every single statistically significant news day could exhibit an abnormal return in the opposite direction indicated by the

[110] VIDEOTAPED DEPOSITION OF STEVEN P. FEINSTEIN, PH.D., CFA, IN RE AMERICAN INTERNATIONAL GROUP, INC. 2008 SECURITIES LITIGATION, MASTER FILE NO. 08-CV-4772-LTS, UNITED STATES DISTRICT COURT SOUTHERN DISTRICT OF NEW YORK, ("AIG deposition"), 135.

[111] Feinstein Report, ¶36 (emphasis added).

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 45 of 370. PageID #: 18872

NAVIGANT

news, and he would still conclude that the test "proved" market efficiency. This is not a matter of the z-test not being a perfect test of market efficiency – it does not test market efficiency at all.

d. Dr. Feinstein's Z-Test Is Unreliable Because It Is Premised on a Series of Statistical Errors and Assumptions That Bias His Results In Favor of a Finding of Market Efficiency.

103. Putting aside the fact that a z-test like Dr. Feinstein's cannot establish market efficiency, Dr. Feinstein's z-test is scientifically unreliable due to a host of fundamental design problems and a series of assumptions (many of which are inconsistent with his methods in prior reports and the FDT article he cites as support for his test) that bias his conclusions in favor of a finding of market efficiency, rendering his test scientifically unreliable.

Flaw 1: Dr. Feinstein's sample size is too small to conduct his z-test reliably according to economic authorities.

104. The first fundamental problem with Dr. Feinstein's z-test design is that, per the rules of statistics, the two proportion z-test formula provides a reliable measure of significance only when both populations being compared – here, news days and non-news days – are calculated from relatively large samples.[112] In order to use the two proportion z-test and draw a valid statistical inference, one basic underlying assumption is that, given a sufficient number of observations, the

[112] Bhattacharyya and Johnson (1977), pp. 308-312. ("Bhattacharyya and Johnson (1977)")

discrete binomial distribution[113] tends to converge to a continuous "normal" distribution.[114] That is, populations being compared must be large, by statistical standards. Dr. Feinstein's z-tests fail to meet these standards.

105. To begin, according to textbooks on which Dr. Feinstein relies, standard procedure calls for at least 30 observations in each of the samples being compared.[115] Dr. Feinstein's analysis, however, relied on a sample of only 9 "news days," far below the 30 observations threshold. For this reason alone, Dr. Feinstein's z-test is unreliable.

106. Further, at his deposition, Dr. Feinstein was asked about a textbook he cited that presented three criteria regarding sample size, each of which must be satisfied for the z-test to be viewed as reliable. Dr. Feinstein admitted that his z-test failed the first two criteria.[116] While he conceded that he did not understand the third criteria, his z-test sample size failed the third criteria as well.

107. In particular, Macfie and Nufrio (2006), a textbook cited by Dr. Feinstein as general

[113] In statistics, a binomial distribution with parameters n and p, is the discrete probability distribution of the number of successes in a sequence of n independent experiments, each asking a yes/no question, where p is the probability of success or "yes."

[114] A binomial distribution can only use discrete values that are whole numbers or ratios of whole numbers. For example, in this case the proportion of significant news days can take the values 0, 1/9, 2/9, 3/9 … to 1. However, by definition a standard normalNormal distribution (think of the common "bell curve") is continuous and can take all possible values (, i.e. [give example]). 0.11111, 0.11112, 0.11113, etc. Hence, in order to approximately match a binomial distribution to a standard normal distribution, and hence to draw a valid statistical inference from the two proportions z-test, a large number of observations (usually greater than or equal to 30) is required. As the number of observations increases, the more a Binomial Distribution resembles a Normal one. See, for example, Bhattacharyya and Johnson (1977), pp. 262-263.

[115] Macfie, Brian P. and Philip M. Nufrio (2006) Applied Statistics for Public Policy, M.E. Sharpe, pg. 323. ("Macfie and Nufrio (2006)") See Also: Penn State Eberly College of Science (2017), Hypothesis Testing, The Pennsylvania State University, Web, https://onlinecourses.science.psu.edu/stat200/node/61.

[116] Feinstein Dep. 206:5-14; 216: 3-6.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 47 of 370. PageID #: 18674

support for a z-test of two proportions, proscribes the following:[117]

> *The following three conditions must be met for both samples to assure the sample sizes are sufficiently large enough to conduct a hypothesis test for differences:*
>
> *1. Both samples need to have at least 30 observations each.*
>
> *2. Both (n1 * p1) and (n2 * p2) have to be greater than five.*
>
> *3. Both [n1 * (1 - p1)] and [n2 * (1 - p2)] have to be greater than five.*

Dr. Feinstein's z-test violates all three of these conditions, as shown in the table below.[118] First, his samples do not both have more than 30 observations. His first sample of "news" days (n1 in the table below) only has 9 observations. Second, both (n1 * p1) and (n2 * p2) are not greater than 5. As shown in the table below, n1 (the total number of news days, 9) * p1 (the proportion of significant news days, 4 / 9) = 4. Finally, his test also violates the third condition, as both [n 1 * (1 - p1)] and [n2 * (1 - p2)] are not greater than 5. [n1 * (1 - p1)] = 5 * 5/9 = 5, as shown in the table below. Note also that Dr. Feinstein's second z-test that excludes November 20, 2007, also violates all three conditions. In this test he only has 3 sample observations of significant news days where n1 = 8 and n1*p1 = 3.

Dr. Feinstein z-test Sample Size Summary (including November 20, 2007)

	Significant	Not Significant	*Totals*	*Proportions*
News Days	n1 * p1 = 4	n1 * (1-p1) = 5	*n1 = 9*	*p1 = 4 / 9*
Non-News Days	n2 * p2 = 19	n2 * (1-p2) = 302	*n2 = 321*	*p2 = 19 / 321*
Totals	*23*	*307*	*330*	

117 Macfie and Nufrio (2006), pg. 323. The relevant excerpt from Macfie and Nufrio (2006) are attached here as Exhibit III.

118 Feinstein Dep., 205:18 – 206:14, 215:8 – 218:8.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 48 of 370. PageID #: 18675

108. Dr. Feinstein also cited another textbook that discusses z-tests of two proportions, Devore (2016), which makes a similar admonition, but requires increased sample size limits. Devore (2016) states that "the test can safely be used as long as [(n1 * p1), (n2 * p2), n1 * (1 - p1), and n2 * (1 - p2)] are all at least 10."[119] As shown in the table above, however, Dr. Feinstein's z-test produces only 4 observations of significant news days, and 5 observations of not significant news days,[120] each far below the required 10 observations.

109. In sum, Dr. Feinstein's news sample observations are too small for this procedure to be statistically valid. This error alone renders Dr. Feinstein's z-test unreliable.

Flaw 2: Dr. Feinstein's z-test identified a total of only four dates with statistically significant abnormal returns.

110. Dr. Feinstein's z-test is also insufficient to establish market efficiency for yet another related reason. Among economists, any test that assesses a relatively small number of dates is not an accepted method of assessing market efficiency (regardless of the sample size requirements that apply to z-tests in particular). As I discussed above, the selection of a single date is not sufficient to establish market efficiency. Likewise, the selection of only a handful of dates is similarly not sufficient to establish market efficiency by using a z-test as Dr. Feinstein does. Here, through his z-test, Dr. Feinstein has, at best, identified only <u>four</u> dates with statistically significant results across the entire 330-day Class Period, not one of which occurred during the first five months of the Class Period. On the basis of these four dates and his comparison of those dates to so-called non-news dates, Dr. Feinstein concludes that the market for Freddie Mac's common stock was

119 Devore (2016), pg. 393. Note that Devore (2016) uses different mathematical notation than Macfie and Nufrio (2006). The Macfie and Nufrio (2006) notation was substituted here for ease of exposition. The relevant exceprt from Devore (2016) are attached here as Exhibit IV.

120 Moreover, as noted below, one of the "news" days (November 20, 2007) should not have been included in the analysis and hence the sample should have included only 3 news days of statistical significance, and 8 observations total.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 49 of 370. PageID #: 18676

NAVIGANT

efficient during an entire 330-day period. Such a conclusion is at odds with well-established economic principles relating to market efficiency (see also discussion above at IV.B.2.b). Moreover, as noted above, in *Polymedica*, the District Court agreed with the Defendants' expert, Dr. Dunbar, and cited to his testimony criticizing the plaintiffs' expert for analyzing cause and effect on only a handful of days.[121] As Dr. Dunbar and his colleagues explained in the FDT Article: "Merely demonstrating a single or small number of cases where there is an apparent cause and effect relationship is not enough, since this measures only one point in time during the class period."[122]

111. The z-test standard proposed by Dr. Feinstein is not capable of "proving" market efficiency as he claims. I am not aware of any economic authority that supports Dr. Feinstein's z-test approach in this case, and, as noted above, even the authors of the law review article that he claims is the basis of this methodology explicitly state that such a conclusion cannot be drawn from the z-test.[123]

Flaw 3: Dr. Feinstein devised an unprecedented and speculative "news day" selection process that has no reasonable probability of discerning which days in the Class Period were in fact "news days" and introduces selection bias.

112. Dr. Feinstein's method of selecting "news days" raises several serious concerns. As discussed above, in stage one of his z-test, it was Dr. Feinstein's intent to identify "news" days

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121 MEMORANDUM AND ORDER IN RE POLYMEDICA CORPORATION SECURITIES LITIGATION, CIVIL ACTION NO. 00-12426-WGY, September 28, 2006 ("Polymedica 2006"), pg. 14.
122 FDT (2004), pg. 128.
123 FDT (2004), pg. 122.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 50 of 370. PageID #: 18677

that had "greater information flow" so that he could compare stock price reactions on those days to "non-news" days. Dr. Feinstein's test for classifying a trading day in the class period as a news day simply asks whether an event relating to Freddie Mac appeared in both *The Wall Street Journal* ("WSJ") and *The New York Times* ("NYT") newspaper articles.[124] In selecting "news days" based upon the appearance of an article in both *The Wall Street Journal* and *The New York Times,* Dr. Feinstein did **not** select the date(s) that the articles were actually published (i.e. the date of the "news"). Rather, Dr. Feinstein selected the date that the event identified in the articles occurred, even if the event occurred before the articles' publication – which occurred on 7 out of the 9 "news days" selected.[125] Dr. Feinstein cites no scientific authority to support the notion that his selection criteria is reliable and I am aware of none. In addition, given that the very point of his test, according to him, is to compare dates with "greater information flow" to other dates without it, his selection approach renders his test results unreliable for this reason alone.

113. Notably, Dr. Feinstein has never used this method for selecting dates previously.[126] In other cases in which he conducted a z-test, he had used earnings dates and/or the dates that a corporation filed a report with the SEC regarding a material event.[127] As he testified, he devised this approach for the first time for this case.[128]

114. Feinstein's method of selecting "news days" renders his test highly speculative. Indeed, Dr. Feinstein's "news" day selection criteria demonstrably fails to select dates with "news" that plainly fits Dr. Feinstein's own definition of a news date (a day with "greater information

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124 Feinstein Report, Exhibit 5.
125 Feinstein Report, Exhibit-5; Feinstein Dep., 232:3-8. Note also, for 2 out of the 9 selected "news days," the *The Wall Street Journal* and *The New York Times* articles ran on different dates entirely.
126 Feinstein Dep. 157:21-23.
127 Feinstein Dep. 157:24-158:5.
128 Feinstein Dep. 157:18-20.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 51 of 370. PageID #: 18678

flow" about the company in question).[129] Two clear examples are Freddie Mac's October 3, 2006 and January 5, 2007 earnings updates when Freddie Mac announced expected profits/losses for Q2 and Q3, 2006 respectively, neither of which were deemed "news days" by Dr. Feinstein's methodology.[130] Freddie Mac also held conference calls on both of these days where executives made extensive presentations and analysts asked detailed questions.[131]

115. There can be little question that these two dates would qualify as trading days with "greater information flow," Dr. Feinstein's definition of a "news day." Indeed, Dr. Feinstein himself recently opined in another matter that "[a] company's financial results and forecasts are among the most important considerations to investors assessing the value of its stock" and that "[c]onsequently, such announcements typically contain material information that could cause the stock price to change."[132] Dr. Feinstein also opined that there are "[n]umerous well-known and highly-regarded academic studies" that "have specifically examined stock price movements caused by earnings announcements, and concurs that earnings announcements are generally

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129 Feinstein Report, ¶139.

130 On Tuesday October 3, 2006 (before market open) Freddie Mac provided an update to investors and announced estimated earnings for the first half of 2006 at $2.7 billion and estimated Q2 2006 earnings of $1.2 billion. ["Freddie Mac says H2 net income $2.7 billion," *Reuters News*, October 3, 2006.] On January 5, 2007 (before market open) Freddie Mac released its financial results for the Q3 2006. Freddie Mac estimated that it sustained a net loss of $550 million in the third quarter compared to a profit of $880 million in the third quarter of 2005. Freddie Mac also stated it expected losses in the fourth quarter. ["Today's Agenda / A look ahead at newsworthy events," *The Wall Street Journal*, January 5, 2007; Marcy Gordon, "Freddie Mac Predicts 3Q, 4Q Losses," *Associated Press*, January 5, 2007.]

131 "FRE - Freddie Mac Market Update," *Thomson Street Events*, October 3, 2006 8:30 AM EST; "FRE – Freddie Mac Market Update," *Thomson Street Events*, January 5, 2006 8:30AM.

132 Declaration and Report of Professor Steven P. Feinstein In Re: CITY OF STERLING HEIGHTS GENERAL EMPLOYEES' RETIREMENT SYSTEM, Individually and on Behalf of All Others Similarly Situated, Plaintiffs vs. Prudential Financial, Inc., et al., Defendants, No. 2:12-cv-05275-SDW-MCA, UNITED STATES DISTRICT COURT DISTRICT OF NEW JERSEY, December 8, 2014, ¶96.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 52 of 370. PageID #: 18679

important information events.[133] Yet, Dr. Feinstein's "news" selection method fails to capture these earnings (and other) dates when new information potentially relevant to Freddie Mac's stock value was released to the market.[134]

116. Notably, according to Dr. Feinstein's market model, Freddie Mac's abnormal return was statistically *insignificant* on both of these omitted days,[135] and including these days as news days with insignificant returns in Dr. Feinstein's z-test would, all else equal, lower his z-statistic.

117. Further, as he admitted, the reason why he devised his WSJ/NYT method of selecting news days was to avoid using the earnings dates that Dr. Hallman had previously tested, which for the most part yielded statistically insignificant results. Dr. Feinstein devised this new approach well aware that, if he conducted testing in the manner that he had in the past (as he asked permission for the Court to do),[136] four or five of the six earnings dates would have yielded statistically insignificant results. I understand from his deposition testimony that Dr. Feinstein knew before he devised his new method for selecting dates that Dr. Hallman opined that only two of six earnings dates yielded statistically significant results.[137] He also had reviewed my report, in

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133 Declaration and Report of Professor Steven P. Feinstein In Re: CITY OF STERLING HEIGHTS GENERAL EMPLOYEES' RETIREMENT SYSTEM, Individually and on Behalf of All Others Similarly Situated, Plaintiffs vs. Prudential Financial, Inc., et al., Defendants, No. 2:12-cv-05275-SDW-MCA, UNITED STATES DISTRICT COURT DISTRICT OF NEW JERSEY, December 8, 2014, ¶97.

134 Further evidence of Dr. Feinstein's arbitrary news day selection criteria and the unreliable basis for his FDT z-test is his failure to analyze the 23 misrepresentation/omission dates mentioned in the TAC. [I discuss these dates in more detail in Appendix V]. If he had performed his FDT z-test using these 23 dates as his "news" dates, he would have found no statistically significant difference in the "incidence rate" of "news" and "non-news" days, suggesting that the market was not efficient per his own methodology. [Appendix VI provides the details of this Feinstein FDT z-test analysis using the 23 TAC "effective" dates.] To perform this analysis, I calculated abnormal returns and statistical significance using Dr. Feinstein's regression market model and used (as he did for the 9 Feinstein "news" dates) indicator variables on the 23 TAC dates. I also considered an alternative version of the analysis without the indicator variables and arrived at similar results.

135 Feinstein Report, Exhibits 4, 6 and 7.

136 Feinstein Declaration, ¶28.

137 Feinstein Dep. 90:13-18.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 53 of 370. PageID #: 18680

NAVIGANT

which I had opined, after correcting Dr. Hallman's errors, that only one of those six dates – November 20, 2007 – yielded a statistically significant result.[138]

118. When asked why he did not use earnings dates, as he had in his previous engagements as an expert, he answered: "Because it was already established that at least four of the six earnings announcements dates were such that they would not be statistically significant. They already established that. So a different rule would provide new information."[139] This is the classic definition of "data snooping" that violates the "axioms of classical statistical analysis" that "future research" should not be "motivated by the successes and failures of past investigations."[140] Thus, Dr. Feinstein's unusual approach to selecting news dates suggests that he engaged in a form of "selection bias," which economists avoid lest their results be deemed unreliable.[141]

119. Putting aside the issue of selection bias, Dr. Feinstein's selection criteria for news versus non-news days bears no connection to the definition of market efficiency. Again, market efficiency is when a stock price promptly absorbs new, material information. That an event was covered in articles in both the *Wall Street Journal* and the *New York Times* says nothing per se

[138] Feinstein Dep. 91:11-18.

[139] Feinstein Dep. 119: 9-121:9.

[140] See, for example, Andrew Lo and A Craig MacKinlay, "Data-Snooping Biases in Tests of Financial Asset Pricing Models," Review of Financial Studies, 1990, vol. 3, issue 3, pp. 431-67. They also note that "when scientific discovery is statistical in nature we must weigh the significance of newly discovered relations in view of past inferences. This is recognized implicitly in many formal statistical circumstances, as in the theory of sequential hypothesis testing." When scientists perform multiple experiments, it is recommended by statisticians that the statistical significance thresholds be adjusted upwards (making it harder to conclude significance), because as the number of experiments increases, the probability of finding a statistically significant result by chance alone increases. As one statistician explains:
The more statistical tests we perform, the more likely we are to reject the null hypothesis when it is true (i.e., a "false alarm," also called a "Type 1" error). This is a consequence of the logic of hypothesis testing: We reject the null hypothesis for rare events, and the larger the number of tests, the easier it is to find rare events which are false alarms. [Herve Abdi, "Holm's Sequential Bonferroni Procedure," in Neil Salkind (Ed.), Encyclopedia of Research Design. Thousand Oaks, CA: Sage. 2010]

[141] This apparent selection bias is also at odds with the Chartered Financial Analyst ("CFA") manual that Dr. Feinstein said he followed in his news selection approach. [See Feinstein Deposition, Day 1, 223:10 – 21.] The CFA Level I curriculum states at section 5.2 that researches should avoid "sample selection bias." The manual notes that "The only advice we can offer at this point is to be aware of any biases potentially inherent in a sample. Clearly, sample selection biases can cloud the results of any study."

about whether the article concerned information that was either new, or material. In other words, Dr. Feinstein's nine purported "news" days are not chosen on the basis that they reflect the disclosure of new, material information. Nor is there anything in the academic literature, or logic, suggesting that an absence of an article running in both publications somehow means that there was *not* new, material information disclosed on that date – i.e., Dr. Feinstein's 321 "non-news" days very well may be days when new, material information was released to the market. Thus Dr. Feinstein's fundamental premise – that his selected "news" days are those with "greater information flow" – is pure speculation.

120. Notably, Dr. Feinstein's WSJ/NYT selection criteria for determining news versus non-news days also directly conflicts with his testimony regarding his purported reason for selecting the last day of the Class Period for his single-date event study. Dr. Feinstein testified that his detailed examination of the flow of information during the entire Class Period resulted in the identification of only a single date on which new, material news was released, November 20, 2007.[142] That is flatly inconsistent with his assertion that his selected "news days" for his z-test are those days with "greater information flow." In any case, putting aside this self-contradiction, the fact remains that Dr. Feinstein's sample selection is not even intended to reflect those days when new, material news was disclosed to the market, and for this reason alone, the test is unreliable for the purpose of proving market efficiency.[143]

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142 Feinstein Dep. 89:2-16.

143 Almost as notable is the reason Dr. Feinstein gave for utilizing his WSJ/NYT rule for separating news versus non-news days. Dr. Feinstein testified that there were over 2,900 articles focused on Freddie Mac during the Class Period, which represented "quite extensive news coverage," and for that reason he chose WSJ/NYT days as those reflecting "extraordinary" information flow. (Feinstein Dep. 156:13-157:17.) In effect, Dr. Feinstein testified that virtually every day in the Class Period is a day with "news" of some sort, and he has simply narrowed the sample to those with "extraordinary" information flow. Market efficiency is not concerned with extraordinary versus normal information flow – it is concerned with the prompt absorption

NAVIGANT

Flaw 4: Dr. Feinstein improperly included a "corrective disclosure day" as a news day.

121. Yet another flaw in Dr. Feinstein's z-test is that he includes within his classification of "news days" the "corrective disclosure" date of November 20, 2007. The FDT article itself dictates that November 20, 2007, should have been excluded from Dr. Feinstein's "news" days to avoid biasing the results.[144] Dr. Feinstein's inclusion of this date, disregarding the instructions of the authors of the FDT Article, biases his results strongly in favor of a finding of statistical significance.

122. The FDT paper that serves as the sole authority for Dr. Feinstein's implementation of the test explicitly states that the FDT z-test analysis must "exclude those days in which a corrective disclosure was made because plaintiffs would normally choose a class period where corrective disclosures coincide with large negative price movements; ***including those days in the analysis would bias the results.***"[145]

123. One of the authors of the FDT article, David Tabak, repeated a similar admonition in a 2016 working paper (also cited by Dr. Feinstein in his Declaration), noting that including the disclosure day at the end of the class period as a "news event" in the FDT test would raise a "selection-bias issue" "as its inclusion [in the class period by the plaintiff] would have been driven,

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of new, material information in a company's stock price. In any case, given Dr. Feinstein's testimony about the "extraordinary" nature of his selected "news" days, his test results showing statistical significance on only 4 out of 9 news days is hardly convincing that the market was efficient. According to the logic of *Polymedica* [which Dr. Feinstein cites as support for his z-test], an objective design of the z-test would include all possible "news" dates. Such a test would be infeasible here because there would be no "non-news" dates according to Dr. Feinstein's Factiva news search.

144 FDT (2004), Footnote 155, pg. 120.

145 FDT (2004), Footnote 155, pg. 120. Emphasis added.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 56 of 370. PageID #: 18683

at least in part, by the fact that there was an associated stock-price movement."[146]

124. Dr. Feinstein cites no authority that allows him to disregard the FDT article's instruction, as there is none. When asked about this departure from the FDT methodology in his deposition, Dr. Feinstein simply stated that this was an area where he and the FDT authors "disagree."[147] Given that FDT appears to be the only authority for even using a z-test to examine market efficiency (subject to FDT's disclaimer that the test is insufficient to prove market efficiency), this means that Dr. Feinstein relies on his opinion alone about the validity of including the corrective disclosure date in his z-test. Obviously, there is no general acceptance in the scientific community of Dr. Feinstein's unique approach when he is alone among economists in adhering to it.

125. Dr. Feinstein's inclusion of November 20, 2007 as a "news day" renders his test biased. Accordingly, the appropriate news day statistical significance "incidence rate" is not 4 days of 9 days, but 3 days out of 8 days.[148] Making this correction yields a z-statistic of 3.53, rather than 4.48 as he claimed.

Flaw 5: Dr. Feinstein failed to use an unpooled approach to calculate standard error estimates.

126. With respect to the two samples being compared in Dr. Feinstein's z- test – news versus non-news days – each sample has a standard error estimate that measures the variability in the sample. Standard error estimates can vary considerably based upon the sample size, among

146 Tabak (2016), fn. 5.
147 Feinstein Dep., 199:15-200:12.
148 Dr. Feinstein appears to acknowledge this flaw in his analysis as he also performs an alternative version of his FDT z-test excluding November 20, 2007 from the analysis. Feinstein Report, ¶144.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 57 of 370. PageID #: 18884

other factors. When sample sizes are similar or are assumed to be similar, statisticians can employ a "pooled" estimate of the standard deviation of the test statistic (or "standard error") based on both populations.[149] This "pooled" estimate combines, or "pools," the standard errors of the two samples and effectively assumes that both sets of data have common variability.

127. However, when dealing with small and unbalanced samples such as those in Dr. Feinstein's z-test, where there are very few "news days" (*i.e.*, 9) and a large number of "non-news days," (*i.e.*, 321), a more conservative approach would be to use separate standard deviation estimates for each population. This is commonly called the "unpooled" approach. In the unpooled approach, there is no assumption that two populations, here the underlying "news days" and "non-news days," have common variability. Despite the considerable imbalance in the number of news days (9) versus non-news days (321), Dr. Feinstein pooled his standard deviation estimates, effectively assuming that both populations have common variability. In his report, Dr. Feinstein does not discuss this assumption or his justification for making it. At deposition, Dr. Feinstein was unable to explain the difference between the pooled approach and the unpooled approach.[150]

128. Notably, the *unpooled* approach is the approach used by FDT in the FDT z-test, not the pooled approach used by Dr. Feinstein in his z-test.[151]

129. Dr. Feinstein's decision to use the pooled approach is not merely an academic issue.

[149] Anderson, David R. et. al. (1984), Statistics for Business and Economics, 2nd edition, West Publishing, pg. 309.

[150] Feinstein Dep., 267:6-268:5

[151] In the FDT article, the authors do not report the formula used for the computation of the z-statistic. However, my replication of the results reported in the paper in the table on pg. 121 matches the reported results when using the unpooled standard error method. Moreover, the authors indicate that they used the unpooled approach, stating that "The test examines whether the means of two samples with potentially different variances are the same and considers both the 0.5% difference in the two figures and the variances (or spread) in the daily returns within each of the two samples." FDT article, n. 158 (emphasis added).

Indeed, the sample standard error for the 9 news days is more than 12 times higher than for the 321 non-news dates.[152] And critically, Dr. Feinstein's deviation from the FDT z-test approach by using a pooled standard error has a strong effect on his calculations, biasing the results in favor of his conclusion of market efficiency. The unpooled standard error is 0.1661, almost double the pooled standard error, of 0.08606. Using the unpooled measure reduces Dr. Feinstein's z-statistic by half, from 4.48 to 2.32 (= 0.3853 / 0.08606 compared to = 0.3853 / 0.1661).

Flaw 6: Dr. Feinstein failed to implement a continuity correction.

130. Another problem with Dr. Feinstein's z-test is that, because the number of observations for the "news-days" and statistically significant "news-days" are relatively small, the z-statistic calculation should have employed a "continuity correction." In statistics, a continuity correction accounts for the difference in the binomial and the normal distributions when the sample size is small by reducing the difference between the sample proportions.[153] When a sample size is statistically large, one can assume that the binomial distribution approximates a normal distribution, which is necessary to draw a reliable inference from the z-test. Not so with a small sample size. When the sample size is small, the statistical model should account for the difference between the binomial distribution (which is discrete) and the normal distribution (which is continuous).[154] In statistical terms, Dr. Feinstein, should have included a "continuity correction"

152 News day SE = 0.1656 (= SQRT (p1(1-p1)/n1)); non-news day SE = 0.0131 (= SQRT (p2(1-p2)/n2). See SAS Institute, SAS STAT manual, version 9.2 at pg. 1755 and Bhattacharyya and Johnson at pp. 309-310. The formula for the test statistic used by FDT in the FDT z-test with an unpooled measure of the standard deviations is given below.

$$Z = \frac{p_1 - p_2}{\sqrt{p_1(1-p_1)(\frac{1}{n_1}) + p_2(1-p_2)(\frac{1}{n_2})}} = \frac{p_1 - p_2}{SE\ (unpooled)}$$

153 Fleiss, Joseph; Levin, Bruce and Cho Paik, Myunghee (2003), "Statistical Methods for Rates and Proportions," Wiley-Interscience, at Section 3.1.

154 See SAS STAT manual at pg. 1754.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 59 of 370. PageID #: 18888

in estimating his FDT Z-statistic to account for the difference between the discrete binomial distribution and the continuous normal distribution.

131. Dr. Feinstein's failure to utilize a continuity correction here has a significant impact upon his z-statistic calculation in favor of a finding of market efficiency. The formula for Dr. Feinstein's z-test, but using an unpooled standard error estimate <u>and</u> continuity correction ("CC") is as follows:

$$Z = \frac{p_1 - p_2 - \left(\frac{1}{n_1} + \frac{1}{n_2}\right)/2}{\sqrt{p_1(1-p_1)\left(\frac{1}{n_1}\right) + p_2(1-p_2)\left(\frac{1}{n_2}\right)}} = \frac{p_1 - p_2 - CC}{SE\ (unpooled)} = \frac{0.444 - 0.0591 - 0.0571}{0.1661} = 1.97$$

132. In other words, using the continuity correction along with the unpooled standard error, reduces Dr. Feinstein's z-statistic to 1.97.

Flaw 7: Dr. Feinstein chooses to "dummy" out variables, unlike his previous implementation of the z-test, a choice that is outcome determinative.

133. Dr. Feinstein's market model "dummies" out, or simply removes, his 9 "news" dates from his regression model estimation periods. This approach, while not necessarily invalid in the abstract, is inconsistent with the model he used in the *Petrobras* case where he also performed the z-test. In that case, when applying his z-test, he did not "dummy" out his selected news dates.[155]

134. In fact, if he had followed his *Petrobras* methodology, and had left all other aspects

[155] *In Re: Petrobras Securities Litigation, Report On Market Efficiency Professor Steven P. Feinstein, October 15, 2015, Exhibit 7a-c, showing the results for his analyses of common ADR, preferred ADRs, and bonds. This report is attached here as Exhibit 1.*

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 60 of 370. PageID #: 18687

of his analysis identical but had not "dummied" out the 9 news dates from his estimation periods, his z-test results for his second z-test (excluding November 20, 2007) would not have been significant, indicating the market was not efficient according to his own flawed logic.[156] This inconsistency indicates that his z-test analysis is not robust to changes in the underlying market model assumptions, indicating the test is unreliable. Regardless of Dr. Feinstein's reasons for his inconsistent approach to dummying out news days, the fact that changing this single aspect of his model determines the outcome of his z-test, demonstrates the unreliability of the test itself as applied here.

Flaw 8: Dr. Feinstein chose to combine estimation periods resulting from a "structural break" in the market, unlike his previous implementation of the z-test.

135. A central assumption in using regression models to predict returns is that the volatility of the stock market – the fluctuation in prices relative to historical averages – remains unchanged over the relevant period. When the time period being analyzed demonstrates a significant change in volatility – i.e., there is a "structural break" – properly constructed event studies (such as the second stage of Dr. Feinstein's z-test) control for changes in in market volatility.[157] Accordingly, economists often use models that control for changing volatility of the stock's returns in particular periods.[158]

156 See Appendix VII for details.
157 Kennedy, Peter, *A Guide to Econometrics*, 5th edition. MIT Press, pp. 47-48.
158 Greene, William H. (2011), Econometric Analysis, 7th edition, NJ: Prentice Hall, Chapter 9, pp. 22, and 279. Also see: [1] Below, Scott D. and Keith H. Johnson (1996), "An Analysis Of Shareholder Reaction To Dividend Announcements In Bull And Bear Markets," Journal of Financial And Strategic Decisions, vol. 9(3), pp. 15 -26; [2] Bommel, J. van, and T. Vermaelen (2003), "Post-IPO capital expenditures and market

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 61 of 370. PageID #: 18688

NAVIGANT

136. Volatility is important when measuring abnormal returns because the statistical significance of the abnormal return is determined using a yardstick called "standard error."[159] In selecting that yardstick, one must account for general market volatility. Under normal circumstances, when the market volatility remains more or less constant, it is appropriate to use the standard regression model with no adjustment for changing volatility. But a sudden change in volatility can affect the standard error in a meaningful way.

137. A simple example involves the measurement of waves. If most of the waves in a placid lake are less than one foot in height, then a two-foot wave may be considered huge, relative to the one-foot waves. However, in a stormy ocean where most waves are 4 feet or higher, the use of the same yardstick of one-foot will lead to the mistaken conclusion that almost all waves are abnormally large, when they are not. One needs to adopt a different yardstick in such a case.[160]

138. Similarly, to isolate properly the impact of unexpected corporate news on a stock price when the volatility in the market has suddenly increased, one must adjust the yardstick – replace the placid-lake yardstick with the stormy-ocean yardstick – to account for that heightened market-wide volatility.

139. Dr. Feinstein agrees. His model for determining abnormal returns accounts for

feedback," Journal of Banking & Finance, vol. 27, pp. 275–305; [3] Henry, Peter Blair (2002), "Is Disinflation Good for the Stock Market?," The Journal of Finance, vol. 57(4), pp. 1617-19648; [4] Arslanalp, Serkan and Peter Blair Henry (2005), "Is Debt Relief Efficient?," The Journal of Finance, vol. 60(2), pp. 1017-1051.

159 Mitchell and Netter (1994), pg. 569.

160 In the Freddie Mac Series Z Preferred Stock class action litigation, Judge Cedarbaum, found it a "convincing critique" of the Plaintiff's expert McCann, that Dr. McCann used a "control period [that] encompassed a time of relative placidity in the relationship between Series Z and the [market index]," and then "calculate[d] his abnormal returns in the 'stormy ocean' of the last third of the class period – a time in which the relationship … was much more volatile," and cited to my conclusion that Dr. McCann "found an excessive number of abnormal returns." [In re Federal Home Loan Mortgage Corp., No. 09 Civ. 832 (MGC), March 27, 2012, at 15].

changing volatility[161] by adjusting for a "structural break" on August 9, 2007, a date he notes (citing to the previous Bajaj 2012 Report in this matter) marked a period of "heightened market-wide volatility."[162] He divides his regression estimation period (the Purported Class Period) into two sub-periods (August 1, 2006 to August 8, 2007, the ["Feinstein Estimation Period 1"] and August 9, 2007 to November 20, 2007, the ["Feinstein Estimation Period 2"]) and tests for abnormal returns independently in each period. He concludes that two out of five news days in the first period and two out of four news days in the second period showed abnormal returns. Critically, he then combines the results to conclude that four out of nine news days showed abnormal returns, which is statistically significant in comparison to the number of non-news days showing abnormal returns.

140. That is not how he performed his z-test in another recent report.[163] There, he presented results from his two estimation periods separately in two z-tests. If he had followed the *Eletrobras* approach in this matter, and properly excluded November 20, 2007 as the FDT z-test requires, he would have found a z-statistic for Estimation Period 2 (August 9, 2007 to November) not to be statistically significant, leaving all other aspects of his analysis unchanged. See Table 1 below. Again, the fact that changes in Dr. Feinstein's subjective application of the z-test prove to be repeatedly outcome determinative demonstrates the unreliability of the test itself.

Flaw 9: Dr. Feinstein failed to correct for a volatility change in February 2007.

141. As noted, Dr. Feinstein agrees that an adjustment must be made for changes in volatility, as his recognition of the structural break on August 9, 2007 demonstrates. Dr. Feinstein, however, fails to recognize that the same logic which compelled him to recognize the structural

[161] Feinstein Report, ¶¶126-127.
[162] Feinstein Report, ¶126.
[163] *In Re: Eletrobras Securities Litigation*, Report on Market Efficiency of Professor Steven P. Feinstein, June 30, 2017, ¶174-180. This report is also attached here as Exhibit II.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 63 of 370. PageID #: 18690

break also requires that he further subdivide the Class Period in light of a second, earlier structural break on February 27, 2007. As explained below, that second structural break requires that he subdivide Feinstein Estimation Period 1 into two sub-periods, from August 1, 2006 to February 26, 2007 ["Feinstein Estimation Period 1a"] and February 27, 2007 to August 8, 2007 ["Feinstein Estimation Period 1b"]. In his deposition, Dr. Feinstein testified that he did not perform any other independent testing to identify structural breaks other than August 9, 2007.[164]

142. There are at least three primary pieces of economic evidence establishing a structural break on February 27, 2007. First, the VIX index[165] and implied volatility[166] for Freddie Mac's stock, starting on February 27, 2007 through August 8, 2007, nearly doubled from their prior levels over the previous year.[167] Second, there were a series of market-wide events that appeared to roil financial markets resulting in a sharp market-wide stock market decline, the largest since 2001, amidst subprime mortgage problems and fears of recession.[168] Third, the use of Dr. Feinstein's Chow test, when applied to Dr. Feinstein's market model to analyze the periods from August 1, 2006 to February 26, 2007 and February 27, 2007 to August 8, 2009, yields a result "statistically significant at the 95% confidence level," demonstrating that "there was a structural change in the

164 Feinstein Dep. 241:23–242:18.

165 VIX is the ticker symbol for the Chicago Board Options Exchange Volatility Index. The index is a "key measure of market expectations of near-term volatility conveyed by S&P 500 stock index option prices."[http://www.cboe.com/products/vix-index-volatility/vix-options-and-futures/vix-index, accessed 8/30/2017.]

166 Implied volatility is the future volatility estimate for Freddie Mac stock. The implied volatility that I report for Freddie Mac stock is computed by OptionMetrics, a well-known provider of such data, through prices of 30-day call options traded on the Company's stock.

167 See Appendix VIII.

168 On February 26, 2007 former Fed Chairman Alan Greenspan warned of the possibility of a recession in 2007. [Burton Frierson, "TREASURIES-Bonds gain on subprime troubles and Iran worry," *Reuters,* February 26, 2007, 09:29.] The next day the Dow Jones industrial Average fell 3.3.% its largest drop since 2001, following a similar sharp decline in China's Shanghai Composite Index. [Tim Annett, "The Evening Wrap: From Bad to Worse," *The Wall Street Journal*, February 27, 2007]

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 64 of 370. PageID #: 18891

regression relationship."[169]

143. If Dr. Feinstein had accounted for this second "structural break" on February 27, 2007 (by subdividing the estimation period as he did for the structural break he identified on August 9, 2007[170]), leaving all other parameters of his model the same (except his inclusion of November 20, 2007) then his FDT z-test result fails to demonstrate statistical significance in any of the periods, even combining the results from each period (unlike his approach in *Eletrobras*).[171] Specifically, if he had applied this volatility-corrected market model, he would have found only **one** of his eight "news" days to be statistically significant (August 30, 2007), which would not support a conclusion of market efficiency.[172]

Flaw 10: Dr. Feinstein does not account for the statistical error inherent in his market model when calculating his z-statistic.

144. Dr. Feinstein's FDT z-test treats his classification of days as having significant abnormal stock price movements as an irrebuttable, exogenously observable variable (like the proportion of "yes" answers for men and women in the baseball survey example above) when in fact, the classification of significant abnormal return days are determined by using the regressions and statistical tests. That is, the "data" that he uses in the z-test are values that are *estimated* using a statistical model, and hence subject to random variation and statistical error.

145. But Dr. Feinstein entirely ignores the statistical error present in his determination of abnormal returns. By ignoring this additional source of statistical error from using estimated

[169] Feinstein Report, ¶126.
[170] See Appendix IX for details. For convenience, I refer to this version of Dr. Feinstein's market model with 3 estimation periods as the "3 Period Feinstein Market Model."
[171] I do not consider his inclusion of November 20, 2007 as one of the "news" dates in his FDT z-test analysis, because as noted above, the FDT paper itself states that alleged disclosure dates should not be included in such a test because it biases the results.
[172] Tellingly, Dr. Feinstein's "diagnostic" tests that he failed to disclose in his report would also fail to find significance even if November 20, 2007 is included in the analysis. See Section IV.B.2.f below.

variables, Dr. Feinstein is most likely inflating the precision of his estimates in his z-test analysis. Especially given the lack of robustness of Dr. Feinstein's z-test that I demonstrate, this additional source of statistical error renders his results unreliable. There is no scientific way that I am aware of to correct for such errors.

e. Correcting Only Three of Dr. Feinstein's Statistical Design Errors Yields a Z-Statistic That Is Statistically Insignificant, Which Cannot Support a Finding of Market Efficiency.

146. In sum, even if a z-test like Dr. Feinstein's could demonstrate market efficiency (and it cannot), Dr. Feinstein's z-test here suffers from numerous design flaws. As noted above, some of these design flaws render the entire z-test unreliable. (See discussions above of sample size and limited events at the subsections discussing Flaws 1 and 2). Others are outcome determinative. (See above discussion of dummying dates and combining results from separate estimation periods at the subsections discussing Flaws 7 and 8). Several other design flaws, when corrected together, are also outcome determinative – the corrected z-test yields statistical insignificance. Notably, all of these flaws benefit Dr. Feinstein's client, OPERS.

147. Several of the more glaring flaws, when taken together, have a combined effect on Dr. Feinstein's z-test analysis that is also outcome determinative. In particular, when I exclude the corrective disclosure date from the classification of news days (Flaw 3), use an unpooled standard error (Flaw 4) and employ a continuity correction (Flaw 5), the z-test yields a **statistically insignificant result, reducing the z-statistic from 4.48 to 1.47, which is below the critical**

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 66 of 370. PageID #: 18693

NAVIGANT

threshold of 1.65[173] that Dr. Feinstein claims is necessary to "prove" market efficiency.[174]

f. Dr. Feinstein's Robustness Checks Do Not Cure the Numerous Flaws In His Report.

148. Following the filing of his report and after the first day of his deposition, Dr. Feinstein produced a series of "robustness" checks on his z-test that were not discussed in his report. The supposed purpose of these "diagnostic" tests is to excuse Dr. Feinstein's failure to comply with the sample size requirements, set forth in the very textbooks that he cites in his Declaration. *See* above, sections []. According to Dr. Feinstein, he was not aware of the results of these tests until after his report, when he was told about them the day before his deposition.[175] While he did not see the results of these reports before his deposition, he was told by one of his assistants that they "indicated there was no problem."[176] His assistants did not maintain the results of those tests, so they were "run … again" following his deposition.[177] These checks included a Fisher's Exact

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173 Dr. Feinstein chose to perform a "one-tailed" test requiring a 1.65 critical value for statistical significance at the 95% level. A more conservative approach would have involved performing a two-tailed test which would have required a higher critical value of 1.96 for statistical significance at the 95% level.

174 This calculation is as follows, where p_1 and p_2 are corrected to be 3/8 and 19/321:

$$Z = \frac{p_1 - p_2 - \left(\frac{1}{n_1} + \frac{1}{n_2}\right)/2}{\sqrt{p_1(1-p_1)\left(\frac{1}{n_1}\right) + p_2(1-p_2)\left(\frac{1}{n_2}\right)}} = \frac{p_1 - p_2 - CC}{SE\ (unpooled)} = \frac{0.3750 - 0.0591 - 0.0641}{0.1717} = 1.47$$

The effect of these corrections on his z-statistic are summarized in the following table:

	pooled (Feinstein)	unpooled	unpooled, continuity corrected	unpooled, continuity corrected, p = 3/8
z statistics	4.48	2.32	1.97	1.47

175 Feinstein Dep. 212:3-21.

176 Feinstein Dep. 40:19-41:2.

177 Feinstein Dep. 408:3-409:18.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 67 of 370. PageID #: 18894

NAVIGANT

test,[178] a "bootstrap" test,[179] and a "binomial" test[180],[181] In his deposition, in response to questions about the small sample issue from which his z-test analysis suffers, Dr. Feinstein claimed that these "diagnostics" tests demonstrated that "the [z-]test is legitimate and the results are reliable," despite his small sample size issues.[182]

149. Although these tests are typically prescribed to address the reliability of statistical tests using small samples, these tests do not demonstrate the reliability of Dr. Feinstein's conclusions in light of the other numerous critiques I present of his z-test, discussed above. Moreover, as I show below, if I correct some of Dr. Feinstein's flawed assumptions, I show that his z-test result is not "robust" even according to his own "diagnostic" tests.

150. **Presenting Results Separately For Regression Estimation Periods, as Dr. Feinstein Did in the Eletrobras Case:** Recall that Dr. Feinstein found a "structural break" on

178 The Fisher's Exact test is a method for testing the statistical dependence between two binary variables. It is a widely used test for relationships that can be expressed as 2x2 contingency tables similar to "Dr. Feinstein's Z-test Significant Day Frequency 2x2 Matrix" above. The test as used by Dr. Feinstein, estimates the 'exact' probability of generating the table above, i.e., 4 significant news days in a random sample of 9 news days taken from a given population of 330 days containing 23 statistically significant days. See for example, Fleiss et al., at Section 3.1.

179 The Bootstrapping test simulates 100,000 random samples of size 9, with the independent probability of 23/330 of choosing a significant date and (1-23/330) probability of a statistically insignificant day. In other words, Dr. Feinstein draws a sequence of length 330 with the 23 news days positioned at random places within the sequence. He independently draws another sequence of length 330, this time with the 9 significant return days randomly positioned within the sequence. Then he counts instances where there are news days and significant returns at the same position. From the 100,000 replications in his simulation, he constructs a distribution of these counts and, ultimately, the percentage of replications for which there were at least 4 instances of a significant returns "news day." [Feinstein Report, Supporting Files, Indep2SmplBoots_FMCC.M.]

180 Dr. Feinstein's "Binomial test" also calculates the probability of generating 4 significant days out of a sample of 9, but under the assumption that the probability of a significant day was 5%. See for example, Devore (2016). It is not clear why Dr. Feinstein assumes a 5% probability of generating a significant day, when the probability over the class period according to his analysis was ~about 7% (=23/330) and about 6% on his "non-news" days (=19/321).

181 See file "2017-08-11 Crowninshield_FreddieMac_CollectiveTest_RobustnessChecks.pdf".

182 Feinstein Dep. 205:18-25.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 68 of 370. PageID #: 18895

August 9, 2007 and estimated his market model regression (Stage 2 of his FDT z-test) over two separate sub-periods spanning the Class Period: Estimation Period 1 (August 1, 2006 to August 8, 2007) and Estimation Period 2 (August 9, 2007 to November 20, 2007), the latter of which Dr. Feinstein referred to as the "systemic liquidity crisis" period.

151. As noted above, while Dr. Feinstein, in at least one recent market efficiency report, presented his FDT z-test results for each estimation sub-period on either side of a "structural break" separately,[183] he did not do so in this matter. (See the discussion of "Flaw 8" above.) Applying the approach Dr. Feinstein used previously to his z-test in this matter, Dr. Feinstein's "robustness" checks would not support a conclusion of a statistically significant difference between the news and non-news "incidence" rates for Estimation Period 2 (i.e., the "systemic liquidity crisis period"). Table 1 below summarizes these results.

[183] REPORT ON MARKET EFFICIENCY BY PROFESSOR STEVEN P. FEINSTEIN, PH.D., CFA. IN RE: ELETROBRAS SECURITIES LITIGATION ¶136-137, 177-180, and Exhibit 11a. No. 15-cv-5754-JGK, UNITED STATES DISTRICT COURT SOUTHERN DISTRICT OF NEW YORK, June 30 2017.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 69 of 370. PageID #: 18696

Table 1: Dr. Feinstein's Z-test and "Robustness" Check Presenting His Results Collectively and Separately for Regression Estimation Periods, as He Did in the Eletrobras Case

Freddie Mac Collective Test Robustness Checks

	WSJ/NYT News Event Days			WSJ/NYT News Event Days Excluding Alleged Corrective Disclosure		
	Dr. Feinstein Estimation Period 1 and 2	*Dr. Feinstein Estimation Period 1*	*Dr. Feinstein Estimation Period 2*	*Dr. Feinstein Estimation Period 1 and 2*	*Dr. Feinstein Estimation Period 1*	*Dr. Feinstein Estimation Period 2*
Feinstein FDT Z-Test Results						
p-value	0.0004%*	0.05%*	0.51%*	0.02%*	*	7.93%
Fisher's Exact Test Results						
p-value	0.18%*	2.88%*	5.69%	1.17%*	*	26.76%
Bootstrap Test Results						
p-value	0.17%*	2.94%*	5.67%	1.15%*	*	26.70%
Binomial Test Results						
p-value	0.06%*	2.99%*	5.75%	0.54%*	*	26.14%

Notes:
"*" Indicates p-values that are statistically significant at the 95% confidence level.
Grey highlighted cells indicate p-values that are not statistically significant at the 95% level according to Dr. Feinstein's one-tail test.
The binomial test results compare the "news" day proportion to the proportion of significant days on "non-news" days.

152. As Table 1 shows, if one properly excludes November 20, 2007, the results are statistically insignificant for all four tests for Estimation Period 2. If one includes November 20, 2007 (which Dr. Feinstein should not have included per FDT), the table shows statistically insignificant results for Estimation Period 2 for all three of his "robustness" checks (the Fisher Exact Test, the Bootstrap test, and the Binomial Test).

153. **Correcting for Dr. Feinstein's Incorrect Regression Model:** The above analysis assumed (counterfactually) that both Dr. Feinstein's market model regression and his news day selection criteria were valid. If I repeat the analysis above, but correct only the regression model (i.e., use the 3 Period Model described above resulting from structural breaks on February 27 and August 8, 2007), then the analysis shows that while Dr. Feinstein's z-test would be statistically significant (over the Class Period), all three of his robustness test would fail to show statistical significance. See Table 2 below. This is significant because Dr. Feinstein testified that if all three

Case: 4:08-cv-00160-BYP Doc #: 385-2 Filed: 09/01/17 70 of 370. PageID #: 18897

NAVIGANT

of his "robustness" checks, which are designed to analyze "small samples," yielded statistically insignificant results, he would conclude that "that the z-test did not indicate that, reliably that the news events were different from the non-news events in terms of their dynamics."[184] The analysis also shows that for Estimation Period 1 (which is the combination of the results for Estimation Period 1a and 1b in the corrected Feinstein 3 Period Model), none of Dr. Feinstein's four tests yield statistically significant results. Note that the results for Estimation Period 2 are unchanged from the prior table as the corrected 3 Period Model did not change Dr. Feinstein's Estimation Period 2 model. Again, these results are summarized in Table 2 below.

184 Feinstein Dep. 458:13-21

NAVIGANT

Table 2: Dr. Feinstein's Z-test and "Robustness" Check Presenting His Results Collectively and Separately for Regression Estimation Periods, as He Did in the Eletrobras Case, But Correcting His Event Study Regression Model

Freddie Mac Collective Test Robustness Checks

	WSJ/NYT News Event Days			WSJ/NYT News Event Days Excluding Alleged Corrective Disclosure		
	Dr. Feinstein Estimation Period 1a, 1b, and 2	*Dr. Feinstein Estimation Period 1a and 1b*	*Dr. Feinstein Estimation Period 2*	Dr. Feinstein Estimation Period 1a, 1b, and 2	*Dr. Feinstein Estimation Period 1a and 1b*	*Dr. Feinstein Estimation Period 2*
Feinstein FDT Z-Test Results						
p-value	2.89%*	70.61%	0.51%*	23.68%	*	7.93%
Fisher's Exact Test Results						
p-value	11.49%	100.00%	5.69%	41.35%	*	26.76%
Bootstrap Test Results						
p-value	11.38%	100.00%	5.71%	41.34%	*	26.45%
Binomial Test Results						
p-value	8.40%	73.39%	5.75%	31.13%	*	26.14%

Notes:
"*" Indicates p-values that are statistically significant at the 95% confidence level.
Grey highlighted cells indicate p-values that are not statistically significant at the 95% level according to Dr. Feinstein's one-tail test.
The binomial test results compare the "news" day proportion to the proportion of significant days on "non-news" days.

154. Dr. Feinstein testified in his deposition that "insignificant results" for his z-test would "erode, to some extent, [his] confidence" in his conclusion of market efficiency.[185] As noted above, he also testified that if all three of his "robustness" checks, which are designed to analyze "small samples," yielded statistically insignificant results, this would lead him to conclude that "that the z-test did not indicate that, reliably that the news events were different from the non-news events in terms of their dynamics."[186] He further testified that if two out of the three "robustness" tests yielded statistically insignificant results, he would have "less confidence" in the z-test results and would put more weight on the "bootstrap" test.[187] Hence, Dr. Feinstein's own analysis, both when

185 Feinstein Dep. 386:13-387:5
186 Feinstein Dep. 458:13-21
187 Feinstein Dep., 458:13-460:4. He further noted that "That would indicate that it's a problem with the z-test and its result was not reliably indicating anything, one way or the other. It's not proof of inefficiency, but I don't think it could be used to support a finding of efficiency."

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 72 of 370. PageID #: 18699

NAVIGANT

presented in the manner he presented it in another recent matter [*Eletrobras* report filed June 2017], and when corrected for an invalid regression model and news day selection criteria, does not demonstrate efficiency over the "severe liquidity crisis period" or the entire Class Period by his own analytical standards.

C. Dr. Feinstein's Failure to Adequately Establish Market Efficiency for Freddie Mac Stock Is Consistent With His Testimony That the Period Experienced a "Systemic Liquidity Crisis" and Market "Dislocation".

155. In sum, Dr. Feinstein fails to prove "cause and effect" (the key fifth *Cammer* factor) through his various empirical tests (event study of a single day, FDT z-test, and "robustness" checks) and thus fails to demonstrate that the market for Freddie Mac stock was efficient over the Class Period. Dr. Feinstein's failure to demonstrate a "cause-and-effect" relationship between Freddie Mac's stock price and new, material information during the Class Period is consistent with his testimony that the historical period he analyzed was "unique."

156. According to Dr. Feinstein, the Class Period after August 8, 2007 experienced a "systemic liquidity crisis" and market "dislocation."[188] The "uniqueness" of the timeframe within the Class Period, marked by a systemic liquidity crisis, further suggests that assuming market efficiency, solely from the standard "structural" factors (as Dr. Feinstein now seems to claim is a sufficient basis[189]), is an insufficient basis to support a conclusion of market efficiency. As I discussed above, the finance literature has documented numerous instances of firms that traded in

[188] Feinstein Report, ¶116 and Feinstein Dep., 243:13-23. See also Feinstein Dep. 244:17–245:4 discussing his finding of a "structural break" and increased volatility in Freddie Mac's stock after August 8, 2007.
[189] Feinstein Dep., 380:13-381:6.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 73 of 370. PageID #: 18960

inefficient markets despite the presence of certain "structural factors."

157. Arbitrage (which in essence requires short-selling the "over-priced" asset and buying the "under-valued" asset with identical payoffs) "plays a critical role in the analysis of securities markets, because its effect is to bring prices to fundamental values and to keep markets efficient."[190] As noted above, in Fama's market efficiency model, arbitrageurs play a key role: "there are large numbers of rational, profit-maximizers [with no capital constraints] actively competing, with each trying to predict future market values of individual securities, and where important current information is almost freely available to all participants."[191] In an efficient market, given competition among such arbitrageurs, the market becomes efficient. It follows that if there are limits to arbitrage, then the market can become inefficient. In an informationally efficient market, where gathering information may be costly, "prices reflect information to the point where the marginal benefits of acting on information (the profits to be made) do not exceed the marginal cost."[192]

158. Real-world limits of arbitrage can render markets inefficient according to several academic studies. For instance, trading institutions (*e.g.*, hedge funds and investment bank trading desks) that engage in arbitrage using outside investors' capital[193] and credit (leverage) offered on a collateralized basis by financial entities such as repo counterparties,[194] can suddenly face a

190 Shleifer, Andrei and Robert W. Vishny (1997), The Limits of Arbitrage, Journal of Finance, Vol. LII, No. 1, pg. 35. ("Shleifer and Vishny (1997)").

191 *Fama, Eugene F., 1965, "Random Walks in Stock Market Prices," Financial Analysts Journal, Vol. 21, No. 5, pg. 56 "Fama (1965)"). As I noted earlier, the Court in Cammer v. Bloom too opined that "the existence of … arbitrageurs would ensure completion of the market mechanism," which Dr. McCann has failed to examine. Cammer, 711 F.Supp. at 1286-87.*

192 Fama (1991), pg. 1575.

193 Shleifer and Vishny (1997), pg. 36.

194 Brunnermeier, Markus, and Lasse H. Pedersen, 2009, "Market Liquidity and Funding Liquidity," Review of Financial Studies 22, ["Brunnermeier and Pedersen (2009)"], pg. 2229.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 74 of 370. PageID #: 18961

liquidity crunch if such outside capital dries up. Such a sudden capital crunch would limit these institutions' ability to undertake arbitrage, and render security markets informationally inefficient.

159. Indeed, the dramatic effect of the 2007/2008 liquidity crunch and ensuing financial crisis on hedge funds performing key arbitrage functions in capital markets was widely noted.[195] As an article in the Economist explained in 2008:

> *A year ago hedge funds were the omnipotent vanguard of financial capitalism. They were uncompromising in their search for returns, and they dominated trading activity in most securities. But the industry has been humbled. The typical fund has fallen by almost a fifth so far this year, according to Hedge Fund Research (HFR), an analysis firm (see chart 1). "Convertible arbitrage" funds—which try to exploit price anomalies among corporate bonds—have lost a staggering 46%. By some margin 2008 has been hedge funds' worst year since HFR began compiling records in 1990. …*

[195] Hedge funds, investment banks and other entities outside the regulated banking sector that perform bank-like functions are considered "shadow banks" that borrow from other financial entities such as money market mutual funds (instead of from depositors like commercial banks do). Such debt is uninsured. To mitigate their counterparties' credit concerns, hedge funds therefore offer creditors collateral through repurchase agreements or "repos." A repo agreement is a form of money like demand deposits at banks. "In a repo transaction, "a "depositor" (e.g., a money market fund) deposits money at a financial institution and receives collateral, valued at market prices. The transaction is short term, so the depositor can "withdraw" the money at any time. The deposit is backed by the bonds received as collateral from the institution where the money is deposited." (Gorton, Gary and Andrew Metrick, "Haircuts," Yale University working paper, May 2010, ("Gorton and Metrick (2010)"), pg. 3).

Liquidity in the short-term repo and commercial paper markets hinge critically on investors' views about the quality of such collateral. As long as hedge funds, and ABCP conduits' collateral is deemed to be of sufficiently high quality to withstand any adverse economic shocks, credit through repo agreements and commercial paper is readily available. However, a significant economic shock can result in a sudden liquidity freeze due to a sudden fear of adverse selection. Such a sudden lack of confidence in counterparties' risk exposures to troubled sub-prime assets did erupt in August 2007, resulting in a "run on the repo." (Gorton and Metrick (2010), pg. 7. See also Gorton, Gary and Andrew Metrick, "Securitized Banking and the Run on Repo," Yale University working paper, November 9, 2010, pg. 1.)

The collapse of the repo market was a systemic event of unprecedented scale as the estimated $12 trillion repo market was larger than the total assets in the U.S. banking system ($10 trillion) and U.S. investment banks funded "roughly half of their assets using repo markets, with additional exposure due to off-balance sheet financing of their customers." (Gorton, Gary and Andrew Metrick, "Securitized Banking and the Run on Repo," Yale University working paper, November 9, 2010, pg. 13.)

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 75 of 370. PageID #: 18962

Over the next few quarters the fallout is likely to be brutal. Between 1990 and last year the industry's assets under management grew almost 50-fold, to nearly $2 trillion (see chart 2). Now industry executives predict that assets could fall by 30-40%, as clients stampede for the exit. The number of funds, which climbed to over 7,000 as a generation of financiers headed for the gold-paved streets of Mayfair in London and Greenwich, Connecticut, could fall by half. …

What is the cause of the fire sales that seem to be at the root of the industry's problems? The obvious answer is a withdrawal of credit, which has in turn forced hedge funds to offload assets. Sceptics have long argued that for all the skill they claim to possess, hedge funds just use cheap money to amplify mediocre returns. By this account they are simply another part of a vast, debt-dependent ecosystem that is now being starved of oxygen. [underline added][196]

160. Former Federal Reserve Chairman Ben Bernanke also commented on the liquidity crisis in October 2007, noting that it led to a deterioration in "market functioning":[197]

[t]he past several months have been an eventful period for the U.S. economy. In financial markets, sharpened concerns about credit quality induced a retrenchment by investors, leading in some cases to significant deterioration in market functioning. …

In these circumstances of heightened volatility and diminished market functioning, banks also became more concerned about the possible risk exposures of their counterparties and other potential contingent liabilities. These concerns prompted banks to become protective of their liquidity and balance sheet capacity and thus to become markedly less willing to provide funding to others, including other banks. As a result, both overnight and term interbank funding markets came under considerable pressure. Interbank lending rates rose notably, and the liquidity in these markets diminished.

161. Given these market realities, it should come as no surprise that Dr. Feinstein's analysis fails to demonstrate a cause-and-effect relationship between material news and the price of Freddie Mac's common stock. Dr. Hallman, Plaintiff's prior expert, also was unable to demonstrate such a cause-and-effect relationship, as he examined six earnings dates, only one of which exhibited a

[196] "Hedge funds in trouble: The incredible shrinking funds", The Economist, October 23, 2008.
[197] Chairman Ben S. Bernanke, "The Recent Financial Turmoil and its Economic and Policy Consequences", At the Economic Club of New York, New York, New York October 15, 2007.

NAVIGANT

statistically significant stock price reaction. Having the benefit of Dr. Hallman's report and my critique of Dr. Hallman's report, Dr. Feinstein was only able to construct a test that yielded a statistically significant result by committing various statistical errors and making numerous assumptions (often inconsistent with authorities on which he relies and his past practice) that biased his test in favor of his conclusion. His inability to construct a valid test that yielded statistically significant results indicating market efficiency is consistent with his and others' observations regarding the market at this time.

V. The Economic Evidence Supports a Finding That The Alleged Misrepresentations and Omissions Had No Impact on Freddie Mac's Common Stock Price.

162. Consistent with the weight of the academic literature regarding the fraud on the market presumption of reliance and underlying hypothesis of an efficient market, the Supreme Court in *Halliburton II* reasoned: "[i]n the absence of price impact, *Basic's* fraud on the market theory and presumption of reliance collapse."[198]

163. In my opinion, the economic evidence demonstrates that the alleged misrepresentations had no impact on Freddie Mac's stock price. As I demonstrate below, using Dr. Feinstein's own market model regression to analyze the alleged misrepresentations in the TAC shows no such price impact.[199] Moreover, utilizing Dr. Feinstein's z-test with the 23 dates of alleged inflationary misrepresentations as news days, the resulting z-statistic evidences no statistically significant variation between news and non-news days.

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[198] *Halliburton II,* 134 S.C.t at 2414.

[199] I reach the same conclusion when using the 3 Period Feinstein Market Model discussed above that more appropriately accounts for the change in volatility during Dr. Feinstein's Estimation Period 1.

164. According to the TAC, the alleged misrepresentations and omissions were made on 23 dates during the Class Period[200] and thus such misrepresentations could potentially impact Freddie Mac stock's price on 23 effective dates.[201] Assuming for the sake of argument that Freddie Mac's stock traded in an efficient market, the Company's stock price would be expected to react quickly and logically to an alleged misrepresentation if it was material. That is, an economist would expect Freddie Mac's abnormal return to be positive and statistically significant on the effective date following an allegedly inflationary misrepresentation (assuming no confounding information).

165. Using Dr. Feinstein's own event study model,[202] I find that Freddie Mac's abnormal

[200] Five of these alleged misstatements purportedly occurred prior to the Proposed Class Period. [TAC, ¶¶ 140-143] However, the Plaintiff does not appear to be claiming any damages on the basis of these five alleged misstatements. The Complaint notes: "As set forth in the certification appended to the Plaintiff's Original Complaint and incorporated by reference herein, Plaintiff purchased shares of common stock of Freddie Mac during the Class Period at artificially inflated prices and has been damaged thereby." [TAC, ¶18.]

[201] Dr. Feinstein assesses Freddie Mac stock's response to unexpected news based on the stock's abnormal return over one day. Therefore, I too have examined the statistical significance of Freddie Mac's abnormal return over one day; namely on the day of the alleged misstatement if it occurred during (or before) market trading hours, or on the following day if the alleged misstatement occurred after trading hours. I refer to these dates as the "effective dates." Note that Plaintiff alleges that on August 1, 2006 misstatements were made during as well as after trading hours. Consequently, I have examined Freddie Mac's stock price reaction to such alleged misstatements by analyzing the statistical significance of the stock's abnormal return on August 1 and 2, 2006. In addition, the Plaintiff alleges that misstatements were made on 26 other dates during or prior to the Proposed Class Period. Therefore, to analyze whether such alleged misstatements on 27 dates had a price impact, I have examined the statistical significance of Freddie Mac's abnormal return on 28 effective dates. Out of the 27 dates, there were 22 dates <u>within</u> the Proposed Class Period when alleged misstatements were made according to the TAC and I refer to these 22 dates as "Alleged Misstatement Dates." See Appendix V for a list of all alleged misstatement dates and corresponding effective dates.

Besides the alleged misstatements that could have impacted Freddie Mac's stock price on these 28 effective dates, TAC also alleges a misstatement that was part of an article published in *Mortgage Risk Magazine* in October 2007, whose effective date cannot be determined because the TAC does not provide the specific date of the alleged misstatement.

[202] I use Dr. Feinstein's event study model to analyze whether Freddie Mac's abnormal returns following alleged misstatements were statistically significant in the logical (positive) direction because, by doing so, the odds of identifying statistically significant abnormal returns is higher (a finding that would support the Plaintiff's claim) than would be the case if I were to employ other, more scientifically reliable models.

return either was *not* statistically significant or was statistically significant in the wrong direction on 22 of the 23 effective dates at issue.[203] Even cumulatively, Freddie Mac's abnormal return was negative over these 23 effective dates.

166. Of the 23 identified dates in the TAC, February 27, 2007 is the only date on which Freddie Mac's abnormal return following an alleged misstatement was statistically significant and positive according to Dr. Feinstein's event study model. However, if I apply the 3 Period Feinstein Market Model that more properly accounts for the change in volatility during Dr. Feinstein's Estimation Period 1 (as discussed in Section IV.B.2.d (Flaw 9) above), Freddie Mac's abnormal return on February 27, 2007 is also no longer statistically significant.

167. In addition, I tested the 23 TAC alleged misrepresentation dates using Dr. Feinstein's flawed FDT z-test, using these dates as the "news days" by which to compare "non-news days." The results of Dr. Feinstein's z-test are statistically insignificant. In other words, Dr. Feinstein's z-test does not find that the TAC misrepresentation dates were more likely to have statistically significant stock returns than the other days of the Class Period. See Appendix VI for details.

168. Additionally, Dr. Feinstein claims that November 20, 2007 was an "allegation related event" on which Freddie Mac announced increased losses "due to a higher provision for credit losses, losses on [its] derivative portfolio and other credit-related items."[204] As part of his

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[203] In order to assess the significance of Freddie Mac's abnormal returns on the 23 effective dates, I used the same market model (*i.e.*, used same market and industry indices and same estimation periods) that Dr. Feinstein used [Feinstein Report, ¶122-127] and include a separate indicator variable to identify each effective date and remove these dates from the model estimation period (as Feinstein did in his Z-test analysis). I found Freddie Mac's abnormal return to be not statistically significant following 20 of the 23 alleged misrepresentations, and statistically significant but *negative* – i.e., in the wrong direction in light of the positive statements -- following 2 dates (June 8, 2007 and August 30, 2007). Even cumulatively, Freddie Mac's abnormal return was negative over these 23 effective dates. See Appendix V detailing these results.

[204] Feinstein Report, ¶116.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 79 of 370. PageID #: 18966

discussion of his event study analysis of a single day (November 20, 2007, the "allegation-related date"), he also claims that "[b]y focusing an event study on the disclosure of the information (or risk materialization date) related to the allegations in the Complaint, one is able to ascertain whether the market for Freddie Mac common stock was efficient, not only generally, but also with respect to the particular information at issue in this case."[205]

169. As I have noted above, my review of the news on November 20, 2007 indicates that the announcements by Freddie Mac were not corrective of previous misstatements or the materialization of alleged prior undisclosed risks, and hence the price decline on this date cannot be indicative of a statistically significant price impact in response to alleged misrepresentations and omissions. Dr. Feinstein has also performed no analysis linking what Freddie Mac reported to investors on that day to the allegation in the TAC, beyond simply claiming this day represented an "allegation-related event."

170. In its conference call the same day, Freddie Mac discussed its previously disclosed subprime exposure in its Retained portfolio (as the TAC acknowledges[206]) and noted that:[207]

> *The credit profile of our [R]etained portfolio* ***remains of the highest credit quality*** *with 57% in agency mortgages and 33% in non-agency securities, of which 97% is triple A rated and does not include any CDOs. … Despite the continued deterioration of the housing market and increases in non-prime delinquencies, we remain comfortable with our risk position on these assets.* ***For the subprime securities****, while we have experienced some downgrades, …* ***no losses are projected on these securities****.*

[205] Feinstein Report, ¶112.

[206] As TAC mentions, at the end of 2006, "Freddie had … about $124 billion of AAA rated subprime exposure in [its] retained portfolio." [TAC, ¶84].

[207] "FRE - Q3 2007 Freddie Mac Earnings Conference Call," *Thomson StreetEvents*, November 20, 2007, p.54 (emphasis added).

Case: 4:08-cv-00160-BYP Doc #: 385-2 Filed: 09/01/17 80 of 370. PageID #: 18967

171. Indeed, the analysts' reports that Dr. Feinstein considered in writing his report, note that Freddie Mac's announced losses of November 20, 2007 were related to the accounting treatment of losses and depressed market prices caused by adverse macroeconomic changes, rather than new revelations about Freddie Mac's business model risks or about Freddie Mac's subprime exposure risks.[208] For example, Morgan Stanley analysts noted that "[u]nfortunately, in the current environment, GAAP losses create a perception that the company's business model is struggling."[209] Similarly, Bears Stearns analysts stated that, "[d]epressed market prices for illiquid assets which in some instances aren't reflective of the Freddie Mac assets being valued are creating economic problems by reducing the capital available to support security purchases."[210] In fact, none of the analysts' reports following November 20, 2007, that were considered by Dr. Feinstein, perceived Freddie Mac's announced losses of November 20 as indicative of a risky business model or disclosure of either additional risks or materialization of previously undisclosed risks.[211] In fact

[208] As noted above, the TAC asserts that, in the November 20, 2007 press release, "[f]or the first time, **Defendants admitted that Freddie Mac's investments in subprime and nontraditional mortgage products had subjected Freddie Mac to significant risk and caused it to sustain substantial losses**." Similarly, the TAC states that "**the November 20, 2007 press release admitted that the Company had been investing in subprime, Alt-A and nontraditional mortgage products**." The TAC also states that "[t]he revelation of November 20, 2007, **was the first indication that the Defendants had misrepresented the truth with regard to Freddie Mac's capital.**" [TAC ¶212] (emphasis added).

[209] Kenneth A. Wagner, and Vivian (Wei) Wang, "Freddie Mac: Reports 3Q07 Loss," *Morgan Stanley Research North America*, November 20, 2007.

[210] David Hochstim, and Michael Nannizzi, "Freddie Mac (FRE-$26.74-Peer Perform): GAAP Loss Reflects Reduced Market Liquidity and Is Restricting Liquidity As Well," *Bear Stearns*, November 20, 2007.

[211] Reports cited by Dr. Feinstein on November 20 or 21, 2007 were as follows: "Freddie Mac (FRE-$26.74-Peer Perform): GAAP Loss Reflects Reduced Market Liquidity and Is Restricting Liquidity As Well," *Bear Stearns & Co*., November 20, 2007; "FNM and FRE: Capital Constrained – Downgrading to Underperform," *Fox-Pitt, Kelton, Inc*., November 20, 2007; "Freddie Mac (FRE): We think capital raise will be dilutive to common shareholders," *Fox-Pitt, Kelton, Inc*., November 20, 2007; "Freddie Mac: Reports 3Q07 Loss," *Morgan Stanley*, November 20, 2007; "First Read: Freddie Mac, Placing Under Review as We Clarify Credit and Capital Outlook," *UBS*, November 20, 2007; "Freddie Mac (FRE - US$ 26.74) 2-Equal weight Change of Earnings Forecast," *Lehman Brothers*, November 21, 2007.

Case: 4:08-cv-00160-BYP Doc #: 385-2 Filed: 09/01/17 81 of 370. PageID #: 18968

none of the analysts' reports over the seven days, November 20-27, 2007 that I reviewed, considered Freddie Mac's disclosures on November 20, 2007 to be indicative of previous undisclosed risks about the business model or materialization of undisclosed risks related to Freddie Mac's subprime exposure.[212] In other words, there is no economic evidence of disclosure of previously undisclosed risks on November 20, 2007, and hence the price decline on that day was not caused by a materialization of allegedly previously undisclosed risks or related to alleged disclosure defects.

VI. Conclusions

172. In my opinion, Dr. Feinstein fails to establish that the market for Freddie Mac common stock was semi-strong form efficient during the Proposed Class Period. His statistical testing, if anything, proves that it was not. Dr. Feinstein's statistical tests are fundamentally flawed for numerous reasons and their results are unreliable.

173. In my opinion, the economic evidence supports a finding that the alleged misrepresentations and omissions had no impact on Freddie Mac's common stock price

[212] These analysts reports include the six mentioned in the previous footnote as well as: "Fitch Places Freddie Mac's 'AA-' Preferred Stock on Rating Watch Negative," *Fitch Ratings*, November 20, 2007; "Freddie Mac (FRE): Losses Spur Need for New Capital; Charter/Mission Intact," *Citi*, November 20, 2007; "FNM and FRE Price Action-Capital Levels and Even More Threatened Business Models," *Miller Tabak*, November 20, 2007; "Freddie Mac: Strong Retained Portfolio Growth in August," *Credit Suisse*, November 25, 2007; "Federal Home Loan Mortg Corp (FRE) – NYSE," *Price Target*, November 25, 2007; "GSE Update: Downgrading GSEs as Credit Pressures Erode EPS and Dividend Outlook," *UBS Investment Research*, November 26, 2007; "Freddie Mac: Announces $6B Preferred Stock Offering; Cuts 4Q Dividend by 50%," *UBS Investment Research*, November 27, 2007; "Freddie Mac (FRE-$24.50-Peer Perform)," *Bear Stearns & Co.*, November 27, 2007; "Freddie Mac (FRE): Divided Halved and $6 Billion of Preferred Raised Stabilizes Current Capital Position," *Credit Suisse*, November 27, 2007; "Mortgage Strategist: Highlights & Recommendations," *UBS Investment Research*, November 27, 2007.

Respectfully submitted,

Mukesh Bajaj

Mukesh Bajaj, Ph. D

September 1, 2017

Appendix I

NAVIGANT

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Professional Summary

Dr. Mukesh Bajaj is a Managing Director and Global Head of the Securities & Finance Practice at Navigant Economics, LLC. Prior to joining Navigant Economics, Dr. Bajaj founded AFE Consulting and served as its President.

Dr. Bajaj advises clients in matters involving economic and financial issues, and has managed hundreds of consulting and litigation support assignments. Dr. Bajaj is an expert in matters relating to securities fraud, valuation of complex derivatives and intellectual property, insider trading, financial market microstructure, intangible assets, transfer pricing, interests in closely-held firms, warrants, restricted stock and other complex contingent securities, and purchase price allocation studies. He has also consulted on financial strategy and acquisition analysis.

Dr. Bajaj has testified in various Federal and State Courts, the Superior Court of California, the State Board of Equalization in California, the U.S. Tax Court, arbitrations, mediations and in IRS Appeals proceedings. He has also testified in Canadian and Australian courts, testified in JAMS arbitration and filed an expert report in the International Center for Settlement of Investment Disputes.

In addition to his work at Navigant, Dr. Bajaj has taught corporate finance, investments, and financial engineering courses in the MBA and Masters in Financial Engineering programs at the Haas School of Business at the University of California at Berkeley. Prior to his consulting practice, Dr. Bajaj was an assistant professor of finance and business economics at the University of Southern California where he taught undergraduate and graduate courses in finance. Dr. Bajaj is the recipient of several teaching awards and scholastic honors and has published several articles in leading academic and applied journals, such as *The Journal of Finance*, *The Journal of Financial Economics*, *The Journal of Financial Research*, *The Journal of Applied Finance*, *International Economic Review*, *Research in Finance*, *The Journal of Corporation Law*, *The Journal of Derivatives* and *Research in Law and Economics*.

Dr. Bajaj holds a PhD in Finance from the University of California at Berkeley and an MBA from the University of Texas at Austin. He earned a Bachelor of Technology in Chemical Engineering from the Indian Institute of Technology, Delhi.

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Professional Experience

2012 – Present — **Navigant Economics**
Managing Director and Global Head of the Securities & Finance Practice

1997 – 2014 — **Haas School of Business, University of California, Berkeley**
Lecturer

2011 – 2012 — **AFE Consulting**
Founder and President

1997 – 2011 — **LECG LLC**
Senior Managing Director and Practice Leader (2007 – 2011)
Member – Executive Management Committee (2007 – 2011)
Member – Management Advisory Committee (2003 – 2007)
Member – Board of Directors (2001 – 2003)
Managing Director (1999 – 2007)
Director (1999)
Affiliate (1998)
Principal (1998)
Senior Economist (1997)

1995 – 1997 — **BVS Inc.**
Senior Associate

1988 – 1995 — **University of Southern California**
Assistant Professor – Finance and Business Economics

- Award from Faculty Research and Innovation Fund, University of Southern California, 1990

1983 – 1988 — **University of California, Berkeley**
Instructor
Graduate Student Instructor

- Graduate Fellowship, University of California, Berkeley (1988)
- Earl F. Cheit Award for Outstanding Teaching, Graduate School of Business, University of California, Berkeley (1986–1987)
- Outstanding Graduate Student Instructor Award, University of California, Berkeley (1986–1987)
- Outstanding Graduate Student Instructor Award, University of California, Berkeley (1985–1986)
- Award for Best Technical Paper published in Indian Chemical Engineer (1983)

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Education

University of California, Berkeley
PhD in Business Administration (Finance)

University of Texas at Austin
MA in Business Administration
* Sword Scholar (Dean's List), University of Texas at Austin

Indian Institute of Technology, Delhi, India
Bachelor of Technology
* National Science Talent Scholar, National Council of Educational Research and Training, India

Expert Testimony on Record

The Tulalip Tribes, et al., v. The State of Washington, et al., Case No. 2:15-cv-00940 in the United States District Court for the District of Washington. Testified in deposition regarding the economic activities that gave rise to disputed taxes, and the state, federal, and tribal contributions to the success of these economic activities. Deposition in July 2017.

In re Allergan, Inc. Proxy Violation Securities Litigation, C.A. 14-cv-02004 (N.D. Cal 2014). Testified in deposition on whether the common stock of Allergan Inc. traded in an efficient market during the Class Period and whether damages in the action are subject to a common formula that can be applied Class-wide (December 2016). Also testified in deposition on the economic materiality of alleged non-public information and damages allegedly caused by insider trading on this information (July 2017). Depositions in December 2016 and July 2017.

OpenGate Capital, LLC, et al., v. Thermo Fisher Scientific Inc., Civil No. 13-475-GMS in the United States District Court for the District of Delaware. Testified in deposition regarding damages allegedly caused by claims of fraudulent misrepresentation in connection with the purchase of a division of Thermo Fisher by OpenGate. Deposition in November 2015.

Continental Industries Group, Inc. v. FTS International Services, LLC, et al., Case Nos. 12-CV-05599 and 12-CV-06966 in the United States District Court, Southern District of New York. Testified in deposition and trial on economic damages resulting from the alleged breaches of two supply agreements between the Plaintiff and Defendants. Deposition in October 2014. Trial in October 2015.

In re: UBS Financial Services, Inc. of Puerto Rico Securities Litigation, Civil Case No.: 3:12-cv-01663-CCC, United States District Court for the District of Puerto Rico. Testified in deposition on economic issues related to certain Puerto Rico closed-end funds for class certification purposes. Deposition in September 2015.

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Expert Testimony on Record (Continued)

In the Matter of the Arbitration between Offshore Exploration and Production LLC, Claimant/Seller, v. Korea National Oil Corporation and Ecopetrol, S.A., Respondents/Purchasers, ICDR Case No. 50 198 T 00825 1 in the International Centre for Dispute Resolution. Testified in arbitration proceedings on the calculation of prejudgment interest related to payments Claimant/Seller asserted should have been made pursuant to an escrow agreement between Claimant/Seller and Respondents/Purchasers. Arbitration in February 2014.

Sekisui America Corporation and Sekisui Medical Co., Ltd. v. Richard and Mary Louise Trudel-Hart, Case No. 12-CIV-03479 in the United States District Court, Southern District of New York. Testified in deposition on damages in an action alleging breach of contract arising out of the sale of a medical diagnostics company. Deposition in September 2013.

Securities and Exchange Commission v. Manouchehr Moshayedi, Case No. 12CV-01179-JVS-JPR in the United States District Court for the Central District of California. Testified in deposition regarding allegations by the SEC of insider trading against the founder and former CEO of STEC, a maker of custom memory solutions. Deposition in August 2013.

In re American International Group, Inc. 2008 Securities Litigation, Master File No. 08-CV-4772-LTS in the United States District Court, Southern District of New York. Testified in depositions and in an evidentiary hearing on market efficiency at class certification stage in a securities fraud class action alleging that Defendants materially misstated the extent to which AIG had accumulated exposure to the subprime mortgage market through its securities lending program and its credit default swap ("CDS") portfolio. Depositions in November 2011 and March 2012. Evidentiary Hearing in April 2013.

Securities and Exchange Commission v. Fabrice Tourre, Case No. 10-CV-3229 (KBF) in the United States District Court, Southern District of New York. Testified in deposition on the economic materiality of the nondisclosure of certain hedge fund positions with respect to a particular synthetic ABS CDO. Deposition in February 2013.

Ohio Public Employees Retirement System, On Behalf of Itself and all Others Similarly Situated, v. Federal Home Loan Mortgage Corporation a/k/a Freddie Mac, Richard F. Syron, Patricia L. Cook, Anthony S. Pizel, and Eugene M. McQuade, Civil Action No. 4:08-cv-160 in the United States District Court for the Northern District of Ohio, Eastern Division (Youngstown). Testified in deposition concerning the efficiency of the market for Freddie Mac's common stock and the economic evidence as it related to the Plaintiff's allegations that alleged misrepresentations and omissions were material. Deposition in January 2013.

Cora E. Bennett v. Sprint Nextel Corporation, Gary D. Forsee, Paul N. Saleh and William G. Arendt, Case No. 09-CV-2122 EFM/KMH in the United States District Court for the District of Kansas. In the class-certification stage of a securities fraud class action, testified in deposition regarding the economic evidence supporting the claim that Sprint bonds traded in efficient markets throughout the Class Period. Deposition in June 2012.

Case: 4:08-cv-00160-BYP Doc #: 385-2 Filed: 09/01/17 88 of 370. PageID #: 18915

NAVIGANT

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Expert Testimony on Record (Continued)

State of New Jersey, Department of Treasury, Division of Investment, on behalf of Common Pension Fund A. v. Merrill Lynch & Co. and Bank of America Corp., Case No. L-3855-09 in the Superior Court of New Jersey. Testified in deposition on the materiality of accounting allegations, loss causation and damages calculations in connection with transactions involving the purchase and subsequent conversion of a convertible preferred security to common stock. Deposition in May 2012.

In re Richard Kirby v Centro Properties Ltd & Ors (VID 326 of 2008), *Richard Kirby v Centro Retail Ltd & Ors* (VID 327 of 2008), and *Nicholas Stott v Pricewaterhouse Securities Ltd* (VID 1028 of 2010), in the Federal Court of Australia. In a pair of securities class action disputes in Australia, testified at trial on alleged disclosure deficiencies of two Australian REITs in connection with certain short-term and long-term debts and the effect of those disclosures on the prices of the REITs' two stapled securities, as well as on the condition of the global credit market during the class period. Trial in May 2012.

In re Lehman Brothers Securities and ERISA Litigation, Case No. 09-MD-2017 (LAK) in United States District Court, Southern District of New York. Testified in deposition on the market for structured products and the market's general awareness of credit risks associated with structured finance products in the class-certification stage of a securities fraud class action alleging materially false and misleading statements and omissions in the offering documents of principal-protected notes. Deposition in April 2012.

In re International Textile Group Merger Litigation, C.A. No. 2009-CP-23-3346 in the Court of Common Pleas for the State of South Carolina, County of Greenville. Testified in deposition on damages and loss causation for class action and derivatives suits arising from a merger. Deposition in April 2012.

Bank of America National Association, and Banc of America Securities LLC v. Bear Stearns Asset Management Inc., Ralph Cioffi, Matthew Tannin, and Raymond McGarrigal, Case No. 1:08-cv-0265-AJN in the United States District Court, Southern District of New York. Testified in deposition on damages related to a securitization transaction and the Defendants' alleged failure to disclose the financial condition of their hedge funds. Deposition in March 2012.

Between: Howard Green and Anne Bell, and Canadian Imperial Bank of Commerce, Gerald McCaughey, Tom Woods, Brian G. Shaw, And Ken Kilgour, No. CV-08-00359335-0000, Ontario Superior Court of Justice. Testified in deposition on loss causation in proceedings under the Class Proceedings Act, 1992 alleging that Defendants made various misrepresentations regarding CIBC's CDO exposure and the extent of impairment of CIBC's CDO positions. Deposition in December 2011.

In re Tronox Inc. Securities Litigation, No. 09 Civ. 06220 (SAS), United States District Court, Southern District of New York. Testified in deposition on market efficiency at class certification stage in a securities fraud class action alleging that Defendants materially misstated the extent of legacy environmental liabilities. Deposition in December 2011.

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Expert Testimony on Record (Continued)

In re Federal Home Loan Mortgage Corp. (Freddie Mac) Securities Litigation, Case No. 1:09-MD-2072 in the United States District Court, Southern District of New York. (Appeal denied by United States Court of Appeals for the Second Circuit, May 31, 2012.) Testified in deposition and hearing before the Court on market efficiency at class certification stage in a securities fraud class action alleging misrepresentations concerning Freddie Mac's capitalization and credit risk exposure. Depositions in August 2011 and November 2011. Court hearing in November and December 2011.

Estate of John F. Koons, III v. Commissioner of Internal Revenue, Docket Nos. 19771-09 and 19772-09, in the United States Tax Court. Testified at trial regarding the valuation of certain membership interests in a limited corporation. March 2011.

In Re Altria Group, Inc. v. United States of America, Case No. 1:06-cv-09430-RJH in the United States District Court, Southern District of New York. Testified in deposition and at trial regarding economic issues affecting tax treatment of certain leveraged lease transactions. Deposition in December 2007. Trial in June-July 2009.

Madison Tyler Holdings, LLC, et al., Claimants v. Financial Asset Trading & Technology of California, LLC, et al., Respondents; *Financial Asset Trading & Technology of California, et al., Cross-Claimants, v. Madison Tyler Holdings, LLC, et al., Cross-Respondents*; and related arbitration *Madison Tyler Holdings, a Delaware Limited Liability Company, et al., Counter-Claimants and Respondents v. Rajashree Karwa, an individual, Counter-Respondent and Claimant*. Arbitration Before JAMS, JAMS Ref. No. 1220038462. Testified in deposition on economic analysis of source of value creation in algorithmic trading strategies. June 2009.

Lawrence E. Jaffe Pension Plan, On Behalf of Itself and All Others Similarly Situated, v. Household International, Inc., et al., Case No. 02-C-5893 in the United States District Court, Northern District of Illinois, Eastern Division. Testified in deposition and at trial on loss causation and damages in a securities class action alleging securities fraud arising from purported accounting irregularities and predatory lending practices to subprime borrowers. Deposition in March 2008. Trial in May 2009.

Guerrero Family Trust, Carmen De Leon Guerrero, Jose T. Tenorio Trust, Estate of Santiago C. Tenorio, Juan T. Guerrero, Jesus T. Guerrero, and AJT Trust, v. Kinki Nippon Tourist Co. LTD, Saipan Hotel Corporation, Pacific Development Inc., Pedro J.L. Igitol, in his official capacity of Secretary of Saipan Hotel Corporation, Morgan Stanley Japan Limited, Marianas Holdings, LLC, and K.K. ING Karuizawa Training Institute, Civil Action No. 04-0574D in the Superior Court of the Commonwealth of the Northern Mariana Islands. Testified in deposition on damages arising from alleged abuse of fiduciary duty and dilution of minority shareholders' stock holding. July 2008.

In the Matter of David A. Finnerty, et al., Administrative Proceeding, File No. 3-11893, Before the Securities and Exchange Commission. Testified in trial regarding the trading patterns of certain NYSE specialists in connection with alleged violations of priority rules and securities laws. February and March 2008.

In Re NYSE Specialists Securities Litigation, Master File No. 03 Civ. 8264 (RWS) in the United States District Court, Southern District of New York. Testified in deposition on class certification issues relating to alleged trading-rule violations by New York Stock Exchange specialist firms. November 2007.

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Expert Testimony on Record (Continued)

Theo Bullmore and Phillip S. Stenger, as Joint Official Liquidators of Beacon Hill Master Ltd. (In Official Liquidation), Plaintiffs v. Ernst & Young Cayman Islands, Ernst & Young LLP, Beacon Hill, Asset Management, LLC, John D. Barry, Thomas Daniels, John Irwin, Mark Miszkiewicz, and ATC Fund Services (Cayman) Limited f/k/a ATC Fund Administrators (Cayman) Limited, Defendants, Index No.: 104314/05 in the Supreme Court of the State of New York, County of New York. Testified in deposition on the causation and alleged damages experienced by the Beacon Hill Master Fund caused by an alleged improper audit by Ernst & Young Cayman Islands. September 2007.

Sterling Savings Association and Sterling Financial Corporation v. United States of America, Defendant, Case No. 95-829C in the United States Court of Federal Claims. Testified in deposition and at trial on damages due to alleged breach of contract as a result of Financial Institutions Reform and Recovery Act of 1989. Depositions in June 2002 and May 2004. Trial in July 2007.

Adelphia Communications Corp., Plaintiff v. Deloitte & Touche LLP, (Defendant) v. John Rigas, Timothy Rigas, Michael Rigas and James Rigas (Additional Defendants), in the Court of Common Pleas, Philadelphia County. Testified in deposition on loss causation and damages issues related to alleged improper conduct by auditor. May 2007.

David S. and Malia A. Litman v. United States of America, Case No. 05-956T; *Robert B. and Michelle S. Diener v. United States of America*, Case No. 05-971T; *Hotels.com Inc. and Subsidiaries (f/k/a Hotel Reservations Network, Inc.) v. United States of America*, Case No. 06-285T. Judge Christine O.C. Miller in the United States Court of Federal Claims. Testified in deposition and at trial on valuation of 9.9 million shares of stock issued to certain former officers of Hotels.com for tax purposes. Deposition in July 2006. Trial in May 2007

Jane Z. Astleford, Donor, Petitioner v. Commissioner of the Internal Revenue, Respondent, Docket No 4342-06 in the U.S. Tax Court. Testified at trial on value of certain interests in a limited partnership. March 2007.

United States of America v. Sanjay Kumar and Stephen Richards, 04-CR-0846 (ILG), in the United States District Court, Eastern District of New York. Testified at trial on loss causation and damages issues in criminal securities fraud matter in which defendants pleaded guilty to improper revenue recognition related accounting irregularities. October 2006.

The Procter and Gamble Company and Subsidiaries & Proctor and Gamble FSC (Barbados) vs. The United States of America, Case number 1:05cv355 in United States District Court for the Southern District of Ohio, Western Division. Testified in deposition on fair market value of certain technologies donated by Proctor and Gamble to various entities in connection with a tax dispute. September 2006.

United States of America v. Richard Volpe, Indictment S1 05 Cr. 390 (SHS) in the United States District Court, Southern District of New York. Testified at trial on liability issues in criminal securities fraud matter alleging illegal trading by certain New York Stock Exchange specialists. August 2006.

United States of America v. Robert Scavone, Indictment S1 05 Cr. 390 (SHS) in the United States District Court, Southern District of New York. Testified at trial on liability issues in criminal securities fraud matter alleging illegal trading by certain New York Stock Exchange specialists. July 2006.

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Expert Testimony on Record (Continued)

United States of America v. Michael Hayward and Michael Stern, Indictment S1 05 Cr. 390 (SHS) in the United States District Court, Southern District of New York. Testified at trial on liability issues in criminal securities fraud matter alleging illegal trading by certain New York Stock Exchange specialists. July 2006.

Commonwealth Holdings, Inc., Profit Sharing Plan & Trust, James T. Waddill, IV et al. v. Salomon Smith Barney, Inc. and John Henry Spatz, in a hearing before NASD. Testified on loss causation and damages aspects of Plaintiffs' claims arising from alleged securities fraud. September 2005.

Messrs. Robert, Charles and John Switzer et al. v. Deutsche Bank et al., in a hearing before NASD. Testified on liability and damages aspects of Plaintiffs' damage claims arising from alleged unsuitable investments in certain leveraged debt obligations. June 2005.

IDT Corp. v Telfonica S.A. et al., Case No. 01 CV 471 in the United States District Court for New Jersey. Testified in deposition on liability and loss causation aspects in a claim of alleged securities fraud. April 2005.

Sherewin I. Ray et al. v. Citigroup Global Markets, Inc. f/k/a Salomon Smith Barney, Inc., Citigroup, Inc. and John Henry Spatz, Case No. 03C3157 in the United States District Court for the Northern District of Illinois, Eastern Division. Testified in deposition on liability and loss causation in a claim of alleged securities fraud. March 2005.

American National Bank and Trust Company of Chicago, as Trustee f/b/o Emerald Investments LP, and Emerald Investments LP, an Illinois Partnership v. Allmerica Financial Life Insurance and Annuity Company. Testified in deposition on certain liability aspects in a breach of contracts claim involving certain mutual fund trading strategies. January 2005.

In re WorldCom, Inc. ERISA Litigation, Master File No. 02 Civ. 4816 (DLC) in the United States District Court, Southern District of New York. Testified in deposition on liability aspects of Plaintiffs' damage claims in an ERISA class action. January 2005.

Xilinx Inc. and Subsidiaries v. Commissioner of Internal Revenue Service, Docket Nos. 004142-01 and 00702-03. Testified in U.S. Tax Court on whether grant date value, or certain spread upon exercise, of employee stock options should be considered part of cost sharing pool in a cost sharing arrangement between Xilinx, Inc. and its Irish affiliate. Trial in July 2004. Submitted affidavit in connection with motion to dismiss in June 2002.

Maxtor Corporation v. Koninklijke Philips Electronics N.V., Philips Semiconductors B.V., Philips Semiconductor International B.V., Philips Electronics North America Corporation, Philips Semiconductors, Inc., Philips Semiconductor Manufacturing, Inc., Philips France, Philips Japan, Ltd., and Does 1 through 25, Case No. CV 808650 in the Superior Court of the State of California, County of Santa Clara. Testified in deposition on damages analysis in connection with alleged design failure of a chip used in manufacturing computer hard drives. March 2004.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 92 of 370. PageID #: 18919

NAVIGANT

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Expert Testimony on Record (Continued)

Mid-Continent Federal Savings Bank v. United States of America, Defendant, Case No. 95-472C in the United States Court of Federal Claims. Testified in deposition and at trial in Court of Federal Claims on damages due to alleged breach of contract as a result of Financial Institutions Reform and Recovery Act of 1989. Deposition in April 2002. Trial in July 2003.

Robert F. Flood v. Bessemer Trust Company, N.A.; *Robert G. Vanneman; and Does 1-25 and Stephen Gorosh v. same defendants*. Testified in deposition on alleged damages due to failure to diversify. November 2002.

Christine P. Rales, Plaintiff v. Steven M. Rales, Defendant, Civil Action No. 02DR166-D in the Superior Court of District of Columbia. Testified in deposition on the fair market value of a block of 19.67 million shares of common stock of Danaher Corporation held by Stephen M. Rales. February 2002.

American National Bank and Trust Company of Chicago, as Trustee f/b/o Emerald Investments LP, and Emerald Investments LP, an Illinois Partnership v. AXA Client Solutions, LLC, The Equitable Life Assurance Society of the United States, and AXA Financial, Inc., Case No. 00 C 6786. Testified in deposition on damages due to alleged breach of contract involving certain mutual fund trading strategies. May 2002.

Statewide Savings Bank, S.L.A., Plaintiff v. United States of America, Defendant, Case No. 95-779C in the United States Court of Federal Claims. Testified in deposition on damages due to alleged breach of contract as a result of Financial Institutions Reform and Recovery Act of 1989. February 2002.

Charles T. McCord and Mary S. McCord, Donors, Petitioners v. Commissioner of Internal Revenue, Respondent, Docket No 7048-00 in the U.S. Tax Court. Testified at trial on value of several interests in a limited partnership. May 2001.

Estate of Elma Middleton Dailey, Deceased, K. Robert Dailey, II, Executor, Petitioner v. Commissioner of Internal Revenue, Respondent, Docket Nos 6251-00 and 6262-00 in the U.S. Tax Court. Testified at trial on value of several interests in a limited partnership. May 2001.

John G. Balletto v. Xoom.com, Inc., Case No. 306798 in the Superior Court of California, San Francisco County. Testified in deposition and at trial relating to damages due to alleged breach of contract. January 2001. Deposition in January 2001. Trial in March 2001.

Edison International (1585456), Mission First Financial (1431482), Edison Capital (1417993), Edison Funding Company (1417994), Renewable Energy Capital Company (0715920), Mission Funding Epsilon (1426267) v. California Franchise Tax Board. Testified before the State Board of Equalization on whether Mission First Financial and its parent company SCEcorp formed a unitary business for tax purposes during 1988 to 1990. December 2000.

Framatome Connectors USA Holdings, Inc. and Subsidiaries, et al. v. Commissioner of Internal Revenue Service, Docket No. 5030-98, 9160-99, 118 T.C. 32 (2002), aff'd, 108 Fed. Appx. 683, (2004). Testified in trial on whether Burndy Japan was a controlled foreign corporation of the petitioner for the years 1988, 1989 and 1992 under section 957(a) (2) of the Internal Revenue Code. October 2000.

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Expert Testimony on Record (Continued)

Barry G. Hittner, Receiver of American Universal Insurance Company v. Sequa Corporation, et al., M.D.L. No. 972, C.A. No. 1:92-512. Testified in deposition on the value of a $50 million note backed by certain real estate. June 2000.

Estate of Richie C. Heck, Deceased, Gary Heck, Special Administrator, Plaintiff v. Commissioner of Internal Revenue Service, Defendant. Docket No. 11619-99 in the U.S. Tax Court. Testified at trial on the valuation of a minority interest in F. Korbel & Bros., Inc. June 2000.

American Heritage Bancorp, Plaintiff v. United States of America, Defendant; *Federal Deposit Insurance Corporation, as successor to the rights of Home Federal Savings Bank, Plaintiff Intervenor v. United States of America, Defendant*. Case No. 90-3982C. Testified in deposition on damages due to alleged breach of contract as a result of Financial Institutions Reform and Recovery Act of 1989. May 2000.

Estate of Robert H. Lurie, deceased, Ann Lurie, Executor v. Commissioner of Internal Revenue, Docket No. 22639-94 in the U.S. Tax Court. Testified at trial on whether certain trusts accumulated assets from investments without transfers for inadequate consideration by the deceased. February 1999.

Joseph K. Mitchell, et al., v. Central Investment Corporation, Case No. A9700035, Special Proceedings in the Court of Common Pleas, Hamilton County, Ohio. Testified in deposition regarding fair cash value of a minority position in stock of a Pepsi-Cola bottling company in connection with a freeze-out merger. March 1998.

Walter L. Gross, Jr. & Barbara H. Gross, Petitioners v. Commissioner of Internal Revenue Service, Respondent, No. 4460-97; Calvin C. Linnemann & Patricia G. Linnemann, Petitioners v. Commissioner of Internal Revenue Service, Respondent, No. 4469-97. Testified at trial regarding the fair market value of a minority interest in a Pepsi-Cola bottling company, which Petitioners had claimed as a gift-tax liability. November 1997.

R. J. R. Nabisco Inc. and Subsidiaries v. Commissioner, Docket No. 3796-95 in the U.S. Tax Court. Testified at trial regarding the nature and useful economic life of cigarette package design for federal income taxation purposes. February 1997.

Clinton, Inc. & Subsidiaries, Petitioner v. Commissioner of Internal Revenue, Respondent, Docket No. 9885-95 in the U.S. Tax Court. Testified in deposition regarding reasonable executive compensation pursuant to Internal Revenue Code Section 162 (a) (1). August 1996.

Fullers Jewelry, Inc., Plaintiff v. Dallas Central Appraisal District, Defendant, Case No. 94-09169-B, District Court, Dallas County, Texas, 44th Judicial District. Testified in deposition regarding the value of merchandise inventory of a retail jewelry chain for purposes of ad valorem taxation. July 1996.

American Marazzi, Inc., Plaintiff v. Dallas Central Appraisal District, Defendant, Cause number 95-07028-B, District Court, Dallas County, Texas, 44th Judicial District. Testified in deposition regarding the value of merchandise inventory of a manufacturer and distributor of ceramic tiles for purposes of ad valorem taxation. June 1996.

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Expert Testimony on Record (Continued)

Terence Dean, et al. v. Dean Security, Inc. et al. Testified in binding arbitration regarding damages due to breach of contract for the sale of a security company. March 1996.

Advertiser's Dynamic Services, Co., Inc., Plaintiff v. United States of America, Defendant, Case No. 3-94-CV-2079-G, District Court, North Dallas, Texas. Testified at trial regarding the nature of contracts between a publisher and salespersons for payroll tax purposes. February 1996.

Publications / Working Papers

Publications

"Economic Consequences: The Real Cost of U.S. Securities Class Action Litigation," 2014, with Nikolai Caswell, Anand Goel, Sumon C. Mazumdar and Rahul Surana, issued by Institute for Legal Reform, U.S. Chamber of Commerce.

"Assessing Market Efficiency for Reliance on the Fraud-on-the-Market Doctrine after *Wal-Mart* and *Amgen*," with Sumon C. Mazumdar and Daniel A. McLaughlin, 2014, in James Langenfeld Ed., *Research in Law and Economics,* Volume 26, 161-207.

"The NUA Benefit and Optimal Investment in Company Stock in 401(K) Accounts," with Sumon C. Mazumdar, Vikram Nanda and Rahul Surana, 2009, in A. H. Chen Ed., *Research in Finance,* Volume 25, 203–227.

"Competition in IPO Underwriting: Time Series Evidence," with Andrew H. Chen and Sumon C. Mazumdar, 2008, in A. H. Chen Ed., *Research in Finance*, Volume 24, 1–25.

"A Matrix-Based Lattice Model to Value Employee Stock Options," with Sumon C. Mazumdar, Rahul Surana and Sanjay Unni, 2006, *Journal of Derivatives* 14, 9–26.

"Mean Reversion in Earnings and the Use of E/P Multiples in Corporate Valuation," with David Denis and Atulya Sarin, 2004, *Journal of Applied Finance* 14, 4–10.

"Securities Class Action Settlements: An Empirical Analysis," with Sumon C. Mazumdar and Atulya Sarin, 2003, *Santa Clara Law Review* 43, 1001–1033.

"Ownership Structure, Agency Costs and Dividend Policy," with Anand M. Vijh and Randolph W. Westerfield, 2002, in A. H. Chen Ed., *Research in Finance*, Volume 19, 1–28.

"The Cost of Raising Preferred Equity Capital," with Sumon C. Mazumdar and Atulya Sarin, 2002, *Journal of Financial Research* 25, 577–592.

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Publications / Working Papers (Continued)

Publications (Continued)

"Firm Value and Marketability Discounts," with David J. Denis, Stephen P. Ferris and Atulya Sarin, 2001, *Journal of Corporation Law* 27, 89–115.

"Transfer Pricing and Foreign Exchange Risk," with Brian Becker and Jonathan Neuberger, *Transfer Pricing Report*, July 1999.

"Dividend Omissions and Forecasts of Future Earnings: Some Positive Evidence on Information Content of Dividends," 1999, in A. H. Chen Ed., *Research in Finance*, Volume 17, 13–39.

"The Relationship Between Ownership, Financing Decisions and Firm Performance: A Signaling Model," with Sudipto Dasgupta and Yuk-She Chan, 1998, *International Economic Review* 39, 723–744.

"Valuation for Smaller Capitalization Companies," with Scott D. Hakala, 1998, in *Financial Valuation: Business and Business Interests* – 1998 update, Warren, Gorham & Lamant.

"Trading Behavior and the Unbiasedness of the Market Reaction to Dividend Announcements," with Anand M. Vijh, 1995, *Journal of Finance* 50. (Abstract reprinted in *Financial Management Collection*, 1996.)

"Beyond Mere Compliance," with Anita S. Agarwal, *Mortgage Banking*, April 1993, 57–61.

"Dividend Clienteles and the Information Content of Dividend Changes," with Anand M. Vijh, 1990, *Journal of Financial Economics* 26, 193–219.

"The Efficient and Inefficient Media for Political Campaign Advertising," with Roland T. Rust and George T. Haley, 1984, *Journal of Advertising* 13, 45–49.

"Modeling of Non-Ideal Residence Time Distribution in a Continuous Flow Stirred Tank Reactor at Zero RPM," with D. Prasanna Rao, 1983, *Indian Chemical Engineer*, 24–27.

"Alternate Criteria for the Comparison of Regression Models," with Roland T. Rust, presented in *Proceedings of the Southwestern Marketing Association*, Spring 1983.

Working Papers

"ESO Expensing Under the Revised FAS 123: A Practitioner's Guide," with Sumon C. Mazumdar, Sanjay Unni, and Anand Vijh, September 2005.

"Investment in Company Stock in 401(k) Accounts," with Sumon C. Mazumdar, May 2005.

"The New Accounting Rules for ESO Expensing: Not Quite As Easy As 123," with Sumon C. Mazumdar and Sanjay Unni, March 2005.

Case: 4:08-cv-00160-BYP Doc #: 385-2 Filed: 09/01/17 98 of 370. PageID #: 18923

Mukesh Bajaj, Ph.D.

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Publications / Working Papers (Continued)

Working Papers (Continued)

"Competition in IPO Underwriting: Time Series Evidence," with Andrew H. Chen, Sumon C. Mazumdar and Atulya Sarin, March 2003.

"Auditor Compensation and Audit Failure: An Empirical Analysis," February 2003.

"The Offer Yield of Preferred Stock," with Sumon C. Mazumdar and Atulya Sarin, March 2002.

"Signaling, Agency Costs and Ownership Structure: Theory and Empirical Implications," with Sudipto Dasgupta and Yuk–Shee Chan, mimeo, Hong Kong University of Science and Technology, 1997.

"Is it Appropriate to Use a Higher Discount Rate to Value Small Firms?" with Martin Hanan, Rick Knoll, and Mark Mitchell.

"Turnover in Equity Ownership, Risk and Return," September 1995.

Professional Affiliations

Member, American Finance Association
Member, Western Finance Association
Member, Financial Management Association
Member, European Finance Association

Appendix II

Appendix II: Documents Considered

Legal Documents

1. *OPERS On Behalf of Itself and all Others Similarly Situated, (collectively, "Plaintiff"), vs. Federal Home Loan Mortgage Corporation a/k/a Freddie Mac, Richard F. Syron, Patricia L. Cook, Anthony S. Piszel, And Eugene M. McQuade (collectively "Defendants").* Third Amended Complaint For Violations Of Federal Securities Laws, United States District Court, Northern District Of Ohio, Eastern Division (Youngstown) Civil Action No. 4:08-cv-160, filed March 28, 2012.
2. Memorandum of Opinion and Order, *Ohio Public Employees Retirement Sys. v. Federal Home Loan Mortgage Corp., et al.*, No. 4:08-cv-160, October 31, 2014.
3. *Ohio Public Employees Retirement Sys. v. Federal Home Loan Mortgage Corp., et al.*, No. 14-4189, 2016 (6th Cir. Jul. 20, 2016).
4. Memorandum In Support Of Lead Plaintiff's Renewed Motion For Class Certification And To Appoint Class Representative And Class Counsel, June 12, 2017.

Declarations, Expert Reports and Testimonies

1. Expert Report of Greg Hallman, *In Re Freddie Mac Securities Litigation*. 4:08-cv-160, August 16, 2012.
2. Expert Report of Mukesh Bajaj, *Ohio Public Employees Retirement System, V. Federal Home Loan Mortgage Corporation A/K/A Freddie Mac, Et Al.*, Civil Action No. 4:08-cv-160, December 14, 2012.
3. Declaration of Professor Steven P. Feinstein, *Ohio Public Employees Retirement System, V. Federal Home Loan Mortgage Corporation A/K/A Freddie Mac, et al.*, Civil Action No. 4:08-cv-160, December 16, 2016.
4. Report on Market Efficiency, Professor Steven P. Feinstein, *Ohio Public Employees Retirement System, V. Federal Home Loan Mortgage Corporation A/K/A Freddie Mac, Et Al.*, Civil Action No. 4:08-Cv-160, June 7, 2017.
5. Deposition of Steven Feinstein on August 10, 2017 and August 17, 2017.

Academic Sources

1. Abdi, Herve (2010), "Holm's Sequential Bonferroni Procedure," *The University of Texas at Dallas.*
2. Anderson, David R. et. al. (1984), *Statistics for Business and Economics*, 2nd edition, West Publishing
3. Arslanalp, Serkan and Peter Blair Henry (2005), "Is Debt Relief Efficient?," *The Journal of Finance*, vol. 60(2), pages 1017-1051
4. A. R. Ansari and R. A. Bradley (1960), "Rank-Sum Tests for Dispersions," *Ann. Math. Statist.* 31(4), pages 1174-1189
5. Bajaj et al. (2014), "Assessing Market Efficiency For Reliance On The Fraud-on-the-Market Doctrine After Wal-Mart and Amgen," *Research in Law and Economics* (26), 161-207
6. Below, Scott D. and Keith H. Johnson (1996), "An Analysis Of Shareholder Reaction To Dividend Announcements In Bull And Bear Markets," *Journal of Financial And Strategic Decisions*, vol. 9(3), pages 15 -26

7. Bhattacharya, Gouri K. and Richard A. Johnson (1977), *Statistical Concepts and Methods*, Wiley
8. Bodie, Zvi, Alex Kane and Alan J. Marcus (2011), *Investments*, McGraw-Hill Irwin, New York, NY
9. Bommel, J. van, and T. Vermaelen (2003), "Post-IPO capital expenditures and market feedback," *Journal of Banking & Finance*, vol. 27, pages 275–305
10. Brav, Alon and J.B. Heaton (2015), "Event Studies in Securities Litigation: Low Power, Confounding Effects, And Bias," W*ashington University Law Review* 93(2)
11. Brunnermeier, Markus, and Lasse H. Pedersen, 2009, "Market Liquidity and Funding Liquidity," *Review of Financial Studies* 22, page 2201-2238
12. Devore, Jay, *Probability and Statistics for Engineering and the Sciences*, 9th edition, Cengage Learning (2016), chapter 9
13. Erenburg, Grigori, Janet Kiholm Smith and Richard L. Smith (2011), "The Paradox of 'Fraud-on-the-Market Theory': Who Relies on the Efficiency of Market Prices?" *Journal of Empirical Legal Studies* 8(2), pages 260-303
14. Fama, Eugene F. (1965), "Random Walks in Stock Market Prices," *Financial Analysts Journal* 21(5), page 55-59.
15. Fama, Eugene F. (1970), "Efficient Capital Markets: A Review of Theory and Empirical Work," *Journal of Finance* 25(2), pages 383-417
16. Fama, Eugene F. (1991), "Efficient Capital Markets: II," *The Journal of Finance* 46(5), pages 1575-1617
17. Fedenia, Mark and Mark Hirschey, (2009), "The Chipotle Paradox," *Journal of Applied Finance*, Issues 1 & 2, pages 1-16
18. Ferrillo, Paul A., Frederick C. Dunbar, and David Tabak (2004), "The 'Less Than' Efficient Capital Markets Hypothesis: Requiring More Proof from Plaintiffs in Fraud-on-the-Market Cases," *St. John's Law Review* 78(1), page 81-129
19. Fischel, Daniel R. (1982), "Use of Modern Finance Theory in Securities Fraud Cases Involving Actively Traded Securities," *The Business Lawyer* 38
20. Fleiss, Joseph; Levin, Bruce and Cho Paik, Myunghee (2003), "Statistical Methods for Rates and Proportions", *Wiley-Interscience*
21. Greene, William H. (2012), *Econometric Analysis*, 7th edition, NJ: Prentice Hall
22. Gorton, Gary and Andrew Metrick, "Securitized Banking and the Run on Repo," *Yale University working paper*, November 9, 2010
23. Gorton, Gary and Andrew Metrick, "Haircuts," *Yale University working paper*, May 2010
24. Hartzmark, Michael L. and H. Nejat Seyhun (2011), "The Curious Incident of the Dog that Didn't Bark and Establishing Effect-and-Cause in Class Action Securities Litigation," *University of Michigan-Ann Arbor*, July 29, 2011
25. Henry, Peter Blair (2002), "Is Disinflation Good for the Stock Market?," *The Journal of Finance*, vol. 57(4), pages 1617-1648
26. Kennedy, Peter, *A Guide to Econometrics*, 5th ed. MIT Press
27. Kaye and Freedman, "*Reference Guide on Statistics*", selection from "*Reference Manual on Scientific Evidence*," (2011) Third Edition
28. Kritzman, Mark P. (1994), "What Practitioners Need to Know About Event Studies," *Financial Analysts Journal*

29. Lamont, Owen, and Richard Thaler, (2003a), "Can the Market Add and Subtract? Mispricing in Tech Stock Carve-Outs," *Journal of Political Economy* 111
30. Lamont, Owen A. and Richard H. Thaler (2003b), "The Law of One Price in Financial Markets," *Journal of Economic Perspectives*, vol. 17(4), pp. 191-202
31. Langevoort, Donald C. (2007), "Basic at Twenty: Rethinking Fraud-on-the-Market," *Georgetown University Law Center*
32. Lo, Andrew and A Craig MacKinlay, "Data-Snooping Biases in Tests of Financial Asset Pricing Models," *Review of Financial Studies*, 1990, vol. 3, issue 3, 431-6
33. Lomax et al. (2012), *Statistical Concepts: A Second Course*, 4th ed., Taylor & Francis
34. Macfie, Brian P. and Philip M. Nufrio (2006), *Applied Statistics For Public Policy*, M.E. Sharpe, chapter 13
35. MacKinlay, A.C., 1997, "Event Studies in Economics and Finance," *Journal of Economic Literature* 35
36. Mitchell, Mark L., and Jeffry M. Netter (1994), "The Role of Financial Economics in Securities Fraud Cases: Applications at the Securities and Exchange Commission," *The Business Lawyer* 49, pages 545-590
37. Mitchell, Mark, Todd Pulvino, And Erik Stafford, 2002, "Limited Arbitrage in Equity Markets," *The Journal of Finance* 57 (2), pages 551 – 584
38. M.J. Fryer, A Note on Barnard's Modification to Wald's Sequential Test for Double Dichotomies, *Journal of the Royal Statistical Society*, Series B (Methodological), Vol. 34, No. 1 (1972), 126-29
39. Shleifer, Andrei and Robert W. Vishny (1997), "The Limits of Arbitrage," *The Journal of Finance* 52(1), page 35-55
40. Tabak, David (2010), "Use and Misuse of Event Studies to Examine Market Efficiency," *NERA Working Paper*
41. Tabak, David (2016), "What Should We Expect When Testing for Price Response to News in Securities Litigation?" *NERA Working Paper*
42. Tabak, David I. and Frederick C. Dunbar (2001), "Materiality and Magnitude: Event Studies in the Courtroom," in *Litigation Services Handbook: The Role of the Financial Expert*, Third Edition, ed. by Roman L. Weil, Michael J. Wagner and Peter B. Frank, John Wiley & Sons, Inc. USA henceforth, "Tabak and Dunbar (2001)" at 19-2.

Analyst Reports

1. "Freddie Mac (FRE-$26.74-Peer Perform): GAAP Loss Reflects Reduced Market Liquidity and Is Restricting Liquidity As Well," *Bear Stearns & Co.*, November 20, 2007
2. "Losses Spur Need for New Capital; Charter/Mission Intact," *Citi*, November 20, 2007
3. "FNM and FRE: Capital Constrained – Downgrading to Underperform," *Fox-Pitt, Kelton, Inc.*, November 20, 2007
4. "Freddie Mac (FRE): We think capital raise will be dilutive to common shareholders," *Fox-Pitt, Kelton, Inc.*, November 20, 2007
5. "Freddie Mac: Reports 3Q07 Loss," *Morgan Stanley*, November 20, 2007
6. "First Read: Freddie Mac, Placing Under Review as We Clarify Credit and Capital Outlook," *UBS*, November 20, 2007
7. "Freddie Mac (FRE - US$ 26.74) 2-Equal weight Change of Earnings Forecast," *Lehman Brothers*, November 21, 2007

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 101 of 370. PageID #: 18928

8. "Fitch Places Freddie Mac's 'AA-' Preferred Stock on Rating Watch Negative," *Fitch Ratings*, November 20, 2007
9. "FNM and FRE Price Action-Capital Levels and Even More Threatened Business Models," *Miller Tabak*, November 20, 2007
10. "Freddie Mac: Strong Retained Portfolio Growth in August," *Credit Suisse*, November 25, 2007
11. "Federal Home Loan Mortg Corp (FRE) – NYSE," *Price Target*, November 25, 2007
12. "GSE Update: Downgrading GSEs as Credit Pressures Erode EPS and Dividend Outlook," *UBS Investment Research*, November 26, 2007
13. "Freddie Mac: Announces $6B Preferred Stock Offering; Cuts 4Q Dividend by 50%," *UBS Investment Research*, November 27, 2007
14. "Freddie Mac (FRE-$24.50-Peer Perform)," *Bear Stearns & Co.*, November 27, 2007
15. "Freddie Mac (FRE): Divided Halved and $6 Billion of Preferred Raised Stabilizes Current Capital Position," *Credit Suisse*, November 27, 2007
16. "Mortgage Strategist: Highlights & Recommendations," *UBS Investment Research*, November 27, 2007

Other Cases

1. Brief of Financial Economists as Amici Curiae in Support of Respondents, Halliburton Co. and David Lesar v., Erica P. John Fund, Inc., February 5, 2014
2. *In re PolyMedica Corp. Securities Litigation*, (1st Cir. 12/13/2005), 1st Cir., 05-1220.
3. *Basic Inc. et al. v. Levinson et al.* 485 U.S. 224 No. 86-279 (1988).
4. *Cammer v. Bloom*, 711 F. Supp. 1264 (D.N.J. 1989).
5. *Krogman v. Sterritt*, 202 F.R.D. 467 (N.D. Tex. 2001).
6. Deposition of Steven Feinstein on February 12, 2014, *In Re: Groupon, Inc. Securities Litigation* Case No. 12 Cv 24050
7. *In re Federal Home Loan Mortgage Corp.*, No. 09 Civ. 832 (MGC), Opinion and Order, March 27, 2012.
8. Halliburton Co. v. Erica P. John Fund, Inc., 134 S. Ct. 2398, 2403, 189 L. Ed. 339 (2014).
9. Opinion, *In Re Polymedica Corporation Securities Litigation*, 453 F. Supp. 2d 260 (D. Mass. 2006).
10. Deposition of Steven Feinstein on July 22, 2011, *In Re American International Group, Inc. 2008 Securities Litigation.*
11. Declaration and Report of Professor Steven P. Feinstein, *City Of Sterling Heights General Employees' Retirement System, et al. vs. Prudential Financial, Inc., et al.*, December 8, 2014.
12. Report On Market Efficiency Professor Steven P. Feinstein, *Alaska Electrical Pension Fund, et al v. Pharmacia Corporation* No. 3:09-1519, June 6, 2011
13. Report On Market Efficiency Professor Steven P. Feinstein, *Claude A. Reese* et al. *v. B.P. plc et al.* No. C08-1008 MJP, March 5, 2015
14. Report on Market Efficiency Professor Steven P. Feinstein, *Silver Wheaton* No. 2:15-cv-05146-CAS, July 8, 2015
15. Report on Market Efficiency Professor Steven P. Feinstein, *Delcath Systems, Inc* No. 13 CIV.3116 (LGS), February 4, 2015

16. Report on Market Efficiency Professor Steven P. Feinstein, *Franz Schleicher* et al No. 02 CV 1332 DFH-TAB, May 2, 2008
17. Report on Market Efficiency Professor Steven P. Feinstein, *The Children's Place Retail* No. 1:07-cv-08252-SAS, November 14, 2008
18. Report on Market Efficiency Professor Steven P. Feinstein, *Groupon, Inc* No. 12 CV 2450, December 4, 2013
19. Report on Market Efficiency Professor Steven P. Feinstein, *JP Morgan* No. 1:12-cv-03852-GBD, February 13, 2015
20. Report on Market Efficiency Professor Steven P. Feinstein, *KBR, Inc* No. 4:14-CV-01287, February 19, 2016
21. Report on Market Efficiency Professor Steven P. Feinstein, *CVS Caremark Corp* No. 1:09-cv-00554-S-DLM, February 6, 2015
22. Report on Market Efficiency Professor Steven P. Feinstein, *Las Vegas Sands* No. 2:10-cv-00765-KJD-GWF, November 7, 2014
23. Report on Market Efficiency Professor Steven P. Feinstein, *Puma Biotechnology, Inc* No. 8:15-cv-00865-AG-JLG, March 15, 2017
24. Report on Market Efficiency Professor Steven P. Feinstein, *Questcor Pharmaceuticals* No. 8:12-cv-01623-DMG(JPRx), August 4, 2014
25. Report on Market Efficiency Professor Steven P. Feinstien, *Proquest Company* No. 2:06-cv-10619, April 11, 2008
26. Report on Market Efficiency Professor Steven P. Feinstein, *Symbol Technologies, Inc* No. 05-CV-3923-DRH, August 22, 2014
27. Report on Market Efficiency Professor Steven P. Feinstein, *Walter Energy, Inc* No. 2:12-cv-00281-VEH, August 15, 2013
28. Report on Market Efficiency Professor Steven P. Feinstein, *World Acceptance Corp* No. 6:14-cv-01606
29. Report on Market Efficiency Professor Steven P. Feinstein, *In Re: Eletrobras Securities Litigation* No. 15-Cv-5754-JGK, June 30, 2017.
30. Report on Market Efficiency Professor Steven P. Feinstein, *Edna Selan Epstein et al. v. World Acceptance Corporation et al.* Civil Action No. 6:14-cv-01606-MGL, September 28, 2016.
31. Report On Market Efficiency Professor Steven P. Feinstein, *In Re: Petrobras Securities Litigation*, October 15, 2015.

News Articles

1. Rucker, Patrick, and Lynn Adler, "WRAPUP 1-Fannie, Freddie to resume current financials," *Reuters News*, 8 June 2007, 12:43.
2. "Freddie Mac says H2 net income $2.7 billion," *Reuters News*, October 3, 2006.
3. "Today's Agenda / A look ahead at newsworthy events," *The Wall Street Journal*, January 5, 2007.
4. "Freddie Mac Issues Q1 2006 Net Income Guidance Above Analysts' Estimates-Conference Call", *Reuters Significant Developments*, August 1, 2006.
5. Burton Frierson, "TREASURIES-Bonds gain on subprime troubles and Iran worry," *Reuters*, February 26, 2007, 09:29.
6. Tim Annett, "The Evening Wrap: From Bad to Worse," *The Wall Street Journal*, February 27, 2007.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 103 of 370. PageID #: 18930

7. "THE FED: Bernanke Says Optimism Not Shaken By Data Or Market," *Dow Jones Business News*, February 28, 2007 12:38.
8. Marcy Gordon, "Freddie Mac Predicts 3Q, 4Q Losses," *Associated Pres*s, January 5, 2007.
9. "Hedge funds in trouble: The incredible shrinking funds," *The Economist*, October 23, 2008.

Freddie Mac

1. "FRE - Freddie Mac Market Update," *Thomson Street Events*, October 3, 2006 8:30 AM EST.
2. "FRE – Freddie Mac Market Update," *Thomson Street Events*, January 5, 2007 8:30AM.
3. Freddie Mac Market Update, *Thomson Newswire*, August 1, 2006 04:30PM.
4. "FRE - Q3 2007 Freddie Mac Earnings Conference Call," *Thomson StreetEvents*, November 20, 2007
5. "Freddie Mac Reports Third Quarter 2007 Net Loss of $2.0 Billion or $3.29 per diluted share," *Freddie Mac Press Release*, November 20, 2007.

Miscellaneous Sources

1. "8.4.2 - Hypothesis Testing," https://onlinecourses.science.psu.edu/stat200/node/61, accessed 8/29/17.
2. "CBOE Volatility Index (VIX)," www.cboe.com/micro/VIX/vixintro.aspx, accessed 10/31/12.
3. SAS STAT manual.
4. Chairman Ben S. Bernanke, "The Recent Financial Turmoil and its Economic and Policy Consequences", At the Economic Club of New York, New York, New York October 15, 2007.
5. 2017-08-11 Crowninshield_FreddieMac_CollectiveTest_RobustnessChecks.pdf.
6. Richard A. DeFusco, CFA, Dennis W. McLeavey, CFA, Jerald E. Pinto, CFA, and David E. Runkle, CFA Second Edition, Reading 10.

Data Sources

1. Bloomberg L.P.
2. OptionMetrics.
3. Factiva.

All other data, documents and sources cited/mentioned in the report and appendices.

Appendix III

Appendix III: Application of Event Study Method

1. The event study method is a widely-accepted technique[1] used by financial economists to analyze the movement of stock prices in response to a specific event or announcement.[2] An event study identifies unanticipated and potentially economically material news and then examines contemporaneous stock price reaction to determine whether the selected unexpected events elicit a response in stock price.[3]

2. The first step in an event study is to identify the "events" through a review of public information from various sources (including company press releases and presentations, newspapers and other publications, websites, newswires and multimedia sources) as well as analyst reports.

3. The next step in an event study is to establish the event window, which is the period over which the potential impact of news on the stock's price is assessed.[4] The event window is essentially the period of time that the information (or event) being analyzed is assumed to be impounded in the stock's price. Typically, for actively traded stocks, financial economists conduct event studies using a one-day event window, *i.e.*, we look at daily stock price returns in response to news events from close of trading on a trading day to the close of trading on the next trading day.

4. The third step in an event study is to analyze the company's stock price reaction to those events. In studying the impact of news on stock price, it is standard practice in event studies to first isolate the effect of market and industry factors on stock returns using a statistical method known as regression analysis. As

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[1] Between 1974 and 2000, more than 500 event studies were published in five leading finance journals. See S.P. Kothari and J.B. Warner (2006), "Econometrics of Event Studies," in *Handbook of Empirical Corporate Finance*, ed. B. Espen Eckbo, North-Holland: Elsevier.

[2] The event study methodology and its various applications have been reviewed in detail by A. Craig MacKinlay in "Event Studies in Economics and Finance," *Journal of Economic Literature*, Vol. XXXV, March 1997, 13-39. For the application of this methodology to securities cases see Mitchell, Mark and Jeffry Netter (1994), "The Role of Financial Economics in Securities Fraud Cases: Applications at the Securities and Exchange Commission," *The Business Lawyer*, February 1994; Cornell, B. and G. Morgan (1990), "Using Finance Theory to Measure Damages in Fraud on the Market Cases," *UCLA Law Review*, 37, No. 2, 883-924; and Tabak, David I. and Frederick C. Dunbar (2001), "Materiality and Magnitude: Event Studies in the Courtroom," *Litigation Services Handbook: The Role of the Financial Expert,* Third Edition, Roman L. Weil, Michael J. Wagner, and Peter B. Frank, John Wiley & Sons, Inc., Chapter 19 (henceforth, "Tabak and Dunbar (2001)").

[3] As discussed in the main body of the report, in an efficient market a stock's price should react quickly to impound new value-relevant information.

[4] Tabak and Dunbar (2001), page 19-4.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 108 of 370. PageID #: 18983

Bodie *et al.* notes, a stock's price on any day could be affected by "a wide range of economic news such as updated forecasts for GDP [Gross Domestic Product], inflation rates, interest rates, or corporate profitability."[5] Therefore, "[i]solating the part of a stock price movement that is attributable to a [company-] specific event is not a trivial exercise."[6]

5. A regression analysis is used to calculate the relationship between changes in a company's stock price and corresponding changes in the value of a market index and an industry index to account for industry-specific factors. This relationship is estimated by running a regression of the stock's returns on market and industry index returns (which is also called the "Market Model") over a period referred to as the "estimation window." The market model and the actual performance of the associated indices are then used to compute the "predicted return" for the stock.

6. The next step in an event study is to calculate the security's "abnormal return," (also referred to as "excess return," or "market-adjusted return") by subtracting the predicted return (using the market model) from the security's observed return. In other words, I calculate abnormal (or "excess") returns by estimating "a predicted stock price return, or percentage change, from the day before the news reaches the market to the day the stock price assimilates the news[,]" using a regression "model that takes into account market and industry effects on stock price returns … [and subtracting] the predicted return from the actual return to compute the so-called abnormal return."[7] A central assumption in using regression models to predict returns is that the volatility remains unchanged over the relevant period.[8]

7. After calculating the stock's abnormal return, I assess its statistical significance to determine if the observed abnormal return is statistically significant relative to normal day-to-day variability reflecting vicissitudes of stock prices.[9] As Tabak and Dunbar (2001) note: "[t]ypically, the predicted return does not

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5 Bodie, Zvi, Alex Kane and Alan J. Marcus, (2011), *Investments*, McGraw-Hill Irwin, New York, NY; henceforth "Bodie *et al.*" at page 353.

6 Bodie *et al.*, page 353.

7 Tabak, David I. and Frederick C. Dunbar, "Materiality and Magnitude: Event Studies in the Courtroom," in *Litigation Services Handbook: The Role of the Financial Expert*, Third Edition, ed. Roman L. Weil, Michael J. Wagner and Peter B. Frank, John Wiley & Sons, Inc. USA, 2001 ("Tabak and Dunbar (2001)").

8 Kennedy, Peter, *A Guide to Econometrics*, 5th edition. MIT Press, pages 47-48.

9 The abnormal or "excess" return could occur as a result of certain company-specific events or purely as a matter of chance, i.e., due to the normal random fluctuations in the stock's price. The excess return is considered "statistically significant" when the likelihood that it occurred due to random fluctuations – and not because of some company-specific event – is 5% or less (the "5% significance level"). The statistical significance is typically assessed by a "t-statistic." In an event study, the t-statistic is a ratio equal to the

exactly equal the actual return even when no event has occurred. To decide whether the difference between the actual and the predicted return [the abnormal return] … results merely from chance, one tests… for statistical significance[.]"[10]

8. When the abnormal return is statistically significant, the study supports a conclusion that the identified news reasonably explains the observed abnormal stock price movement, and therefore that the information was value relevant to investors and hence was economically material.

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estimated excess return divided by its "standard error" (a measure of error in estimating the excess return). A t-statistic of 1.96 or greater (in absolute value) is considered statistically significant at the 5% significance level. On the other hand, if the likelihood that the excess return occurred purely as a matter of chance is more than 5%, the excess return is considered "statistically insignificant."

A test of statistical significance is typically performed by checking whether the test statistic is too extreme in its deviation from a distribution. If the objective of the event is to test whether the returns are significantly different than zero (i.e., either significantly higher than or significantly lower than zero) then a 2-tailed test is appropriate. In the 2-tailed test, the analyst is agnostic to the hypothesized price direction, and, using a 95% level, a t-statistic with a value greater than 1.96, or less than -1.96, is considered statistically significant. A 2-tailed test is typically not appropriate where the direction of the price movement is hypothesized as an aspect of the test, for example, where one is looking specifically for positive statistically significant movements (i.e., price increases).

In such cases a 1-tailed test, wherein the deviation is in one direction compared to the mean of the distribution – for example, too large or too small – is appropriate. Robert Stine and Dean Foster, "Statistics for Business," 2nd Edition, Pearson, 2014, at 386-387. The 1-tailed test is appropriate when the direction of the deviation from the mean is specified beforehand, for example, in order to test if one drug is significantly better than a placebo. *See, e.g.*, abstract of J.E. Overall, "Tests of one-sided versus two-sided hypotheses in placebo-controlled clinical trials," *Neuropsychopharmacology* Vol. 3 No. 4 (Aug. 1990): 233-5 ("The p value associated with a test of significance is supposed to represent the probability of observed results given that the null hypothesis is actually true. In evaluating the efficacy of a new drug against placebo, regulatory considerations focus on superiority of the new drug over placebo… Thus, the only probability of concern is the probability that apparent superiority of drug over placebo is a chance finding, and that is the probability associated with a one-tailed test.") (available at: https://www.ncbi.nlm.nih.gov/pubmed/2400542). In that case, using the test to identify drugs that perform significantly better or significantly worse than a placebo would be meaningless and incorrect. In a 1-tailed t-test, a t-statistic of 1.65 is statistically significant at a 95% level of confidence.

10 Tabak and Dunbar (2001), page 19-3.

Appendix IV

Appendix IV: Number of Exact Statistically Significant News Days Needed to get a Z-Statistic of 1.65 per Dr. Feinstein's Z-test by Varying News Days

[A]	[B]	[B] / [A] x 100
Number of News Days	***Number of Exact Significant News Days***	***Percentage***
3	0.933	31%
4	1.114	28%
5	1.281	26%
6	1.438	24%
7	1.588	23%
8	1.731	22%
9	1.870	21%
10	2.005	20%
11	2.137	19%
12	2.265	19%
13	2.391	18%
14	2.514	18%
15	2.635	18%
16	2.755	17%
17	2.872	17%
18	2.988	17%
19	3.102	16%
20	3.215	16%
21	3.327	16%
22	3.437	16%
23	3.547	15%
24	3.655	15%
25	3.762	15%
26	3.868	15%
27	3.974	15%
28	4.078	15%
29	4.182	14%
30	4.285	14%
31	4.387	14%
32	4.489	14%
33	4.590	14%
34	4.690	14%
35	4.790	14%
36	4.889	14%
37	4.987	13%
38	5.085	13%
39	5.182	13%
40	5.279	13%

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 110 of 370. PageID #: 18937

Appendix IV: Number of Exact Statistically Significant News Days Needed to get a Z-Statistic of 1.65 per Dr. Feinstein's Z-test by Varying News Days

[A]	[B]	[B] / [A] x 100
Number of News Days	***Number of Exact Significant News Days***	***Percentage***
41	5.375	13%
42	5.471	13%
43	5.566	13%
44	5.661	13%
45	5.756	13%
46	5.850	13%
47	5.943	13%
48	6.036	13%
49	6.129	13%
50	6.221	12%
51	6.313	12%
52	6.405	12%
53	6.496	12%
54	6.587	12%
55	6.678	12%
56	6.768	12%
57	6.858	12%
58	6.947	12%
59	7.037	12%
60	7.126	12%
61	7.214	12%
62	7.303	12%
63	7.391	12%
64	7.478	12%
65	7.566	12%
66	7.653	12%
67	7.740	12%
68	7.827	12%
69	7.913	11%
70	7.999	11%
71	8.085	11%
72	8.170	11%
73	8.256	11%
74	8.341	11%
75	8.426	11%
76	8.510	11%
77	8.595	11%
78	8.679	11%

Appendix IV: Number of Exact Statistically Significant News Days Needed to get a Z-Statistic of 1.65 per Dr. Feinstein's Z-test by Varying News Days

[A]	[B]	[B] / [A] x 100
Number of News Days	***Number of Exact Significant News Days***	***Percentage***
79	8.763	11%
80	8.847	11%
81	8.930	11%
82	9.013	11%
83	9.096	11%
84	9.179	11%
85	9.262	11%
86	9.344	11%
87	9.427	11%
88	9.509	11%
89	9.590	11%
90	9.672	11%
91	9.753	11%
92	9.835	11%
93	9.916	11%
94	9.996	11%
95	10.077	11%
96	10.157	11%
97	10.238	11%
98	10.318	11%
99	10.398	11%
100	10.477	10%
101	10.557	10%
102	10.636	10%
103	10.715	10%
104	10.794	10%
105	10.873	10%
106	10.952	10%
107	11.030	10%
108	11.109	10%
109	11.187	10%
110	11.265	10%
111	11.342	10%
112	11.420	10%
113	11.497	10%
114	11.575	10%
115	11.652	10%
116	11.729	10%

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 112 of 370. PageID #: 18959

Appendix IV: Number of Exact Statistically Significant News Days Needed to get a Z-Statistic of 1.65 per Dr. Feinstein's Z-test by Varying News Days

[A]	[B]	[B] / [A] x 100
Number of News Days	***Number of Exact Significant News Days***	***Percentage***
117	11.806	10%
118	11.882	10%
119	11.959	10%
120	12.035	10%
121	12.111	10%
122	12.187	10%
123	12.263	10%
124	12.339	10%
125	12.414	10%
126	12.490	10%
127	12.565	10%
128	12.640	10%
129	12.715	10%
130	12.790	10%
131	12.865	10%
132	12.939	10%
133	13.013	10%
134	13.088	10%
135	13.162	10%
136	13.236	10%
137	13.309	10%
138	13.383	10%
139	13.456	10%
140	13.530	10%
141	13.603	10%
142	13.676	10%
143	13.749	10%
144	13.822	10%
145	13.894	10%
146	13.967	10%
147	14.039	10%
148	14.111	10%
149	14.183	10%
150	14.255	10%
151	14.327	9%
152	14.398	9%
153	14.470	9%
154	14.541	9%

Appendix IV: Number of Exact Statistically Significant News Days Needed to get a Z-Statistic of 1.65 per Dr. Feinstein's Z-test by Varying News Days

[A]	[B]	[B] / [A] x 100
Number of News Days	***Number of Exact Significant News Days***	***Percentage***
155	14.612	9%
156	14.683	9%
157	14.754	9%
158	14.825	9%
159	14.895	9%
160	14.966	9%
161	15.036	9%
162	15.106	9%
163	15.177	9%
164	15.246	9%
165	15.316	9%
166	15.386	9%
167	15.455	9%
168	15.525	9%
169	15.594	9%
170	15.663	9%
171	15.732	9%
172	15.801	9%
173	15.869	9%
174	15.938	9%
175	16.006	9%
176	16.074	9%
177	16.142	9%
178	16.210	9%
179	16.278	9%
180	16.346	9%
181	16.413	9%
182	16.481	9%
183	16.548	9%
184	16.615	9%
185	16.682	9%
186	16.749	9%
187	16.815	9%
188	16.882	9%
189	16.948	9%
190	17.015	9%
191	17.081	9%
192	17.147	9%

Appendix IV: Number of Exact Statistically Significant News Days Needed to get a Z-Statistic of 1.65 per Dr. Feinstein's Z-test by Varying News Days

[A]	[B]	[B] / [A] x 100
Number of News Days	***Number of Exact Significant News Days***	***Percentage***
193	17.212	9%
194	17.278	9%
195	17.343	9%
196	17.409	9%
197	17.474	9%
198	17.539	9%
199	17.604	9%
200	17.669	9%
201	17.733	9%
202	17.798	9%
203	17.862	9%
204	17.926	9%
205	17.990	9%
206	18.054	9%
207	18.118	9%
208	18.181	9%
209	18.245	9%
210	18.308	9%
211	18.371	9%
212	18.434	9%
213	18.497	9%
214	18.559	9%
215	18.622	9%
216	18.684	9%
217	18.746	9%
218	18.808	9%
219	18.870	9%
220	18.931	9%
221	18.993	9%
222	19.054	9%
223	19.115	9%
224	19.176	9%
225	19.237	9%
226	19.297	9%
227	19.358	9%
228	19.418	9%
229	19.478	9%
230	19.538	8%

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 115 of 370. PageID #: 18942

Appendix IV: Number of Exact Statistically Significant News Days Needed to get a Z-Statistic of 1.65 per Dr. Feinstein's Z-test by Varying News Days

[A] *Number of News Days*	[B] *Number of Exact Significant News Days*	[B] / [A] x 100 *Percentage*
231	19.598	8%
232	19.657	8%
233	19.716	8%
234	19.776	8%
235	19.835	8%
236	19.893	8%
237	19.952	8%
238	20.010	8%
239	20.068	8%
240	20.126	8%
241	20.184	8%
242	20.242	8%
243	20.299	8%
244	20.356	8%
245	20.413	8%
246	20.470	8%
247	20.527	8%
248	20.583	8%
249	20.639	8%
250	20.695	8%
251	20.751	8%
252	20.806	8%
253	20.861	8%
254	20.916	8%
255	20.971	8%
256	21.026	8%
257	21.080	8%
258	21.134	8%
259	21.188	8%
260	21.241	8%
261	21.295	8%
262	21.348	8%
263	21.400	8%
264	21.453	8%
265	21.505	8%
266	21.557	8%
267	21.609	8%
268	21.660	8%

Appendix IV: Number of Exact Statistically Significant News Days Needed to get a Z-Statistic of 1.65 per Dr. Feinstein's Z-test by Varying News Days

[A]	[B]	[B] / [A] x 100
Number of News Days	***Number of Exact Significant News Days***	***Percentage***
269	21.711	8%
270	21.762	8%
271	21.812	8%
272	21.863	8%
273	21.912	8%
274	21.962	8%
275	22.011	8%
276	22.060	8%
277	22.108	8%
278	22.157	8%
279	22.204	8%
280	22.252	8%
281	22.299	8%
282	22.345	8%
283	22.392	8%
284	22.437	8%
285	22.483	8%
286	22.528	8%
287	22.572	8%
288	22.616	8%
289	22.660	8%
290	22.703	8%
291	22.746	8%
292	22.788	8%
293	22.829	8%
294	22.870	8%
295	22.911	8%
296	22.951	8%
297	22.990	8%
298		0%
299		0%
300		0%
301		0%
302		0%
303		0%
304		0%
305		0%
306		0%

Appendix IV: Number of Exact Statistically Significant News Days Needed to get a Z-Statistic of 1.65 per Dr. Feinstein's Z-test by Varying News Days

[A]	[B]	[B] / [A] x 100
Number of News Days	***Number of Exact Significant News Days***	***Percentage***
307		0%
308		0%
309		0%
310		0%
311		0%
312		0%
313		0%
314		0%
315		0%
316		0%
317		0%
318		0%
319		0%
320		0%
321		0%
322		0%
323		0%
324		0%
325		0%
326		0%
327		0%
328		0%
329		0%

Notes:

1. Number of Statistically Significant Days = 23.
2. Number of Days in the Class Period = 330

Appendix V

Appendix V: Alleged Class Period Misstatements and Freddie Mac Stock's Dividend Adjusted Return and Its Abnormal Return Following Such Alleged Misstatements

	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
								Feinstein Event Study Model[5]		
No	**Summary of Third Amended Complaint's (TAC) Allegations**	**¶**	**Date in TAC**	**Effective Date[1]**	**Freddie Mac Stock Return[2]**	**Market Return[6]**	**Industry Return[6]**	**Abnormal Return[3]**	**t-Statistics[4]**	**Two-Tail P-Value**
1.	On August 1, 2006, Freddie Mac issued a press release entitled "Freddie Mac Voluntarily Adopts Temporary Limited Growth for Retained Portfolio." In it, Defendants falsely portrayed Freddie Mac's exposure to risk.	145	8/1/2006	8/1/2006	-0.57%	-0.53%	-0.25%	-0.37%	-0.481	63.06%
2.	At a Freddie Mac Market Update, Defendants repeated the substance of that press release, making the following knowingly false statements. "I would emphasize that our underlying business fundamentals are really unchanged from our last call on May 30th. Our low level of interest rate and credit risk is unchanged."	146	8/1/2006	8/2/2006	-0.24%	0.67%	0.12%	-0.38%	-0.491	62.37%
3.	On September 8, 2006, Defendant Syron falsely stated in his prepared remarks at Freddie Mac's Annual Shareholder Meeting that Freddie Mac has a "strong capital position".	214	9/8/2006	9/8/2006	0.51%	0.24%	0.56%	-0.10%	-0.131	89.57%
4.	On September 12, 2006, Defendant Syron's prepared remarks at a Lehman Brothers Financial Services Conference contained the following misrepresentations: Despite these pressures, Freddie Mac has managed to increase shareholder value and sustain our guarantee portfolio market share of the GSE market, while keeping our traditional risks low.	147	9/12/2006	9/12/2006	0.60%	1.13%	1.21%	-0.66%	-0.850	39.61%
5.	On September 18, 2006, Senior Vice President and Treasurer of Freddie Mac, Timothy Bitsberger made false statement that Freddie Mac had a "strong and growing capital base" at the Bank of America 36th Annual Investment Conference.	216	9/18/2006	9/18/2006	-0.51%	0.15%	-0.19%	-0.34%	-0.447	65.50%
6.	On October 3, 2006, Freddie Mac issued a press release entitled "Freddie Mac Provides Market Update; Estimated Net Income for First Half of 2006 of $2.7 Billion; Company Maintains Strong Capital Position and Continued Solid Risk Management Performance" that include the false statement from Syron regarding Freddie Mac's prudent risk management and strong capital base. During a Freddie Mac Market Update Conference Call on that date, Defendants McQuade and Syron falsely assured the Company's risk management, capital position and financial condition.	148 149 217 218	10/3/2006	10/3/2006	1.34%	-0.04%	1.04%	0.22%	0.283	77.70%
7.	On October 18, 2006, Defendant Syron publicly made the following misrepresentations: It's why we have purchased the newer, more untraditional loans in a very prudent and balanced way – with very low credit losses as an added benefit.	150	10/18/2006	10/19/2006	-1.94%	0.24%	-0.75%	-1.19%	-1.522	12.90%
8.	During a January 5, 2007 Market Update Conference Call, Defendant Piszel falsely claimed that Freddie Mac displayed very low and well-managed interest rate and credit exposures. The press release on the same day provided false assurances of Freddie Mac's capital position.	151 219	1/5/2007	1/5/2007	-1.51%	-0.73%	-0.83%	-0.71%	-0.923	35.66%

Appendix V: Alleged Class Period Misstatements and Freddie Mac Stock's Dividend Adjusted Return and Its Abnormal Return Following Such Alleged Misstatements

	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
								Feinstein Event Study Model[5]		
No	**Summary of Third Amended Complaint's (TAC) Allegations**	**¶**	**Date in TAC**	**Effective Date**[1]	**Freddie Mac Stock Return**[2]	**Market Return**[6]	**Industry Return**[6]	**Abnormal Return**[3]	**t-Statistics**[4]	**Two-Tail P-Value**
9.	On January 30, 2007, Defendant Cook presented the following prepared remarks at a Citigroup Financial Services Conference, falsely downplaying Freddie Mac's risk exposure: This story is played out in our current credit risk measures as well. Through the third quarter, Freddie's total single-family delinquencies and credit losses have stayed at very low, manageable levels.	152	1/30/2007	1/30/2007	-0.16%	0.58%	0.51%	-0.71%	-0.920	35.82%
10.	On February 8, 2007, Defendant McQuade presented prepared remarks at Credit Suisse Financial Services Conference, echoing Defendant Cook's January 30th misrepresentations: In addition, despite the changing credit environment and volatile financial markets, Freddie maintained very low interest-rate and credit risk exposures throughout the year.	153	2/8/2007	2/9/2007	0.00%	-0.69%	-0.98%	0.96%	1.242	21.53%
11.	On February 27, 2007, Freddie Mac issued a press release entitled "Freddie Mac Announces Tougher Subprime Lending Standards to Help Reduce the Risk of Future Borrower Default." where Defendants made false assurance of purchasing only higher quality subprime mortgages. During two interviews on February 27, 2007, Defendant Syron again falsely claimed that Freddie Mac had no credit exposure to subprime mortgages.	154 155 156	2/27/2007	2/27/2007	-1.91%	-3.47%	-3.62%	1.69%	2.080	3.83%
12.	On March 23, 2007, in Freddie Mac's 2006 Annual Report, Defendants misrepresented Freddie Mac's exposure to subprime mortgages, credit risk management, financial condition and capital adequacy. In a press release that accompanied its Annual Report for 2006 Financial Results, Defendants falsely emphasized that Freddie Mac did not face risk from its nontraditional mortgage exposure and managed capital prudently. During a March 23, 2007 earnings conference call, Defendant Syron discussed Freddie Mac's February 27, 2007 announcement that it would cease buying certain subprime mortgages, falsely claiming that the step was not due to increased risk concerns. On the same conference call, Defendant Syron also made false statement regarding Freddie Mac's capital strength. A March 23, 2007 article by Bloomberg News quoted Defendant McQuade's remark that "[w]e don't think we'll lose any money at all on subprime."	156 157 159 160 161 162 164 220	3/23/2007	3/23/2007	0.21%	0.17%	0.17%	0.00%	0.006	99.53%

Appendix V: Alleged Class Period Misstatements and Freddie Mac Stock's Dividend Adjusted Return and Its Abnormal Return Following Such Alleged Misstatements

	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
								Feinstein Event Study Model[5]		
No	**Summary of Third Amended Complaint's (TAC) Allegations**	**¶**	**Date in TAC**	**Effective Date[1]**	**Freddie Mac Stock Return[2]**	**Market Return[6]**	**Industry Return[6]**	**Abnormal Return[3]**	**t-Statistics[4]**	**Two-Tail P-Value**
13.	In an April 16, 2007 *Business Week* article entitled "How Big is the Bite on Freddie and Fannie? The mortgage giants' exposure to risky loans could be bigger than they say," Defendant Piszel gave false assurance of Freddie Mac's subprime exposure and dismissed concerns raised in the article that GSEs are underplaying their exposure to subprime risk.	165	4/16/2007	4/16/2007	3.36%	1.04%	2.10%	1.18%	1.504	13.34%
14.	On May 14, 2007, Syron provided still further unqualified false assurances regarding FRE's ability to purchase subprime loans without risk: Last month we announced that we will purchase up to $20 billion in fixed rate and hybrid ARM subprime products structured to limit payment shock on borrowers.	166 167 168	5/14/2007	5/14/2007	-0.99%	-0.27%	-0.57%	-0.45%	-0.586	55.82%
15.	At the May 17, 2007 Lehman Brothers 10th Annual Financial Services Conference, Defendant Cook echoed Defendant Syron's remarks of May 14, 2007 when she falsely assured investors that, "at the end of 2006, Freddie had basically no subprime exposure in our guarantee business, and about $124 billion of AAA rated subprime exposure in our retained portfolio."	170	5/17/2007	5/17/2007	-0.22%	-0.05%	-0.30%	0.05%	0.065	94.79%
16.	On June 8, 2007, Defendant Syron's prepared remarks delivered at Freddie Mac's Annual Stockholders' Meeting misleadingly stated: Your company ended the year in a good position to weather the current housing downturn. . .	173	6/8/2007	6/8/2007	-1.20%	1.07%	1.15%	-2.39%	-3.099	0.21%
17.	On June 14, 2007, Freddie Mac published its Financial Report for the Three Months Ended March 31, 2007 (the "IQ07 Report"), where Freddie Mac did not quantify its subprime exposure. Freddie Mac did not include any statement regarding its exposure to subprime loans in its Single Family credit guarantee portfolio. On June 14, 2007, Freddie Mac issued a press release entitled "Freddie Mac Releases First Quarter 2007 Financial Results; Company Resumes Quarterly Reporting," which provided a false financial outlook. During Freddie Mac's first quarter 2007 financial results conference call, Defendants Syron and Piszel made false statements regarding Company's exposure to subprime, risk management and financial condition.	174 175 177 178 179	6/14/2007	6/14/2007	-0.94%	0.53%	-0.21%	-0.74%	-0.949	34.35%
18.	On July 2, 2007, *Bloomberg News* quoted Freddie spokesperson Sharon McHale. In response to an estimate of Freddie Mac's mortgage exposure, Ms. McHale denied that a loss by Freddie Mac in the neighborhood of $3 billion would deplete Freddie Mac's mandatory capital reserve requirements. McHale falsely stated that Freddie Mac was not exposed to losses of that magnitude and that loss estimates of that size were "absurd."	180	7/2/2007	7/3/2007	0.26%	0.34%	0.55%	-0.33%	-0.434	66.46%
19.	On July 28, 2007, the *Wall Street Journal* published an article entitled "Fannie, Freddie Are Said to Suffer in Subprime Mess." In it, Citigroup Inc. analysts estimated that falling prices on subprime mortgage bonds have cut the value of such securities held by Fannie Mae and Freddie Mac by $4.7 billion, $3.2 billion of which was attributed to Freddie Mac. A Freddie Mac spokeswoman falsely denied the analysts' estimate and called it "mistaken."	181	7/28/2007	7/30/2007	2.52%	0.93%	1.22%	1.24%	1.612	10.80%

Appendix V: Alleged Class Period Misstatements and Freddie Mac Stock's Dividend Adjusted Return and Its Abnormal Return Following Such Alleged Misstatements

	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
								Feinstein Event Study Model[5]		
No	**Summary of Third Amended Complaint's (TAC) Allegations**	**¶**	**Date in TAC**	**Effective Date[1]**	**Freddie Mac Stock Return[2]**	**Market Return[6]**	**Industry Return[6]**	**Abnormal Return[3]**	**t-Statistics[4]**	**Two-Tail P-Value**
20.	On August 30, 2007, Freddie Mac published its Financial Report for the Three and Six Months Ended June 30, 2007 (the "2Q07 Report"), where it falsely stated Freddie Mac's exposure to subprime mortgages. At the Freddie Mac second quarter 2007 financial results conference call on that date, Defendant Piszel falsely characterized Freddie Mac's risk.	182 183	8/30/2007	8/30/2007	-5.16%	-0.38%	-1.12%	-3.48%	-2.025	4.37%
21.	On September 10, 2007, at the Lehman Brothers 5th Annual Financial Services Conference, Defendant Cook further falsely assured investors that Freddie Mac's investments in nontraditional mortgages were safe and would not subject Freddie Mac to write-downs.	184	9/10/2007	9/10/2007	-0.95%	-0.29%	-0.09%	-0.37%	-0.216	82.90%
22.	On September 17, 2007, at the Bank of America Securities 37th Annual Investor Conference, Defendant Piszel again misrepresented Freddie Mac's risk exposure and falsely denied that Freddie Mac would write down any of its subprime portfolio – a denial that was proven to be untrue just two months later.	185	9/17/2007	9/17/2007	-1.44%	-0.58%	-0.75%	-0.07%	-0.038	96.97%
23.	On November 7, 2007, Freddie Mac issued a press release entitled "Freddie Mac Statement in Response to New York Attorney General Subpoenas," which continued to conceal the fraud by stating in part: Accurate appraisals are fundamental to our effective credit risk management as well as to the long-term success of the homebuyers we are charted to serve.	187	11/7/2007	11/7/2007	-9.02%	-2.72%	-5.19%	-2.19%	-1.209	22.74%

							Cumulative Abnormal Return[7]	**-8.88%**		

Notes:

1. If the event in question occurs before the market closes (4:00 P.M. ET), then the event date and effective date are the same. However, if the event occurs after the close of market, then the effective date is the following trading date.
2. Freddie Mac Stock Return is the stock's daily dividend adjusted logrithmic return, taken from Feinstein's Supporting Files for his Exhibit 4.
3. The abnormal return is considered statistically significant at the confidence level of 95% if the t-statistic's absolute value is greater than or equal to 1.96. [See *e.g.*, MacKinlay, A.C. "Event Studies in Economics and Finance," Journal of Economic Literature 35, March 1997, pages 13-39]
4. The t-Statistic is a ratio of the abnormal return to the regression standard error, which is the average daily fluctuation in the abnormal return observed over the estimation window. See e.g., MacKinlay, A.C. "Event Studies in Economics and Finance," *Journal of Economic Literature 35*, March 1997, pages 13-39.
5. Predicted and residual returns were calculated using Dr. Feinstein's event study model as described in Exhibit 7 of the Feinstein Report, but substituting the "news" dummy dates with Third Amended Complaint ("TAC") misstatement (effective) dates.
6. Market and Industry Returns were taken from Dr. Feinstein's Supporting Files for his Exhibit 6.
7. The cumulative abnormal return across these 23 days is calculated as [(1+excess return on day 1) * (1+excess return on day 2) …. (1+excess return on day 23) – 1].

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 123 of 370. PageID #: 18950

Appendix VI

Appendix VI: Dr. Feinstein's Z-Test for 23 Alleged Misrepresentation/Omission Dates[1] mentioned in the Third Amended Complaint

	Third Amended Complaint Alleged Misrepresentation/Omission Dates as News Event Days
General Parameters	
Trading Days During Class Period [2]	*329*
# of News Days	23
# of Non-News Days	306
Z-Test Results	
% of Statistically Significant News Days[3]	13.04%
% of Statistically Significant Non-News Days	6.21%
Z-statistic	1.27
95% Critical Threshold	1.645
P-value[4]	10.33%

Notes:

1. The dates used for this analysis are effective dates (see Appendix V). If the Third Amended Complaint (TAC) event in question occurs before the market closes (4:00 P.M. ET), then the effective date is the same as the event date. However, if the TAC event occurs after the close of market, then the effective date is the following trading date.

2. There are 329 trading days because November 20, 2007, the alleged corrective disclosure date, is excluded from this analysis.

3. Statistically significant days were determined using a 95% confidence level (in a two-tailed test) as per the market model described in Appendix V. That is, abormal returns were calculated using Dr. Feinstein's event study model as described in Exhibit 7 of the Feinstein Report, but substituting the "news" dummy dates with Third Amended Complaint ("TAC") misstatement (effective) dates.

4. The p-value is the one-tail (right tail) p-value. The two-tailed p-value is twice this value: 20.66%.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 125 of 370. PageID #: 18952

Appendix VII

Appendix VII: Dr. Feinstein's Z-Test Using Results from Dr. Feinstein's Regression Market Model, but Without "News" Date "Dummy" Variables

	Feinstein's WSJ/NYT Dates as "News" Event Days (excluding November 20, 2007)
General Parameters	
Trading Days During Class Period[1]	*329*
# of News Days	8
# of Non-News Days	321
Z-Test Results	
% of Statistically Significant News Days	12.50%
% of Statistically Significant Non-News Days	5.92%
Z-statistic	0.77
95% Critical Threshold	1.645
P-value[2]	22.11%

Notes:

1. Dr. Feinstein performs a version of his z-test excluding November 20, 2007 (see paragraph 144 of the Feinstein Report). This is consistent with the FDT paper, footnote 155 which notes that the Z-test must ““exclude those days in which a corrective disclosure was made because plaintiffs would normally choose a class period where corrective disclosures coincide with large negative price movements; including those days in the analysis would bias the results,” as I have also discussed in the report.

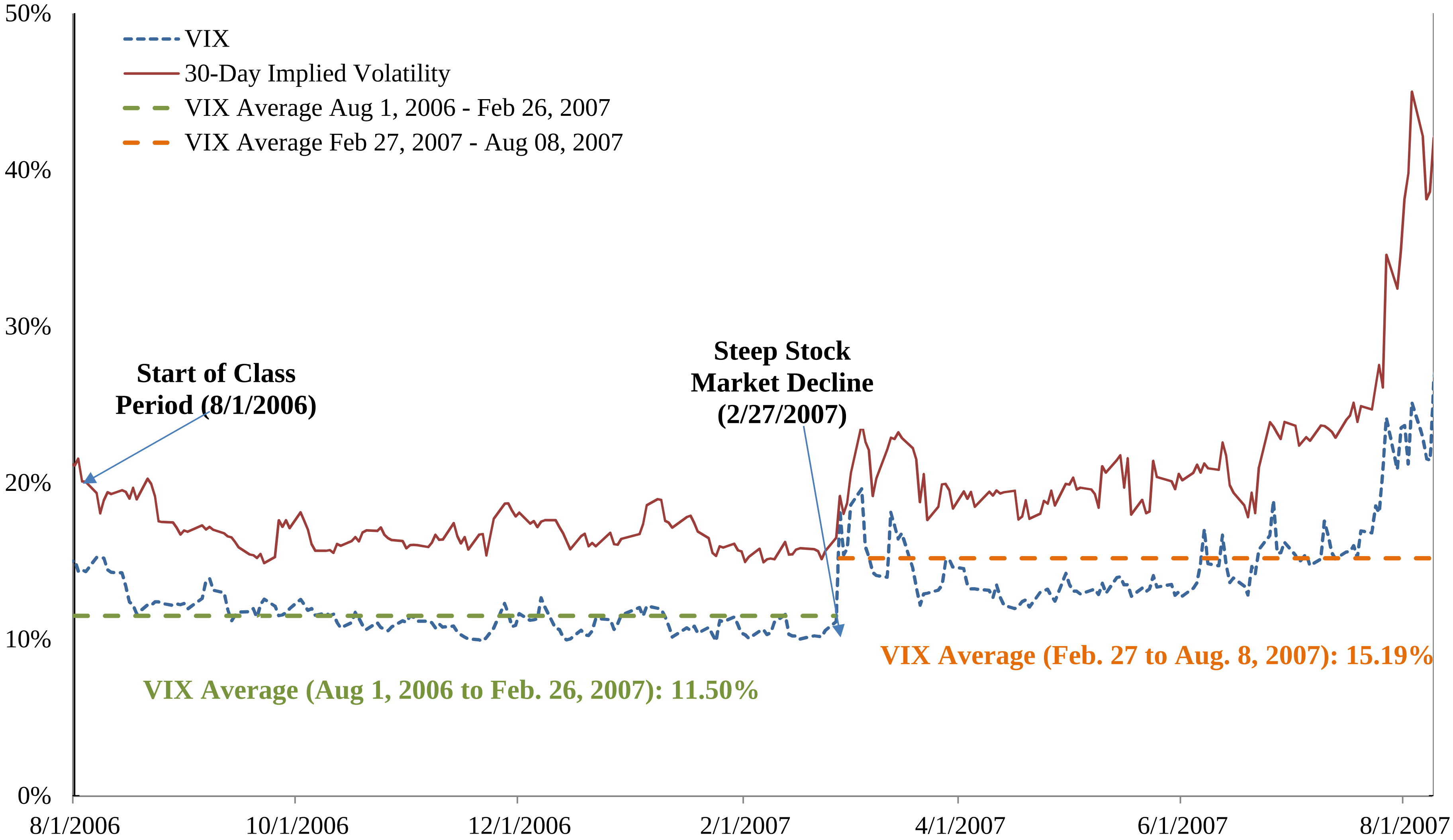
2. The p-value is the one-tail (right tail) p-value. The two-tailed p-value is twice this value: 44.2167%.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 127 of 370. PageID #: 18954

Appendix VIII

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 128 of 370. PageID #: 18955

Appendix VIII: VIX Index and Freddie Mac's Implied Volatility



Sources: Bloomberg and OptionMetrics

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 129 of 370. PageID #: 18958

Appendix IX

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 130 of 370. PageID #: 18957

Appendix IX: 3 Period Feinstein Model and Number of Statistically Significant Days by Estimation Period

Panel A: Regression Models

Estimation Period 1a: August 1, 2006 to February 26, 2007

Regression Statistics	
R Squared	0.315
Adjusted R Squared	0.306
Standard Error	0.69%
Observations	143

	Coefficients	**Standard Error**	***t-statistic***
Intercept	0.01%	0.06%	0.18
Market Index	0.271	20.37%	1.33
Peer Index	0.656	18.44%	3.56

Estimation Period 1b: February 27, 2007 to August 08, 2007

Regression Statistics	
R Squared	0.699
Adjusted R Squared	0.679
Standard Error	0.85%
Observations	114

	Coefficients	**Standard Error**	***t-statistic***
Intercept	0.06%	0.08%	0.68
Market Index	-0.436	26.22%	-1.66
Peer Index	1.368	19.47%	7.03
February 27, 2007	1.46%	0.94%	1.56
April 18, 2007	1.40%	0.88%	1.59
May 22, 2007	0.04%	0.86%	0.05
June 14, 2007	-0.47%	0.87%	-0.54
August 8, 2007	-0.69%	0.87%	-0.79

Estimation Period 2: August 9, 2007 to November 19, 2007

Regression Statistics	
R Squared	0.709
Adjusted R Squared	0.687
Standard Error	1.66%
Observations	72

	Coefficients	**Standard Error**	***t-statistic***
Intercept	-0.34%	0.21%	-1.65
Market Index	0.328	35.14%	0.93
Peer Index	1.132	23.46%	4.83
August 30, 2007	-3.42%	1.68%	-2.04
September 27, 2007	0.02%	1.68%	0.01
November 8, 2007	-3.10%	1.69%	-1.84

Appendix IX: 3 Period Feinstein Model and Number of Statistically Significant Days by Estimation Period

Panel B: Number of Significant Days Per Period

Period	No. of Significant Days	No. of days in the period	% Significant (in Period)
Aug 1, 2006 - Feb 26, 2007	9	143	6.3%
Feb 27, 2007 - Aug 08, 2007	5	114	4.4%
Aug 09, 2007 - Nov 19, 2007	7	72	9.7%

Source: Dr. Feinstein's provided returns data for the Class Period.

<u>Exhibit I</u>

Professor Steven P. Feinstein's Report on Market Efficiency

IN RE: PETROBRAS SECURITIES LITIGATION (No. 14-cv-9662 (JSR))

UNITED STATES DISTRICT COURT
SOUTHERN DISTRICT OF NEW YORK

IN RE: PETROBRAS SECURITIES LITIGATION	No. 14-cv-9662 (JSR)

REPORT ON MARKET EFFICIENCY

PROFESSOR STEVEN P. FEINSTEIN, PH.D., CFA

October 15, 2015

Case: 1:14-cv-09662 Doc #: 508-2 Filed: 07/03/18 134 of 370. PageID #: 18961

TABLE OF CONTENTS

SCOPE OF PROJECT AND REPORT

1. I was asked by Pomerantz LLP, Lead Counsel for the Lead Plaintiff, to determine whether the securities of Petróleo Brasileiro S.A. (“Petrobras” or the “Company”), including: 1) common American Depositary Receipts (the “common ADRs” or the “Petrobras common ADRs”); 2) preferred American Depositary Receipts (the “preferred ADRs” or the “Petrobras preferred ADRs”); and 3) certain debt securities[1] (the “Bonds” or “Petrobras Bonds”) traded in an efficient market during the period from 22 January 2010 to 28 July 2015 (the “Class Period”).
2. In addition, I have been asked to opine on whether damages under Section 10(b) of the Exchange Act of 1934 and SEC Rule 10b-5 adopted thereunder (collectively, “Section 10(b)”) can be computed in this matter for all Class members using a common class-wide methodology that is consistent with the Plaintiff’s allegations of liability.
3. Toward these ends, I analyzed the markets for Petrobras’ securities, the price behavior of the securities, and the factors that are generally accepted to be indicative of market efficiency for publicly-traded securities. I examined Company press releases, conference call transcripts, security analyst reports, news articles, SEC filings, security prices, trading volume, the performance of the overall stock market, and the performance of Petrobras’ peer groups, as well as other pertinent data and documents. I also read Lead Plaintiff’s Consolidated Third Amended Class Action Complaint (“Complaint”) dated 1 September 2015, and considered the allegations therein. Exhibit-1 lists the documents I considered in preparing this report and arriving at the opinions expressed herein.
4. This report presents my methodology, findings, and conclusions.
5. I understand that discovery is ongoing in this matter. I reserve the right to amend, refine, or modify my opinion and report, including in the event any additional information or analyses become available.

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[1] From Bloomberg I obtained a list of twenty-two U.S. dollar denominated bonds issued by Petrobras Global Finance (or its predecessor PifCo) prior to 2015 with the following CUSIPs: 71645WAT8, 71645WAR2, 71645WAS0, 71645WAM3, 71645WAP6, 71645WAQ4, 71645WAU5, 71647NAC3, 71647NAA7, 71647NAE9, 71647NAB5, 71647NAF6, 71647NAJ8, 71647NAH2, 71647NAM1, 71645WAH4, 71645WAN1, 71645WAL5, 71647NAL3, 71647NAD, 71647NAK5, and 71647NAG4 (collectively referred to herein as the “Petrobras Bonds” or the “Bonds”).

CREDENTIALS

6. I am an Associate Professor of Finance at Babson College, and the founder and president of Crowninshield Financial Research, Inc., a financial economics consulting firm.
7. I hold a Ph.D. in Economics from Yale University, a Master of Philosophy degree in Economics from Yale University, a Master of Arts in Economics from Yale University, and a Bachelor of Arts degree in Economics from Pomona College. I also hold the Chartered Financial Analyst ("CFA") designation, granted by the CFA Institute.
8. At Babson College, I have taught undergraduate and MBA level courses in Valuation, Capital Markets, Investments, Equity Analysis, Fixed Income Analysis, Financial Management, Risk Management, Quantitative Methods, and Security Valuation. I have also taught executive courses on investments and corporate financial management for numerous corporations. Other courses I have taught are listed in my curriculum vitae, which is attached as Exhibit-2.
9. At Babson College, I have held the Chair in Applied Investments and served as the Director of the Stephen D. Cutler Investment Management Center, a research and education center dedicated to the study and teaching of investments and capital markets.
10. Prior to my joining the faculty at Babson College, I taught finance at Boston University. Previously, I was an Economist at the Federal Reserve Bank of Atlanta where my primary responsibilities were to monitor financial markets, analyze proposed regulation, and advise the Bank President in preparation for his participation in meetings of the Federal Open Market Committee – the government body responsible for monetary policy in the United States.
11. I have published extensively in the field of finance. My finance articles have appeared in the *Atlanta Federal Reserve Bank Economic Review*, *Derivatives Quarterly*, *Derivatives Weekly*, *The Engineering Economist*, *The Journal of Risk*, *The American Bankruptcy Institute Journal*, *The Journal of Financial Planning*, *The Journal of Forensic Economics*, *Managerial Finance*, *Risk Management*, and *Primus*. I am the author of *Finance and Accounting for Project Management*, published by the American Management Association. I wrote two chapters in the book *The Portable MBA in Finance and Accounting* – one on corporate financial planning and the other on risk management. I have presented research at the annual conventions of the American Finance Association,

the Academy of Financial Services, the Multinational Finance Society, the Financial Management Association, the Taxpayers Against Fraud Education Fund Conference, and the International Conference on Applied Business Research. Co-authored papers of mine have been presented at the Eastern Finance Association meetings and the Midwestern Finance Association meetings. A list of all the publications I authored in the previous ten years can be found in my curriculum vitae, which is attached as Exhibit-2.

12. I have been selected to review papers for numerous finance journals and conferences, and I have reviewed finance textbook manuscripts for Prentice-Hall, Elsevier, Blackwell, and Southwestern Publishing. I have been quoted on matters relating to finance and investments in *The Wall Street Journal*, *The Washington Post*, *The New York Times*, *The Financial Times*, *The Boston Globe*, and *Bloomberg News*, and my research relating to financial analysis and valuation has been discussed in *The Wall Street Journal*, *Bond Buyer*, and *Grant's Municipal Bond Observer*.

13. I am a member of the American Finance Association, the Financial Management Association, the North American Case Research Association, the National Association of Forensic Economics, the CFA Institute, and the Boston Security Analysts Society, where I have served as a member of the education committee and ethics subcommittee. I served on the Fixed Income Specialization Examination Committee of the CFA Institute.

14. The CFA designation is the premier credential for financial analysts worldwide. In order to receive this credential, applicants must pass a series of three exams covering such topics as economics, equity analysis, financial valuation, business analysis, quantitative methods, investment analysis, portfolio management, risk management, financial accounting, and ethical and professional standards. For over ten years I taught in the Boston University CFA Review Program and the Boston Security Analysts Society CFA Review Program – two of the leading review programs that prepared candidates for the CFA exams. In both of these programs I taught candidates at the most advanced level.

15. In addition to my teaching, research, CFA, and academic community responsibilities, I practice extensively as a financial consultant. Past and present clients include the United States Securities and Exchange Commission, the Internal Revenue Service, the Attorney General of the State of Illinois, and the National Association of Securities Dealers, now known as FINRA. As a financial consultant, I have conducted analyses and presented

opinions related to markets, valuation, and damages in over 70 cases. Exhibit-3 lists my prior testimony appearances over the past four years.

16. I am the president and founder of the consulting firm Crowninshield Financial Research, which receives compensation for the work performed by me and the analysts who assist me on this case. My firm is being compensated at a rate of $750 per hour for my work. My compensation is neither contingent on my findings nor on the outcome of this matter.

CONCLUSIONS

17. The Petrobras securities traded in efficient markets over the course of the Class Period. The Petrobras securities satisfied the factors set forth in *Cammer v. Bloom*, 711 F. Supp. 1264, 1273 (D.N.J. 1989) and *Krogman v. Sterritt*, 202 F.R.D. 467 (N.D. Tex. 2001), which, consistent with financial economic principles and empirical research, indicate market efficiency.

18. The Petrobras common ADRs satisfied all of the *Cammer* and *Krogman* factors by wide margins. In particular, with a high degree of statistical certainty, empirical tests prove that there was a cause and effect relationship between the release of new, Company-specific information and movements in the Petrobras common ADRs' price. Additional statistical tests examining the behavior of Petrobras common ADRs on important announcement dates further indicate that the Petrobras common ADRs responded to new Company-specific information throughout the Class Period. The empirical analyses not only indicate market efficiency, but prove that the common ADRs demonstrate the essence of market efficiency.

19. The Petrobras preferred ADRs traded in an efficient market over the course of the Class Period. The Petrobras preferred ADRs satisfied all *Cammer* and *Krogman* factors. Statistical tests prove that there was a cause and effect relationship between the release of new Company-specific information and appropriate movements in the Petrobras preferred ADR prices, which is the essence of market efficiency.

20. The Petrobras Bonds traded in an efficient market over the course of the Class Period. The Petrobras Bonds satisfied all of the *Cammer* and *Krogman* factors, with the possible exception of the *Krogman* bid-ask spread factor. The bid-ask spread factor could not be analyzed for the bonds due to the unavailability of data. Nonetheless, the other *Cammer*

and *Krogman* factors, including high trading volume, indicate no impediments to trading, which is what the bid-ask factor also addresses. Moreover, statistical tests prove that there was a cause and effect relationship between the release of new information expected to be material to the bonds and movements in the prices of the Bonds. The Bond prices moved appropriately in response to Company-specific information and in response to changes in market interest rates continuously over the course of the Class Period.

21. Based on the foregoing, I conclude that the markets for the Petrobras common ADRs, Petrobras preferred ADRs, and the Petrobras Bonds were efficient over the course of the Class Period.

22. I further conclude that damages in this matter can be computed on a class-wide basis for all securities and for all Class Members using a common methodology that is consistent with the Plaintiff's allegations of liability.

BACKGROUND

A. About the Company

23. Prior to and throughout the Class Period, Petrobras was the state-controlled Brazilian oil and gas company headquartered in Rio de Janeiro.[2] The Company described itself as "an integrated oil and gas company that is one of the largest companies in Latin America in terms of revenue."[3] As of 31 December 2014, the Brazilian government owned 28.67% of the Company's outstanding capital stock and 50.26% of the voting shares.[4]

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[2] Petróleo Brasileiro S.A. – Petrobras Form 20-F for the fiscal year ended 31 December 2014, filed 15 May 2015, p. 32; and Petróleo Brasileiro S.A. – Petrobras Form 20-F for the fiscal year ended 31 December 2010, filed 25 May 2011, p. 29.

[3] Petróleo Brasileiro S.A. – Petrobras Form 20-F for the fiscal year ended 31 December 2014, filed 15 May 2015, p. 32; and Petróleo Brasileiro S.A. – Petrobras Form 20-F for the fiscal year ended 31 December 2010, filed 25 May 2011, p. 29.

[4] Petróleo Brasileiro S.A. – Petrobras Form 20-F for the fiscal year ended 31 December 2014, filed 15 May 2015, p. 32.

Case: 1:14-cv-09662-JSR Document 267-19 Filed 10/15/15 Page 7 of 38 PageID #: 413969

24. Operating in Brazil and 16 other countries, the Company categorized its business into six areas: 1) Exploration and Production; 2) Refining, Transportation and Marketing; 3) Distribution; 4) Gas and Power; 5) Biofuel; and 6) International.[5]

25. For fiscal years 2010, 2011, 2012, 2013, and 2014, Petrobras reported sales revenue of \$120.5 billion, \$145.9 billion, \$144.1 billion, \$141.5 billion and \$143.7 billion, respectively.[6] The Company reported total assets for fiscal years 2010, 2011, 2012, 2013, and 2014 of \$308.7 billion, \$320.0 billion, \$327.4 billion, \$321.4 billion, and \$298.7 billion, respectively.[7]

B. About the Petrobras Common and Preferred ADRs

26. Petrobras' common and preferred shares trade on the São Paulo Stock Exchange under the symbols PETR3 and PETR4, respectively.[8] In the U.S., the Company's stock is traded in the form of an American Depositary Receipt ("ADR"), which is also referred to as an American Depositary Share ("ADS"). According to the SEC, "An ADR is a negotiable certificate that evidences an ownership interest in ADSs which, in turn, represent an interest in the shares of a non-U.S. company that have been deposited with a U.S. bank. It is similar to a stock certificate representing shares of stock. The terms ADR and ADS are often used interchangeably by market participants."[9] This report refers to the Petrobras shares listed on the New York Stock Exchange ("NYSE") as ADRs.

27. JPMorgan Chase Bank, N.A. serves as depositary for both the common and preferred Petrobras ADRs.[10]

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[5] Petróleo Brasileiro S.A. – Petrobras Form 20-F for the fiscal year ended 31 December 2014, filed 15 May 2015, p. 33.

[6] Petróleo Brasileiro S.A. – Petrobras Form 20-F for the fiscal year ended 31 December 2014, filed 15 May 2015, p. 15.

[7] Petróleo Brasileiro S.A. – Petrobras Form 20-F for the fiscal year ended 31 December 2014, filed 15 May 2015, p. F-4; Petróleo Brasileiro S.A. – Petrobras Form 20-F for the fiscal year ended 31 December 2013, filed 30 April 2014, p. F-6;

[8] Petróleo Brasileiro S.A. – Petrobras Form 20-F for the fiscal year ended 31 December 2010, filed 25 May 2011, p. 136.

[9] "Investor Bulletin: American Depositary Receipts," SEC Office of Investor Education and Advocacy, August 2012, p. 1.

[10] Petróleo Brasileiro S.A. – Petrobras Form 20-F for the fiscal year ended 31 December 2010, filed 25 May 2011, p. 136.

28. The Company's common ADRs, each representing two Petrobras common shares, have traded on the New York Stock Exchange (NYSE) under the symbol PBR since 2000.[11] The Company's preferred ADRs, each representing two Petrobras preferred shares, have traded on the NYSE under the symbol PBRA since 2001.[12] Holders of the preferred ADRs and common ADRs receive dividends from the respective ADRs.[13]

29. During the Class Period, the Petrobras common ADR price peaked at $47.10 per common ADR on 12 March 2010, according to price data obtained from FactSet. On 28 July 2015, the last day of the Class Period, the Petrobras common ADR price had fallen to $6.59 per common ADR, representing declines of 86.0% from the Class Period peak.

30. During the Class Period, the total outstanding value of the common ADRs peaked at $33.2 billion on 1 April 2011, common ADR outstanding data obtained from the Company's Investor Relations website.[14] On 28 July 2015, the last day of the Class Period, the total outstanding value of the common ADRs had fallen to $5.26 billion, representing a decline of 84.1% from the Class Period peak, respectively.

31. During the Class Period, the Petrobras preferred ADR price peaked at $42.18 on 16 March 2010, according to price data obtained from FactSet. On 28 July 2015, the last day of the Class Period, the common ADR price fell to $6.59 and the preferred ADR price fell to $5.95, representing declines of 86.0% and 85.9% from the Class Period peaks, respectively.

32. During the Class Period, the total outstanding value of the preferred ADRs peaked at $28.6 billion on 8 April 2011, preferred ADRs outstanding data obtained from the Company's Investor Relations website.[15] On 28 July 2015, the last day of the Class

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[11] Petróleo Brasileiro S.A. – Petrobras Form 20-F for the fiscal year ended 31 December 2010, filed 25 May 2011, p. 136.

[12] Petróleo Brasileiro S.A. – Petrobras Form 20-F for the fiscal year ended 31 December 2010, filed 25 May 2011, p. 136.

[13] Petróleo Brasileiro S.A. Petrobras Form 424B2 Prospectus Supplement, filed 28 September 2010, p. S-17.

[14] Petróleo Brasileiro S.A., "Capital Ownership Composition and Capital Ownership History," Investor Relations website accessed at http://www.investidorpetrobras.com.br/en/corporate-governance/capital-ownership on 13 October 2015.

[15] Petróleo Brasileiro S.A., "Capital Ownership Composition and Capital Ownership History," Investor Relations website accessed at http://www.investidorpetrobras.com.br/en/corporate-governance/capital-ownership on 13 October 2015.

Period, the total outstanding value of the preferred ADRs had fallen to $4.08 billion, representing a decline of 85.8% from the Class Period peak.

C. About the Petrobras Bonds

33. The twenty-two (22) Petrobras Bonds I examined have face values ranging from $500 million to $5.2 billion and a total face value of $41.4 billion. The maturities for the Bonds range from 3 years to 30 years. Each of the Bonds are senior unsecured securities meaning that they are senior to the Petrobras common ADRs and preferred ADRs in the corporate capital structure. Therefore, Bond interest must be paid before common stock dividends are distributed, and in the event of a bankruptcy, bond principal must be repaid to investors before common stock holders receive any asset liquidation proceeds.

34. Corporate bonds (like the Petrobras Bonds) have characteristics different from common stocks and preferred shares. Those distinguishing characteristics generally cause bonds to trade with less frequency than common or preferred stock.

> "Corporate bonds will likely trade less frequently than stocks because outside macro-economic and internal financial factors generally both have smaller effects on bond pricing than on stock pricing. Unlike common stocks, corporate bonds have predictable cash flows, predictable terminal values, fixed upside opportunities—namely, redemption at par value or $100 in our example—and priority on the corporation's assets. As such, many corporate bonds are close substitutes for one another. On the other hand, corporate equity does not have predictable cash flows, predictable terminal values, fixed upside opportunities, or priority on the corporate assets."
> **"Fraud on the Market: Analysis of the Efficiency of the Corporate Bond Market," by Michael L. Hartzmark, Cindy A. Schipani and H. Nejat Seyhun, *Columbia Business Law Review*, Volume 2011, Number 3.**

35. Nonetheless, despite their various distinguishing features, published peer-reviewed empirical research has concluded that the market for corporate bonds with characteristics similar to those of the Petrobras bonds is an efficient market:

> "[T]he informational efficiency of corporate bond prices is similar to that of the underlying stocks. We find that stocks do not lead bonds in reflecting firm-specific information. We further examine price behavior around earnings news and find that information is quickly incorporate into both

Case: 1:16-cv-01036-BYP Doc #: 36-12 Filed: 10/01/16 145 of 370. PageID #: 41959

> bond and stock prices, even at short return horizons. Finally, we find that measures of market quality are no poorer for the bonds in our sample than for the underlying stocks."
> **"The Informational Efficiency of the Corporate Bond Market: An Intraday Analysis," by Edith Hotchkiss and Tavy Ronen, *The Review of Financial Studies*, 2002.**

36. Appendix-1 provides more detailed descriptions of the Petrobras Bonds.

D. Summary of Allegations

37. The Plaintiff's contend that "during the Class Period, the Exchange Act Defendants made materially false and misleading statements by misrepresenting facts and failing to disclose a multi-year, multi-billion dollar money-laundering and bribery scheme. Specifically, Petrobras' senior executives inflated the value of the Company's construction contracts for the sole purpose of receiving kickbacks from companies that were awarded the contracts illegally."[16] It is alleged that these misdeeds caused the Company to "overstate the [Property, Plant, and Equipment] line item on its balance sheet because the overstated amounts paid on inflated third party contracts were carried as assets on Petrobras' balance sheets."[17] Petrobras delayed its financial reporting and stated that the delay was a consequence of an ongoing investigation, conducted by Brazilian Federal Police, dubbed Operation Lava Jato.[18]

38. Many Petrobras employees and politicians were under criminal investigation. In fact, "as of February 13, 2015, it was revealed that *two thousand employees of Petrobras [were] now under investigation*, reflecting the rampant widespread nature of the fraud."[19] Similarly, "prosecutors have asked [Brazil's] Supreme Court to investigate 34 sitting politicians, including the speakers of both houses of Congress, for allegedly receiving bribe money."[20]

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[16] Complaint, ¶202.

[17] Complaint, ¶202.

[18] Complaint, ¶4.

[19] Complaint, ¶7.

[20] Complaint, ¶14.

39. The investigations into Petrobras' alleged wrongdoings were not limited to Brazil. "Petrobras is currently the subject of civil and criminal investigations by the U.S. Securities and Exchange Commission ('SEC') and by the U.S. Department of Justice ('DOJ')."[21]

40. The Plaintiff contends that the Company's misrepresentations and omissions related to the alleged corruption caused the market prices of the Company's securities to be artificially inflated. As the information about the alleged corruption and investigation was released to the public, all of Petrobras' securities saw substantial declines in value, as described below.[22]

EFFICIENT MARKET DEFINED

41. The definition of market efficiency set forth by Judge Alfred J. Lechner, Jr. in the 1989 *Cammer v. Bloom* decision is often cited as a legal authority on the meaning of market efficiency and is consistent with the definition generally-accepted by the academic finance community:

> "As relevant here, courts have permitted a rebuttable presumption of reliance in the case of securities traded in 'efficient markets' (*i.e.*, markets which are so active and followed that material information disclosed by a company is expected to be reflected in the stock price)."
> ***Cammer v. Bloom* Opinion, 711 F. Supp. 1264, 1273 (D.N.J. 1989).**

42. Judge Lechner also cited the definitions offered by commentators Alan R. Bromberg and Lewis D. Lowenfels, and by finance professor Eugene Fama:

> "An efficient market is one which rapidly reflects new information in price."
> **Alan Bromberg & Lewis Lowenfels, *Securities Fraud and Commodities Fraud*, §8.6 (Aug. 1988); *see also Cammer*, 711 F. Supp. at 1276.**

[21] Complaint, ¶16.

[22] I arrived at this finding, which is consistent with the Plaintiff's allegations, in the course of conducting this market efficiency study. The current study presented in this report, however, is not a complete and comprehensive loss causation or damages analysis, neither of which I have been asked to conduct at this juncture.

> "A market in which prices always 'fully reflect' available information is called 'efficient.'"
> **"Efficient Capital Markets: A Review of Theory and Empirical Work," by Eugene Fama, *Journal of Finance*, 1970, cited in *Cammer*, 711 F. Supp. at 1280.**

43. Professor Fama elaborated and refined his definition in a *Halliburton II* amici curiae that he co-authored:

> "But economists do ***not*** generally disagree about whether market prices respond to new material information. In particular, there is little doubt that the stock price will increase reasonably promptly after favorable news about a company is released and decline after unfavorable news. Our conclusion that prices generally move reasonably promptly in the predicted direction in response to unexpected material public information (favorable or unfavorable) is perfectly consistent with the view that there are sometimes anomalies in the way markets process information and that bubbles can exist."
> **Brief of Financial Economists as Amici Curiae in Support of Respondents, Halliburton Co. and David Lesar v., Erica P. John Fund, Inc., FKA Archdiocese of Milwaukee Supporting Fund, Inc. 5 February 2014, p. 3 (emphasis in original).**

44. The Supreme Court in the *Basic v. Levinson* decision focused on the same important characteristic at the heart of these definitions of market efficiency:

> "The fraud on the market theory is based on the hypothesis that, in an open and developed securities market, the price of a company's stock is determined by the available material information regarding the company and its business … ."
> ***Basic v. Levinson*, 485 U.S. 224, 243,108 S. Ct. 978, 988-89, 99 L. Ed. 2d 194 (1988); *see also Cammer*, 711 F. Supp. at 1276.**

45. The *Amgen* decision defined market efficiency similarly:

> "The fraud-on-the market premise is that the price of a security traded in an efficient market will reflect all publicly available information about a company ... ."
> ***Amgen Inc. v. Conn. Ret. Plans & Trust Funds, _U.S._*, 133 S. Ct. 1184, 1190 (2013), 185 L. Ed. 2d 308 (2013).**

46. In its recent *Halliburton II* decision, the Supreme Court addressed the cause and effect relationship at the center of market efficiency thusly:

> "Even the foremost critics of the efficient-capital-markets hypothesis acknowledge that public information generally affects stock prices. … Debates about the precise *degree* to which stock prices accurately reflect public information are thus largely beside the point. 'That the . . . price [of a stock] may be inaccurate does not detract from the fact that false statements affect it, and cause loss,' which is 'all that *Basic* requires.'"
> ***Halliburton Co. v. Erica P. John Fund, Inc.*, 134 S. Ct. 2398, 2410, 189 L. Ed. 339 (2014) (emphasis in original).**

47. An efficient market, as defined by *Cammer*, *Basic*, *Amgen*, Bromberg and Lowenfels, and Fama, is a market in which publicly-available information is rapidly incorporated into the price of a security such that the trading price reflects all publicly-available information. As these cases and *Halliburton II* recognized, market efficiency is relevant to a securities case as it addresses the question of whether false information (e.g., in the form of an alleged misrepresentation or omission) would likely have impacted the prices at which investors bought and sold securities, and which were relied upon.

A. The *Cammer* Factors

48. The *Cammer* opinion lays out five factors that generally indicate the market for a security is efficient. As described below, economic rationales support each factor as an indicator of market efficiency. The five factors are: 1) trading volume, 2) coverage by securities analysts, 3) number of market makers, 4) eligibility for S-3 registration, and 5) empirical evidence that the security price reacts to new, company-specific information.

49. Empirical research has confirmed that volume, number of market makers, and analyst coverage are indicative of market efficiency:

> "Consistent with the efficiency indicators used recently by the courts, the inefficient firms have lower mean trading volume, fewer market makers, lower analyst following, and lower institutional ownership (number and percentage) than efficient firms."
> **"The Fraud-on-the-Market Theory and the Indicators of Common Stocks' Efficiency," by Brad M. Barber, Paul A. Griffin, and Baruch Lev, *Journal of Corporation Law*, 1994, p. 302.**

50. Barber, et al., also found that high institutional ownership was indicative of market efficiency.

51. With respect to the empirical factor, Barber, et al., used empirical tests as the standard for market efficiency by which to judge the significance of the other variables. Consequently, they acknowledge the importance of the empirical factor.

52. Consistent with financial economic theory and empirical research, the language used by the *Cammer* court describes the factors not as five ***necessary*** factors, but rather as indicative of the degree to which the market for a security is expected to be efficient:

> "There are several different characteristics pertaining to the markets for individual stocks which are probative of the degree to which the purchase price of a stock should reflect material company disclosures."
> ***Cammer*, 711 F. Supp. at 1283.**

53. The *Cammer* opinion describes the nature of the five factors as follows:

> "There are several types of facts which, if alleged, might give rise to an inference that [the company] traded in an efficient market. It is useful to set forth an explanation of how the existence of such facts would cause the understanding that disclosed company information (or misinformation) would be reflected in the company's stock price, the underpinning of the fraud on the market theory.
> **Id. at 1285-86, *Peil, supra, 806 F.2d at 1160* (footnote omitted).**

> "First, plaintiffs could have alleged there existed an average weekly trading volume during the class period in excess of a certain number of shares."
> **Id. at 1286.**

> "Second, it would be persuasive to allege a significant number of securities analysts followed and reported on a company's stock during the class period."
> **Id.**

> "Third, it could be alleged the stock had numerous market makers."
> **Id.**

> "Fourth, as discussed it would be helpful to allege the company was entitled to file an S-3 Registration in connection with public offerings…"
> **Id. at 1287.**

Case: 4:15-cv-00060-5B62-DSC #: 563-2 Filed: 07/01/18 10/25/15 70 PageID #: 413977

> "Finally, it would be helpful to a plaintiff seeking to allege an efficient market to allege empirical facts showing a cause and effect relationship between unexpected corporate events or financial releases and an immediate response in the stock price."
> **Id.**

> "As previously noted, one of the most convincing ways to demonstrate efficiency would be to illustrate, over time, a cause and effect relationship between company disclosures and resulting movements in stock price."
> **Id. at 1291.**

B. The *Krogman* Factors

54. In addition to the five *Cammer* factors that indicate market efficiency, the district court in *Krogman v. Sterritt*, 202 F.R.D. 467 (N.D.Tex. 2001), and the Fifth Circuit Court of Appeals in *Unger v. Amedisys*, 401 F.3d 316 (5th Cir. 2005), concluded that three additional factors were also indicative of market efficiency.

55. These additional factors, the *Krogman* factors, are: 1) the company's market capitalization, 2) the stock's float, and 3) the typical bid-ask spread.

56. Market capitalization is the total value of all outstanding shares. It equals the number of shares outstanding times the price per share. Reasonably, the larger a firm's market capitalization, the more prominent and well known the company will be. Larger companies tend to attract more analyst and news media coverage, and gain the attention of greater numbers of investors, including large institutional investors. All of these characteristics, which accompany a large market capitalization, promote market efficiency.

57. The stock's float is the number of shares outstanding, less shares held by insiders and affiliated corporate entities. It is generally the number of shares available for trading by outside investors in the open market. Float is highly correlated with market capitalization, but it focuses on the shares available for trading rather than all outstanding shares. Stocks with large floats tend to trade more actively, attract more analyst and news media coverage, and garner the attention of greater numbers of investors, including large institutional investors. All of these characteristics, that accompany a high float level, promote market efficiency.

58. The bid-ask spread is the difference between the price at which market makers are offering to buy a security and the price at which they are offering the security for sale. If a security is actively traded and information about the security is readily available, the bid-ask spread will tend to be narrow. Moreover, a narrow bid-ask spread makes trading in the security less costly for investors, and thereby tends to attract greater interest, greater coverage, and greater volume, which in turn are factors that are generally considered to promote market efficiency.

EFFICIENCY OF THE MARKET FOR PETROBRAS COMMON ADRS

59. To assess whether the market for Petrobras common ADRs was efficient during the Class Period, I analyzed the market for, and behavior of, Petrobras common ADRs, focusing on the factors that are generally accepted to be indicative of market efficiency.

A. *Cammer* and *Krogman* Factor Analysis

1. Trading Volume

60. Throughout the Class Period, Petrobras common ADRs traded regularly and actively. On average, 21.3 million common ADRs changed hands daily.[23] On one day, 27 October 2014, over 164 million common ADRs traded. Petrobras' common ADR trading data are presented in Exhibit-4a.

61. In addition to average daily trading volume, another volume metric to consider in determining market efficiency is the percentage of outstanding shares that turn over each week. During the Class Period, the average weekly trading volume of Petrobras common ADRs was approximately 106.4 million ADRs, or 14.1% of all common ADRs outstanding.[24] This level of trading activity is well above levels accepted by courts as being indicative of market efficiency for common stock.[25] In *Cammer*, the court cited the conclusion of Alan R. Bromberg and Lewis D. Lowenfels that "weekly trading of 2% or

[23] Data obtained from FactSet.

[24] Estimated by averaging the ratio of the daily trading volume to the number of common ADRs outstanding, and multiplying by 5 (the number of trading days in a typical week).

[25] *Cammer*, 711 F. Supp. at 1286.

Case 1:14-cv-09662-JSR Document 287 Filed 10/05/15 Page 15 of 370 PageID #: 18579

more of the outstanding shares would justify a strong presumption that the market for the security is an efficient one; 1% would justify a substantial presumption."[26] The trading volume for Petrobras common ADRs during the Class Period was far more than the threshold for a strong presumption of market efficiency.

62. Both in terms of average daily trading volume and also on the basis of the percentage of outstanding shares traded weekly, the market for Petrobras common ADRs was very active. Consistent with the *Cammer* opinion, economic theory, and empirical research, the active trading volume in Petrobras common ADRs is strong evidence of the efficiency of the market for Petrobras common ADRs over the course of the Class Period.

2. <u>Analyst Coverage and Other Avenues of Information Dissemination</u>

Analyst Coverage

63. Securities analysts disseminate and interpret information about the companies they cover. They conduct research and provide valuation opinions, helping market participants acquire relevant information and understand the implications of that information for valuation and investment decisions. Consequently, securities analysts facilitate the flow of information and the digestion of information within the marketplace. These functions promote market efficiency.

64. Petrobras was the subject of broad analyst coverage throughout the Class Period. From the Thomson Research database, I obtained analyst reports on Petrobras published during the Class period from 40 firms including Barclays, Credit Suisse, Deutsche Bank, ITAU, HSBC, Jeffries, JP Morgan, Morgan Stanley, Planner Corretora, Raymond James, and UBS.[27]

65. Transcripts of Petrobras' conference calls conducted during the Class Period reveal that at least 12 additional firms also followed Petrobras: Banco Espirito Santo, Bank of America Merrill Lynch, Bradesco, Citi, Goldman Sachs, FMC, Hudson Bay Capital, Northwestern, Spectari, UNUM Group, and Tudor, Pickering, Holt & Co.

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[26] Id., at 1293.

[27] The complete list of analyst reports I obtained is presented in Exhibit-1.

Case: 1:14-cv-09662-JSR Document 567-14 Filed 10/01/15 Page 18 of 37 PageID #: 13980

66. Consequently, at least 52 firms covered Petrobras during the Class Period.

67. Barber, et al., found that coverage by one or two analysts strengthened the presumption of efficiency for a publicly traded stock.[28] Consistent with the *Cammer* opinion, financial economic principles, and published research the widespread coverage of Petrobras by professional securities analysts is evidence of the efficiency of the market for Petrobras common ADRs during the Class Period.

Institutional Ownership and Buy-Side Analysis

68. FactSet provides data on institutional ownership of Petrobras common ADRs. The data are compiled from the Form 13-F filings that major investment institutions are required to submit to the SEC. Major institutions are defined as firms or individuals that exercise investment discretion over the assets of others in excess of $100 million. Large investment firms generally employ financial analysts who conduct their own research on the stocks they buy. According to the FactSet data, a total of 1,598 major institutions owned Petrobras common ADRs during the Class Period.[29]

News Coverage

69. News media also facilitate the flow of material information to the marketplace, thereby promoting market efficiency. In the case of Petrobras, such coverage was extensive. A Factiva database search established that approximately 20,000 articles were published about the Company during the Class Period.[30]

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[28] Brad M. Barber et al., "The Fraud-on-the-Market Theory and the Indicators of Common Stocks' Efficiency," *Journal of Corporation Law*, 1994.

[29] According to filings that reported holdings, there were 1,598 institutions that held Petrobras common ADR as of the end of each quarter during the class period (31 March 2010, 30 June 2010, 30 September 2010, 31 December 2010, 31 March 2011, 30 June 2011, 30 September 2011, 31 December 2011, 31 March 2012, 30 June 2012, 30 September 2012, 31 December 2012, 31 March 2013, 30 June 2013, 30 September 2013, 31 December 2013, 31 March 2014, 30 June 2014, 30 September 2014, 31 December 2014, 31 March 2015, and 30 June 2015). There may have been additional institutions that held Petrobras common ADRs during the Class Period, though not on the quarterly reporting dates.

[30] Based on a Factiva search for articles published during the Class Period where "Petroleo Brasileiro SA" was the "Company" search field parameter, the language was "English," and the sources were limited to newswires, the *Wall Street Journal*, or "Major News and Business Sources."

70. The articles obtained from Factiva include published news articles and press releases. Information also emerged throughout the Class Period in the form of SEC filings, conference calls, and presentations.

71. During the Class Period, therefore, information about Petrobras was readily available to market participants as there was a consistent flow of news provided by news media, analysts, and various other sources. This extensive news coverage is further evidence of the efficiency of the market for the Petrobras common ADRs.

72. Petrobras was not an obscure company, escaping the notice of analysts and investors. Rather, the Company was large, well known, widely covered, and widely held. These facts strongly support a finding that the market for Petrobras common ADRs was an efficient market during the Class Period.

3. Market Makers and Listing on the New York Stock Exchange

73. The number of market makers is one of the factors the *Cammer* court determined indicates market efficiency. Market makers are financial intermediaries who trade in a particular security, standing ready to buy and sell with individual investors, institutions, and other market makers. A large number of market makers implies that many market participants are trading that particular stock, which generally results in a high degree of liquidity and a narrower bid-ask spread. With a large number of market makers, it is generally easy for investors to execute trades in a timely fashion and with reasonable transaction costs.

74. The subject company in the *Cammer* case, Coated Sales, Inc., was listed on the NASDAQ, an over-the-counter market consisting of multiple competing market makers, using electronic systems to make quotes and effect trades.

75. The *Cammer* court's understanding that the market-making infrastructure of a stock market is indicative of its efficiency, or lack thereof, makes the fact that Petrobras common ADRs traded on the venerable New York Stock Exchange during the Class Period highly relevant. The NYSE is one of the most renowned, most liquid, and most efficient forums for trading stocks in the world. Stocks on the NYSE are traded under the supervision of a lead market maker or "Designated Market Maker" ("DMM"), formerly

known as a specialist.[31] DMMs are responsible for maintaining a fair and orderly market for each security in which they are registered.[32]

76. In fact, citing Bromberg and Lowenfels, the *Cammer* court explicitly acknowledged the importance of an NYSE listing and the implications of such a listing for market efficiency.

> "We think that, at a minimum, there should be a presumption – probably conditional for class determination – that certain markets are developed and efficient for virtually all the securities traded there: the New York and American Stock Exchanges, the Chicago Board Options Exchange and the NASDAQ National Market System."
> ***Cammer*, 711 F. Supp. at 1292 (quoting Bromberg & Lowenfels, *Securities Fraud and Commodities Fraud*, §8.6 (1988)).**

77. At the time of the *Cammer* opinion the NYSE and NASDAQ were distinctly separate exchanges. NASDAQ market makers did not make markets for NYSE-listed securities such as the Petrobras common ADRs. However, since that time, the stock markets have evolved dramatically. Beginning in April 2005, NASDAQ enabled trading in most NYSE-listed stocks through its market maker structure.[33] This NASDAQ market making activity is in addition to the principal market for listed stocks on the NYSE.

78. From February 2010 through June 2015, there were at least 575 market makers for Petrobras common ADRs, including well known firms such as: Goldman Sachs, JP Morgan, Citigroup, and Morgan Stanley.[34]

79. The facts that it traded on the NYSE and had a large number of market makers are strong evidence that the Petrobras common ADRs traded in an efficient market throughout the Class Period. That the Petrobras common ADRs were listed on the NYSE gave its common ADRs access to a highly developed network of brokers with its market overseen by the NYSE DMM. These facts are compelling evidence of the efficiency of the market for Petrobras common ADRs.

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[31] "Fact Sheet; Designated Market Makers," NYSE Euronext, 2012.

[32] "Organization and Functioning of Securities Markets," by Frank Reilly and Keith Brown, in *Equity and Fixed Income CFA Program Curriculum*, vol. 5, Pearson Custom Publishing, 2008.

[33] "Nasdaq To Enable Customers To Trade NYSE Stocks," *Reuters*, March 28, 2005.

[34] Market maker data obtained from Bloomberg.

4. F-3 Registration Eligibility

80. The *Cammer* court noted that S-3 registration eligibility is indicative of market efficiency because the filing requirement ensured that financial data were available to market participants, and the "public float" requirement indicated that many market participants would have examined the information.[35]

> "The 'public float' aspect of the Form S-3 requirements ensures that enough investors have in fact read the previously filed document."
> **Id. at 1285.**

> "Again, it is the number of shares traded and value of shares outstanding that involve the facts which imply efficiency."
> **Id. at 1287.**

81. F-3 registration is for foreign companies listed on U.S. exchanges what S-3 registration is for domestic companies. A U.S. company is eligible for S-3 registration, and a foreign company is eligible for F-3 registration, when, among other things, it has filed Exchange Act reports for a specified length of time and has outstanding float above a certain sizable value. At the time of the *Cammer* opinion, the conditions for S-3 registration were that a company had filed financial reports with the SEC for 36 months, and had outstanding float over $150 million held by non-affiliates, or $100 million of such float coupled with annual trading volume exceeding 3 million shares. Eligibility for Form F-3 registration is the same as for S-3, except that the company must be a foreign private issuer, i.e. not a U.S. company.[36]

82. In 1992, the SEC amended its requirements for S-3 registration eligibility to 12 months of filings and at least $75 million of float. Since 2007, the SEC has allowed domestic companies with less than $75 million of float to file an S-3 registration so long as the company has been filing financial reports for at least a year, has "a class of common equity securities listed and registered on a national securities exchange, and the issuers do

[35] *Cammer*, 711 F. Supp. at 1284-85.

[36] "Eligibility of Smaller Companies to Use Form S-3 or F-3 for Primary Securities Offerings," SEC website, accessed at www.sec.gov/info/smallbus/secg/s3f3-secg htm, 28 January 2008.

not sell more than the equivalent of one-third of their public float in primary offerings over any period of 12 calendar months."[37] However, despite the fact that the $75 million requirement has been loosened, courts continue to focus on this $75 million benchmark when analyzing this *Cammer* factor.[38]

Float

83. A company's float is the number or value of shares that can potentially trade freely in the marketplace. It is generally defined as the number or value of outstanding shares, minus insider holdings and shares owned by affiliated corporate entities.[39]

84. I computed Petrobras' common ADR float using data on shares outstanding and insider holdings presented on the Company's website, and ADR price data obtained from FactSet.

85. Petrobras common ADR float averaged $16.9 billion during the Class Period, far exceeding the level required for F-3 registration. During the Class Period, float ranged between $3.96 billion and $33.2 billion, always far exceeding the minimum requirement for F-3 registration eligibility.

Financial Filings

86. Petrobras regularly filed financial reports with the SEC throughout the Class Period. The financial information in the SEC filings, supplemented by information provided by analysts and news coverage, provided investors with access to financial information about the Company on a continuous basis.

87. In the latter part of the Class Period, however, Petrobras announced that it would have to delay its 3Q 2014 financial filing for the fiscal quarter ended 30 September 2014. According to the Company, the consolidated 3Q 2014 filing was delayed because of, "the accusations and investigations of the 'Lava Jato Operation' being conducted by the

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[37] "Revisions To The Eligibility Requirements For Primary Securities Offerings On Forms S-3 And F-3," SEC Release No. 33-8878, 19 December 2007.

[38] See, e.g., *Vinh Nguyen v. Radient Pharm. Corp.*, 287 F.R.D. 563, 573 (C.D. Cal. 2012).

[39] For a discussion of the generally accepted definitions of shares outstanding and float, see "Float Adjustment Methodology," *S&P Dow Jones Indices*, July 2012.

Brazilian Federal Police."[40] Despite meeting the minimum float requirements for F-3 registration, the Company was ineligible for F-3 registration on account of its delay in filing the consolidated 3Q 2014 financials. By the Company's own admission, the filing delay was related to the Company's misdeeds and circumstances that are at the core of this litigation.

88. On 14 November 2014, Petrobras announced that it would have to delay its Q3 2014 filing. The Company cured the delinquency on 23 April 2015 with the filing of its audited Q3 2014 financials. In the interim, unconsolidated financial information was available in the SEC filings which, supplemented by information provided by analysts and news coverage, provided investors with access to financial information about the Company on a continuous basis.

89. F-3 registration eligibility indicates company characteristics associated with market efficiency, in particular characteristics of size, transparency, and the availability of relevant financial information. Petrobras possessed those particular characteristics throughout the Class Period, with the exception of the period near the end of the Class Period, when there was a filing delinquency that is among the bases for the claims in this case.

90. Not only was Petrobras eligible for F-3 registration, but the Company did file F-3 Registration Statements prior to the Class Period on 11 December 2009 and during the Class Period on 29 August 2012.[41] Consistent with the *Cammer* opinion, Petrobras' eligibility to file an F-3 registration is indicative of the efficiency of the market for Petrobras common ADRs during the Class Period.

5. <u>Market Capitalization</u>

91. Market capitalization is the total value of a firm's outstanding common stock. The larger a firm's market capitalization, the more prominent and well known the company will be. Larger companies tend to attract more analyst and news media coverage, and gain the attention of greater numbers of investors, including large institutional investors. All of

[40] Petrobras, "Third quarter 2014 financial statements," Form 6-K filed on 14 November 2014.

[41] Form F-3 Registration Statement, filed by Petróleo Brasileiro S.A. on 11 December 2009; Form F-3 Registration Statement, filed by Petróleo Brasileiro S.A. on 29 August 2012.

these characteristics, which accompany a large market capitalization, promote market efficiency.

92. I measured the market capitalization of Petrobras inclusive of all its outstanding common shares, as well as the market capitalization comprised by just the common ADRs in isolation. Both values are extremely large. During the Class Period, the market capitalization of Petrobras common stock averaged $78.3 billion. The value of Petrobras' common stock is greater than the market capitalization values of at least 90% of all publicly-traded companies in the U.S.[42]

93. On average during the Class Period, there were 756.1 million common ADRs outstanding. The aggregate market value of the common ADRs averaged $16.9 billion during the Class Period. This aggregate value placed Petrobras' common ADRs within the 1st decile of U.S. companies – meaning that the aggregate value of Petrobras' common ADRs alone was larger than the market capitalizations at least 90% of all publicly-traded companies in the U.S.

94. Consistent with the *Krogman* court's opinion, Petrobras' large market capitalization – for the common ADRs in insolation as well as for the Company as a whole – throughout the Class Period is further evidence of the efficiency of the market for Petrobras common ADRs.

6. Float

95. While float would exclude shares held by insiders and affiliated corporate entities, according to the Company's SEC filings, none of the Petrobras common ADRs were held by insiders or affiliated corporate entities.[43] Petrobras' common ADR float is thus equal to its common ADR market capitalization, which, averaging $16.9 billion during the Class Period, was larger than the total market capitalizations of at least 90% of all publicly-traded companies in the U.S. The size of Petrobras' common ADR float

[42] This calculation is based on averaged month-end data from CRSP for January 2010 through December 2014. I grouped public companies into deciles, so that the 1st decile contains the largest 10% of all public companies listed on the NYSE, Amex, and NASDAQ, while the 10th decile contains the smallest 10%.

[43] Based on a review of Petrobras' annual reports on form 20-F filed with the SEC for the Fiscal Years 2010 – 2015 as well as the prospectus supplement dated 23 September 2010.

indicates it handily surpassed the standard of the second *Krogman* factor for market efficiency.

96. Float can also be analyzed as a percentage of total common ADRs outstanding, as well as in share and value terms. On average during the Class Period, there were 756.1 million common ADRs in Petrobras' float and the same number of common ADRs outstanding, resulting in a float of 100% of common ADRs outstanding.

97. Petrobras' substantial common ADR float (by any measure) is indicative of the efficiency of the market for the common ADRs during the Class Period.

7. Bid-Ask Spread

98. I obtained data on daily closing bid and ask quotes for Petrobras common ADRs during the Class Period from FactSet. I measured the percentage bid-ask spread as the difference between the bid and ask quotes, divided by the average of the bid and ask quotes, which is the standard way of measuring percentage bid-ask spreads in the finance literature. Exhibit-4a presents Petrobras' common ADR bid-ask spread data.

99. The average bid-ask spread for Petrobras common ADRs over the course of the Class Period was 0.09%. By comparison, the average month-end bid-ask spread over the course of the Class Period for all stocks in the CRSP database was 0.59%. Petrobras' common ADR bid-ask spreads were substantially narrower than the mean level among all other CRSP stocks – which are the stocks traded on the NYSE, Amex, NASDAQ, and NYSE Arca.

100. In dollar terms, Petrobras' common ADR bid-ask spread during the Class Period averaged $0.01 per common ADR. For all stocks in the CRSP database, the average bid-ask spread during the Class Period was $0.08 per share.

101. The average bid-ask spread in the market for Petrobras common ADRs over the course of the Class Period was well below the typical bid-ask spreads exhibited by other publicly-traded stocks in the United States. Petrobras' narrow bid-ask spread strongly supports a conclusion of market efficiency.

EMPIRICAL EVIDENCE OF MARKET EFFICIENCY

102. Of the five *Cammer* factors, the empirical factor was cited by the *Cammer* court as "one of the most convincing ways to demonstrate efficiency":

> "As previously noted, one of the most convincing ways to demonstrate efficiency would be to illustrate over time, a cause and effect relationship between company disclosures and resulting movements in stock price."
> ***Cammer*, 711 F. Supp. at 1291.**

103. The special importance the *Cammer* court placed on the empirical factor is justified by economic principles, as the empirical factor focuses on the essence of market efficiency whereas the other four factors are indicators that generally signal market efficiency.

104. I conducted four sets of empirical tests of the efficiency of the market for Petrobras common ADRs using the Company's 6-K filings during the Class Period.

105. The first empirical test was an event study that investigates whether the Petrobras common ADRs exhibited statistically significant movements in reaction to allegation-related disclosures more frequently than on all other trading days. If the frequency of statistically significant returns is greater for an objectively selected sample of allegation-related disclosure events than within the population of all other days, this result would indicate a cause and effect relationship between the release of case-related information and ADR price movements. Such a finding would indicate market efficiency generally, and also market efficiency with respect to the specific information at issue in this case.

106. As described in greater detail below, for this portion of the study, I identified Company 6-K filings that mentioned "corruption" in the 6-K filing in text that conveyed new information rather than repeated boilerplate language. Tested event dates were the respective dates the information cited in the 6-Ks was first received by the market, making proper adjustments if the news was received after the close of trading. Selecting case-related or corruption-related events in this manner is an objective, replicable event selection methodology that requires no assumptions regarding the merits of Plaintiff's allegations.

107. The second empirical test was an event study that investigates whether the Petrobras common ADRs exhibited statistically significant movements in reaction to earnings announcements more frequently than on all other days. If the frequency of statistically significant returns is greater within the sample of earnings announcement events than within the population of all other days, this result would indicate a cause and effect relationship between the release of Company-specific information and ADR price movements throughout the Class Period. Such a finding would indicate market efficiency over the course of the Class Period.

108. The third empirical test combined the first two tests, examining the behavior of the common ADRs in reaction to case-related disclosures and earnings announcement dates collectively. More frequent statistically significant security price movements in reaction to these events as compared to the frequency of statistically significant price movements on all other days would be compelling proof of market efficiency.

109. The fourth empirical test was an event study that examined a broad set of news events that occurred over the course of the estimation period. The events tested were all the Company's 6-Ks filed with the SEC. I examined news articles to determine when the information announced in the Company's 6-K was first received by the market, which established the event dates to be tested for statistical significance. If the 6-K days more frequently exhibited statistically significant common ADR returns than all other days, this finding would indicate a cause and effect relationship between the release of information and the common ADR's price movements, which would be additional compelling empirical proof of market efficiency.

A. Event Study Test of Market Efficiency

110. The event study is the paramount tool for testing market efficiency, as renowned financial economist and Nobel laureate Eugene Fama attests:

> "The cleanest evidence on market-efficiency comes from event studies, especially event studies on daily returns. When an information event can be dated precisely and the event has a large effect on prices, the way one abstracts from expected returns to measure abnormal daily returns is a second-order consideration. As a result, event studies give a clear picture of the speed of adjustment of prices to information."
> **"Efficient Capital Markets: II," by Eugene F. Fama, *Journal of Finance*, 1991, p. 1607.**

111. Event study analysis is one of the most commonly used analytic methodologies employed by finance researchers. Campbell, Lo, and MacKinlay [1997] present an excellent description and examples of the methodology and write about how it is generally accepted and widely used in academic research.[44] Crew, et al., [2012] write about how the methodology is generally accepted and widely used in forensic applications.[45]

112. An event study measures how much a security price rises or falls in response to new, company-specific information. Statistical regression analysis determines how much of a security price change is explained by market factors, rather than company-specific information, so that those influences can be statistically factored out. The portion of a security price change that cannot be attributable to market factors is called the residual price movement or "residual return." The event study isolates the residual return and also tests whether the residual return can reasonably be explained as merely a random fluctuation.

113. If the security return over an event period is statistically significant, it indicates that the price movement cannot be attributed to market factors, or to random volatility, but rather was caused by new, company-specific information. Such proof of a cause and effect relationship between the release of information and movement in the security price establishes market efficiency.

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[44] Chapter 4 of *The Econometrics of Financial Markets*, by John Y. Campbell, Andrew W. Lo, and A. Craig MacKinlay, Princeton University Press, 1997.

[45] "Federal Securities Acts and Areas of Expert Analysis," by Nicholas I. Crew, et al., in Chapter 24 of the *Litigation Services Handbook*; *The Role of the Financial Expert*, 5th ed., edited by Roman L. Weil, Daniel G. Lentz, and David P. Hoffman, John Wiley & Sons, Inc., 2012.

Case: 1:16-cv-05362-... Document #: ... Filed: ... Page ... of ... PageID #: ...

1. A Caveat about Non-Significant Security Price Movements

114. It is important to note that an event study tests the joint hypothesis that the security trades in an efficient market and that the valuation impact of the information disseminated on an event date is of such large magnitude as to exceed the threshold for statistical significance. Therefore, a finding of statistical significance indicates market efficiency, but a finding of non-significance does not necessarily establish inefficiency, as a modest non-significant price reaction may be the appropriate and efficient reaction to a particular disclosure.

115. For example, if a company reports earnings that are in line with the prior expectations of analysts and investors, even though the announcement would be important, the mix of information may not have changed sufficiently on that date to elicit a statistically significant security price reaction. In this example, a modest movement or even no movement at all may be the appropriate price reaction. In such a case, an event study finding that a return was non-significant would not indicate inefficiency. On the contrary, the appropriately non-significant price movement would show that the security is behaving as it should in an efficient market.

116. Similarly, when a company misleads analysts and investors by concealing important information, the effect of the concealment would generally not be a significant security price movement at the time of the concealment and over its duration. The concealment would maintain the mix of information as it previously was, so the appropriate price reaction would be a maintenance of the price level where it previously was.

117. Therefore, appropriate candidate events or groups of events for inclusion in a market efficiency event study should include events on which company-specific information was released that is new, unexpected, and may reasonably contain information of such import as to be expected to elicit a price reaction over the threshold for statistical significance. It is important to note, however, that especially when selecting groups of events using an objective screen capturing all events of a certain type (i.e., 6-Ks that mention corruption, earnings announcements, or all 6-Ks), it is not only possible but likely that many of the events thusly selected will not contain information that should elicit a large security price reaction. For this reason, for an event study of this type, the event results should be

evaluated collectively, on the basis of sample frequency of statistically significant returns.[46]

2. Selection of Corruption-Related 6-K Events

118. Not only did the *Cammer* court single out the empirical factor as "one of the most convincing ways to demonstrate efficiency," but it also recognized the special importance of the specific information allegedly misrepresented that is the subject of the litigation:

> "The central question under the fraud on the market theory is whether the stock price, at the time a plaintiff effected a trade, reflected the 'misinformation' alleged to have been disseminated."
> ***Cammer*, 711 F. Supp. at 1282 (emphasis in original).**

119. By focusing an event study on disclosures of information related to the allegations in the Complaint, one is able to ascertain whether the market for the Petrobras common ADR was efficient, not only generally, but also with respect to the particular information at issue in this case. Consequently, the empirical behavior of the Petrobras common ADR following case-related announcements is important for determining whether the market for the Company's common ADR was efficient during the Class Period.

120. An event study testing market efficiency does not require a comprehensive identification of all events during the Class Period, including all of those cited in the Complaint,[47] on which new material allegation-related information was disclosed. An objective screen may capture only some of those events. Further, because of the high threshold for statistical significance, it is also important to note that new information may be economically significant without being statistically significant.

121. To identify potential allegation-related event dates, I conducted a search of the 863 6-Ks filed by the Company's during the Class Period[48] which contained the search term:

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[46] See, for example: "The 'Less Than' Efficient Capital Markets Hypothesis: Requiring More Proof from Plaintiffs in Fraud-on-the-Market Cases," by Paul A. Ferrillo, Frederick C. Dunbar and David Tabak, *78 St. John's L. Rev. 81*, 119-22 (2004).

[47] A comprehensive identification of all disclosures of information related to the alleged fraud is beyond the scope of this report, and is properly addressed in an analysis of loss causation and damages.

[48] Petrobras Form 6-Ks were obtained from FactSet.

"corrupt*".[49] This search identified 49 6-Ks. I then removed from this sample 41 dates where the term "corrupt*" was used only in boilerplate language.[50] Finally, I reviewed each of the remaining 6-Ks to determine whether the information disclosed in the 6-K was new valuation-relevant information.[51] Based on this criteria, I removed two additional dates from the sample.[52]

122. The list of the 6-K allegation-related disclosure event dates I identified based on the above criteria is as follows: 27 October 2014; 14 November 2014; 15 December 2014; 28 January 2015; 30 January 2015; and 23 April 2015 (the "corruption-related 6-K dates"). The news on these dates included the following information:

i. 27 October 2014 – Petrobras filed a 6-K titled "Internal Steps Taken by Petrobras in Response to 'Lava Jato Operation'."[53]

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[49] The term "Corrupt*" covers all iterations of the term "Corrupt" such as "Corruption" or "Corruptible". All 6-Ks identified by the complete screening process, however, referred to information related to the corruption scandal at issue in this case.

[50] *See*¸ for example: "The Company's actual results could differ materially from those expressed or forecast in any forward-looking statements as a result of a variety of assumptions and factors. These factors include, but are not limited to, the following: (i) failure to comply with laws or regulations, including fraudulent activity, corruption, and bribery; (ii) the outcome of ongoing corruption investigations and any new facts or information that may arise in relation to the "Lava Jato Operation"; (iii) the effectiveness of the Company's risk management policies and procedures, including operational risk; and (iv) litigation, such as class actions or proceedings brought by governmental and regulatory agencies. A description of other factors can be found in the Company's Annual Report on Form 20-F for the year ended December 31, 2013, and the Company's other filings with the U.S. Securities and Exchange Commission" (Petrobras Form 6-K, filed 10 February 2015); and "Lastly, Petrobras reiterates its commitment to business ethics and transparency and the need to adopt consolidated compliance measures in the Petrobras Corruption Prevention Program (PPPC), which expressly provides for the application of sanctions against suppliers that breach its Code of Ethics as well as other items of the PPPC" (Petrobras Form 6-K, filed 9 March 2015).

[51] The determination of whether the information disclosed in the 6-K was new valuation-relevant information was based solely on the contents of the filing itself. There may have been additional information that was revealed that day other than the contents of the filing.

[52] The dates of the removed 6-Ks were 17 March 2015 and 8 July 2015.

[53] Petrobras Form 6-K, filed 27 October 2014.

ii. 14 November 2014 – Petrobras filed a 6-K titled "Third Quarter 2014 Financial Statements," in which it stated "that it will not file its third quarter 2014 financial statements."[54] According to the Company, the consolidated 3Q 2014 filing was delayed because of, "the accusations and investigations of the 'Lava Jato Operation' being conducted by the Brazilian Federal Police."[55]

iii. 15 December 2014 – Petrobras filed a 6-K titled "Third Quarter 2014 Condensed Information," in which it stated that "in light of new facts that occurred after November 13, 2014, directly or indirectly related to the 'Lava Jato Operation', it decided not to file its consolidated interim financial statements for the 3rd quarter 2014 not reviewed by the independent auditors."[56]

iv. 28 January 2015 – Petrobras filed a 6-K titled "Third Quarter Of 2014 Results Not Reviewed By Independent Auditors," in which it stated that "the Company understands that it will be necessary to make adjustments at the financial statements to correct the values of fixed assets that may have been impacted by amounts related to misconducts made by suppliers, politicians, Petrobras employees and other groups in the context of the 'Lava Jato Operation'."[57]

v. 30 January 2015 – Petrobras filed a 6-K titled "Moody's reviews Petrobras' Global Ratings," in which it stated that Moody's had lowered the Company's credit rating from Baa2 to Baa3 reflecting "concerns with the ongoing investigation of improper payments and possible liquidity pressures that may arise if the Company is not able to timely deliver its audited financial statements."[58]

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[54] Petrobras Form 6-K, filed 14 November 2014.

[55] Petrobras, "Third quarter 2014 financial statements," Form 6-K filed on 14 November 2014.

[56] Petrobras Form 6-K, filed 15 December 2014.

[57] Petrobras Form 6-K, filed 28 January 2015.

[58] Petrobras Form 6-K, filed 30 January 2015.

vi. 23 April 2015 - Petrobras filed four 6-Ks for its audited financial results for 3Q 2014, which included additional information concerning the impact of the corruption-related issues.[59]

3. Selection of Earnings Announcement Dates

123. A company's financial results and forecasts are among the most important considerations to investors assessing the value of its securities. Consequently, such announcements typically contain material information that could cause the security price to change.

> "No other figure in the financial statements receives more attention by the investment community than earnings per share. The relationship between accounting earnings and security prices is probably the single most important relationship in security analysis, and its prominence is reflected in the attention given to price-earnings ratios."
> ***Financial Reporting an Accounting Revolution*, 3rd ed., William H. Beaver, 1998, p. 38.**

> "Analysts, investors, senior executives, and boards of directors consider earnings the single most important item in the financial reports issued by publicly held firms."
> **"Earnings Management to Exceed Thresholds," Francois Degeorge, Jayendu Patel, and Richard Zeckhauser, *Journal of Business*, 1999, p. 1.**

124. Numerous well-known and highly-regarded academic studies (for example, Beaver [1968], Ball and Brown [1968], Ball [1978], Watts [1978], Patell and Wolfson [1984], and Ball and Kothari [1991]) have specifically examined stock price movements caused by earnings announcements, and concur that earnings announcements are generally important information events.

125. Consequently, a pattern of more frequent statistically significant security price movements on earnings announcement days as compared to all other days is indicative of market efficiency.

126. Table-1 below presents the 23 dates on which Petrobras announced earnings during the Class Period and their respective test dates:

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[59] Petrobras Form 6-K, filed 23 April 2015. The Company filed four 6-Ks on this date, all of these contained information regarding the Company's audited financial results for 3Q 2014 and passed the screen for the term "corrupt*".

Table – 1: Earnings and Guidance Dates During the Class Period

	Announcement Date	Announcement Time	Effective Event Test Date	Title of Company Press Release	Source
[1]	3/24/2010	10:00 AM	3/24/2010	Petrobras Announces Fiscal Year 2009 Results	*FactSet*
[2]	5/18/2010	11:00 AM	5/18/2010	Petrobras Announces First Quarter 2010 Results	*FactSet*
[3]	8/17/2010	12:00 PM	8/17/2010	Petrobras Announces First Half 2010 Results	*FactSet*
[4]	11/16/2010	9:00 AM	11/16/2010	Petrobras Announces Third Quarter 2010 Results	*FactSet*
[5]	3/1/2011	11:00 AM	3/1/2011	Petrobras Announces Fiscal Year 2010 Results	*FactSet*
[6]	5/17/2011	10:30 AM	5/17/2011	Petrobras Announces First Quarter 2011 Results	*FactSet*
[7]	8/17/2011	10:30 AM	8/17/2011	Petrobras Announces Second Quarter 2011 Results	*FactSet*
[8]	11/16/2011	9:00 AM	11/16/2011	Petrobras Announces Third Quarter 2011 Results	*FactSet*
[9]	2/14/2012	9:00 AM	2/14/2012	Petrobras Announces Fiscal Year 2011 Results	*FactSet*
[10]	5/17/2012	10:00 AM	5/17/2012	Petrobras Announces First Quarter 2012 Results	*FactSet*
[11]	8/8/2012	9:00 AM	8/8/2012	Petrobras Announces First Half 2012 Results	*FactSet*
[12]	10/29/2012	8:00 AM	10/31/2012*	Petrobras Announces Third Quarter 2012 Results	*FactSet*
[13]	2/5/2013	8:00 AM	2/5/2013	Petrobras Announces Fourth Quarter 2012 Results	*FactSet*
[14]	4/29/2013	9:00 AM	4/29/2013	Petrobras Announces First Quarter 2013 Results	*FactSet*
[15]	8/12/2013	9:00 AM	8/12/2013	Petrobras Announces First Half 2013 Results	*FactSet*
[16]	10/28/2013	8:00 AM	10/28/2013	Petrobras Announces Third Quarter 2013 Results	*FactSet*
[17]	2/26/2014	8:00 AM	2/26/2014	Petrobras Announces Fourth Quarter 2013 Results	*FactSet*
[18]	5/12/2014	9:00 AM	5/12/2014	Petrobras Announces First Quarter 2014 Results	*FactSet*
[19]	8/11/2014	9:00 AM	8/11/2014	Petrobras Announces First Half 2014 Results	*FactSet*
[20]	11/17/2014	8:00 AM	11/17/2014	Petrobras Announces Third Quarter 2014 Results	*FactSet*
[21]	1/29/2015	11:00 AM	1/29/2015	Petrobras Announces Third Quarter 2014 Results	*FactSet*
[22]	4/23/2015	10:00 AM	4/23/2015	Petrobras Announces Fourth Quarter 2014 Results	*FactSet*
[23]	5/18/2015	11:00 AM	5/18/2015	Petrobras Announces First Quarter 2015 Results	*FactSet*

Note:

* On 29 October 2012, Hurricane Sandy caused the U.S. stock markets to close for two trading days, 10/29/2012 and 10/30/2012. The effective test date for the announcement of Third Quarter 2012 Results is 31 October 2012, the first trading day after the markets reopened.

4. Selection of 6-K events

127. According the SEC, publically traded U.S. companies “must report certain material corporate events on a more current basis. Form 8-K is the ‘current report’ companies must file with the SEC to announce major events that shareholders should know about.”[60] For foreign companies that trade on a U.S. exchange, the form 6-K is used to the same effect as the form 8-K.[61] As such, one would expect that there would generally be a greater flow of information on event dates referred to in 6-Ks as compared more ordinary days. Therefore, a pattern of more frequently statistically significant price movements on

[60] www.sec.gov/answers/form8k htm

[61] https://www.sec.gov/about/forms/form6-k.pdf

6-K dates as compared to all other days would indicate that the security reacts to new information, and would thus indicate market efficiency.

128. There are several instances where the effective testing date for the 6-K event is different from the 6-K filing date. For example, when the Company announces the same information contained in the 6-K during trading hours of a particular day, but files the 6-K on the next day or on a different day, the effective day would be the trading day when the Company first made the announcement.

129. During the Class Period, Petrobras issued 863 6-Ks on 503 unique days. Exhibit-5 presents the filing dates for all 863 6-Ks and the 503 unique test dates thusly identified.

5. <u>Isolating the Impact of Company-Specific Information</u>

130. Event study analysis determines how much of the Company's security return following each of the events was driven by Company-specific information as opposed to market and peer group factors. This method, which is generally accepted and widely used in econometric modeling, involves running a regression to determine how the price of the Petrobras common ADR typically behaved in relation to the overall stock market, its peer group, and in this case, the value of the Brazilian currency. The regression results are then used to determine how much of each event day's actual price return is explained by the market, peer group, and currency factors ("the explained return"). The explained return is then subtracted from the actual return, to isolate the residual return, which is the ADR's return after controlling for the other effects.

131. I ran a regression modeling the return of Petrobras common ADR as a function of: 1) a constant term, 2) the returns of the overall stock market, 3) a peer group index return net of the market return, and 4) the percentage change in the value of the Brazilian real in terms of US dollars.[62]

[62] The net return I refer to with respect to the Peer Index is the result of orthogonalizing the Peer Index returns relative to the Market Index returns. Orthogonalizing is a statistical procedure for removing the component of one explanatory variable that is redundantly captured by another explanatory variable, so as to better observe the relationships between the dependent variable and the unique information content of the explanatory variables.

132. For the overall stock market factor I used the CRSP NYSE/AMEX/NASDAQ Market Index ("Market Index"), which is a generally accepted and widely used measure of the overall stock market performance. The Market Index appropriately incorporates payment of dividends by the constituent companies.

133. I constructed a value-weighted index from the market capitalizations and returns of the constituent companies in the NYSE Arca Oil Index (XOI, formerly known as the Amex Oil Index) excluding Petrobras (the "Peer Index").[63] I selected the constituents of this index for the peer group because Petrobras itself used this index for comparison purposes in its annual reports issued during the Class Period.[64]

134. Each trading day's return for the Peer Index is the value-weighted average of the constituent company returns, inclusive of dividends. All returns used in the construction of the Peer Index and in the regressions are logarithmic returns – that is, the natural logarithm of the ratio of the current day's closing price plus dividends to the previous day's closing price. Logarithmic returns are commonly used in event studies and equity analysis. Analysts and researchers generally use logarithmic returns instead of percent price changes because of various computational advantages.[65]

135. For the currency factor, I used the daily logarithmic returns computed from the daily values of the Brazilian real expressed in US dollars.

136. Petrobras' common ADR prices, trading volume, and returns are shown in Exhibit-4a. Market Index, Peer Index, and Brazilian exchange data are presented in Exhibit-6.

137. Given the length of the Class Period, and so that the regression estimates would be relatively contemporaneous for events being tested, I ran regressions on daily returns covering the entire Class Period broken down into five separate sub-periods. Each sub-period is approximately one year in length, which is a typical estimation period length for event study regressions. The regression estimation sub-periods were constructed by beginning at the end of the Class Period and going one year back. The penultimate

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[63] The constituents of the Amex Oil Index were obtained from www marketwatch.com.

[64] *See*, Petrobras - Report of the Administration for the fiscal year 2011; Petrobras - Report of the Administration for the fiscal year 2012, Petrobras - Report of the Administration for the fiscal year 2013; Petrobras - Report of the Administration for the fiscal year 2014.

[65] Appendix-2 presents the mathematical formula for the logarithmic return and a discussion of the measure.

regression estimation sub-period extends one year further back from that last regression estimation sub-period. I continued that process of dividing the Class Period into regression estimation sub-periods, combining the remaining data at the beginning of the Class Period into a larger sub-period so that no sub-period would be less than one year in length.

138. For the common ADRs, the five periods thusly constructed were: 22 January 2010 to 28 July 2011; 29 July 2011 to 27 July 2012; 30 July 2012 to 26 July 2013; 29 July 2013 to 28 July 2014; and 29 July 2014 to 28 July 2015. Because the Class Period started in January 2010, I included the approximately 6 month period from January 2010 to July 2010 in the first regression period. Each period was one year in length, except for the sub-period at the start of the Class Period.

139. The choice of using the Class Period for the regression estimation period is a widely used and generally accepted methodology in event study analysis.

> "Three general choices for the placement of an estimation window are before the event window, surrounding the event window, and after the event window."
> **"Materiality and Magnitude: Event Studies in the Courtroom," by David I. Tabak and Frederick C. Dunbar in *Litigation Services Handbook, The Role of the Financial Expert*, 3rd ed., edited by Roman L. Weil, Michael J. Wagner, and Peter B. Frank, John Wiley & Sons, Inc., 2001, p. 19.5.**

140. The regression results are presented in Exhibit-7a.

141. I computed the explained portion of the Petrobras common ADR's return on each event date by adding: 1) the estimated regression intercept term, 2) the respective day's Market Index return multiplied by the Market Index coefficient estimated by the regression, 3) the respective day's Peer Index return multiplied by the regression's Peer Index coefficient; and 4) the foreign exchange return multiplied by the regression's foreign exchange factor coefficient.

142. I then computed the residual returns by subtracting each day's explained return from the actual return.

6. *t*-Test

143. For each event, a statistical test called a *t*-test was conducted to determine whether the residual return of the Petrobras common ADR was statistically significant. Statistical significance means that the event return after controlling for the market, peer group, and foreign exchange effects was of such magnitude that it cannot be explained by random volatility, but alternatively must have been caused by new, Company-specific information. A *t*-test compares the residual return on an event date to the typical residual return exhibited over the corresponding regression estimation sub-period. If the event date residual return is far greater (positively or negatively) than the typical residual return, the *t*-test indicates that the residual return is statistically significant.[66]

144. The daily event study results for the Petrobras common ADR are presented in Exhibit-8a.

7. Collective Event Study Results

145. One can assess market efficiency by observing whether a particular security has a greater proportion of statistically significant price movements on days with greater information flow than on more typical days. That is, if the security's price movements are generally more frequently statistically significant among the collection of news event days than among all other days, this result would establish that there is a cause and effect relationship between the flow of information and the security price movements. This finding indicates market efficiency.

146. Based on this principle, I conducted four sets of proportionality tests (or "z-tests") using the event study results from the three sets of event dates identified above (i.e. corruption-related 6-K days, earnings announcement days, and all 6-K days) compared against all other days in the Class Period. The z-test is a commonly used and widely

[66] The test is called the *t*-test because it involves the computation of a *t*-statistic, which is the event day residual return divided by the standard deviation of residual returns from the control period, *i.e.*, the regression estimation data comprising all other days. If the absolute value of the *t*-statistic is greater than the critical *t*-statistic value (1.96 for large samples), the likelihood that the residual return could have been caused by random volatility alone is less than 5%, which is generally accepted to be so unlikely that the random volatility explanation can be rejected, and the security return for that day is deemed statistically significant.

accepted methodology for testing if the difference in the proportion of statistically significant observations for two samples is statistically significant.[67]

147. As shown in Exhibit-8a, there were a total of 1,388 days during the Class Period on which the Petrobras common ADR traded and, of the 1,388 days, 75 days had statistically significant residual returns. The next section describes how many of these statistically significant residual returns occurred on the tested event dates versus how many occurred on non-event dates.

Z-test on corruption-related 6-K days vs. all other days

148. As shown in Exhibit-13c, there were 3 statistically significant corruption-related 6-K days out of the total of 6 corruption-related 6-K days for the Petrobras common ADR. There were 72 of the remaining 1,382 non-event days were statistically significant. Accordingly, the proportion of statistically significant days in the corruption-related 6-K days is 50% and the proportion of statistically significant days in all other days is 5.4%. There was therefore a much greater frequency of statistically significant days within the event day group than within the non-event group.

149. This difference in proportions is associated with a z-score of 4.84, which is greater than the critical z-statistic threshold of 1.65 for significance at the 95% confidence level, and indicates that the difference between the two samples is too severe to accept the notion that the price of the Petrobras common ADR behaves no differently on days with a greater flow of information than all other days. The likelihood of obtaining a proportional difference of this magnitude and associated z-score given that particular explanation is only 0.00006%. Therefore, this proportional difference is deemed statistically significant.

150. This statistical test result indicates that there was a cause and effect relationship between the release of new, Company-specific information and reactions in the Petrobras common ADR price, which therefore establishes that the Petrobras common ADR traded in an efficient market during the Class Period.

[67] "The 'Less Than' Efficient Capital Markets Hypothesis: Requiring More Proof from Plaintiffs in Fraud-on-the-Market Cases," by Paul A. Ferrillo, Frederick C. Dunbar and David Tabak, *78 St. John's L. Rev. 81*, 119-22 (2004); and *In re PolyMedica Corp. Sec. Litig.*, 453 F. Supp. 2d 266 (D. Mass. 2006).

Z-test on earnings announcement days vs. all other days

151. As shown in Exhibit-13c, there were 8 statistically significant earnings announcement days out of the total of 23 earnings announcement days for the Petrobras common ADR. There were 67 statistically significant other days out of a total of 1,365 other days. Accordingly, the proportion of statistically significant days in the earnings announcement days is 34.78% and the proportion of statistically significant days in all other days is 4.91%.

152. This difference in proportions is associated with a z-score of 6.28, which is greater than the critical z-statistic threshold of 1.65 for significance at the 95% confidence level, and indicates that the difference between the two samples is too severe to accept the notion that the price of the Petrobras common ADR behaves no differently on days with a greater flow of information than all other days. The likelihood of obtaining a proportional difference of this magnitude and associated z-score given that particular explanation is virtually nil. Therefore, this proportional difference is deemed statistically significant.

153. This statistical test result indicates that there was a cause and effect relationship between the release of new, Company-specific information and reactions in the Petrobras common ADR price, which therefore establishes that the Petrobras common ADR traded in an efficient market during the Class Period.

Z-test on corruption-related 6-K days and earnings announcement days vs. all other days

154. As shown in Exhibit-13c, there were 11 statistically significant corruption-related 6-K and earnings announcement days out of the combined total of 28 corruption-related 6-K and earnings announcement days[68] for the Petrobras common ADR. There were 64 statistically significant other days out of a total of 1360 other days. Accordingly, the proportion of statistically significant days in the corruption-related 6-K and earnings announcement days is 39.29% and the proportion of statistically significant days in all other days is 4.71%.

[68] Because 23 April 2015 was both an earnings announcement date and a corruption-related 6-K date, the total number of corruption-related 6-K and earnings announcement days is 28.

155. This difference in proportions is associated with a z-score of 8.01, which is greater than the critical z-statistic threshold of 1.65 for significance at the 95% confidence level, and indicates that the difference between the two samples is too severe to accept the notion that the price of the Petrobras common ADR behaves no differently on days with a greater flow of information than all other days. The likelihood of obtaining a proportional difference of this magnitude and associated z-score given that particular explanation is virtually nil. Therefore, this proportional difference is deemed statistically significant.

156. This statistical test result indicates that there was a cause and effect relationship between the release of new, Company-specific information and reactions in the Petrobras common ADR price, which therefore establishes that the Petrobras common ADR traded in an efficient market during the Class Period.

Z-test on all 6-K days vs. all other days

157. As shown in Exhibit-13c, there were 39 statistically significant 6-K days out of the total of 503 6-K days for the Petrobras common ADR. Therefore, there were 36 statistically significant other days out of a total of 885 other days. Accordingly, the proportion of statistically significant days among the 6-K days is 7.75% and the proportion of statistically significant days among all other days is 4.07%.

158. This difference in proportions is associated with a z-score of 2.92, which is greater than the critical z-statistic threshold of 1.65 for significance at the 95% confidence level, and indicates that the difference between the two samples is too severe to accept the notion that the price of the Petrobras common ADR behaves no differently on days with a greater flow of information than all other days. The likelihood of obtaining a proportional difference of this magnitude and associated z-score given that particular explanation is only 0.18%. Therefore, this proportional difference is deemed statistically significant.

159. This statistical test result indicates that there was a cause and effect relationship between the release of new, Company-specific information and reactions in the Petrobras common ADR price, which therefore establishes that the Petrobras common ADR traded in an efficient market during the Class Period.

8. Collective Event Study Summary

160. The event study results show that for all three sets of events identified as appropriate candidates for inclusion in a market efficiency event study for the Petrobras common ADR, there was a statistically significantly greater frequency of statistically significant price reactions on the news event dates as compared to the frequency on non-event dates.

161. All four of the empirical proportion tests I conducted demonstrated a cause and effect relationship between the release of new, Company-specific information and reactions in the Petrobras common ADR price. These results are compelling evidence of the efficiency of the market for the Petrobras common ADR during the Class Period.

B. Petrobras Common ADR Market Efficiency Summary and Conclusion

162. Petrobras common ADRs traded on the NYSE and numerous market makers facilitated trading in the security. The Company was widely covered by analysts and news media. Institutional ownership of Petrobras common ADRs was widespread, trading was active, and market capitalization and float were high (both for the Company as a whole and for the security independently). The security's bid-ask spread was narrow. Current and historical financial information about the Company was readily available to investors and analysts, and the Company qualified for F-3 registration throughout the Class Period, with the exception of one interval when the Company's financial filing was delayed on account of the alleged misdeeds at issue in this case.

163. Not only did the Petrobras common ADR satisfy the *Cammer* and *Krogman* factors that indicate market efficiency, but it also satisfied the empirical *Cammer* factor, which demonstrates the essence of market efficiency.

164. Given these facts, I conclude that the Petrobras common ADR traded in an efficient market over the course of the Class Period.

EFFICIENCY OF THE MARKET FOR THE PETROBRAS PREFERRED ADR

165. To assess whether the market for Petrobras Preferred ADRs was efficient during the Class Period, I analyzed the market for, and behavior of, Petrobras Preferred ADRs,

focusing on the factors that are generally accepted to be indicative of market efficiency for a publicly-traded security.

166. Preferred stock is a hybrid security with both bond-like and equity-like characteristics. Like a bond, preferred stock traditionally delivers investors fixed periodic payments, called preferred dividends, equal to a specified rate times a specified face value.

> "Preferred stock has features similar to both equity and debt. Like a bond, it promises to pay to its holders a fixed amount of income each year. In this sense preferred stock is similar to an infinite-maturity bond, that is, a perpetuity. It also resembles a bond in that it does not convey voting power regarding the management of the firm. Preferred stock is an equity investment, however. The firm retains discretion to make the dividend payments to the preferred stockholders; it has not contractual obligation to pay those dividends. Instead, preferred dividends are usually cumulative; that is, unpaid dividends cumulate and must be paid in full before any dividends may be paid to holders of common stock."
> ***Investments*, 8th edition, by Zvi Bodie, Alex Kane, and Alan J. Marcus, McGraw-Hill Irwin, 2009, p. 37.**

167. Preferred stock is a senior claim to common stock, but generally junior to bonds. Preferred dividends must be paid in full before common stock dividends can be paid, and in the event of a bankruptcy or liquidation, preferred stock investors must be repaid the face value before any liquidated asset proceeds are distributed to common stock investors. On the other hand, obligations to senior bond holders must be satisfied before preferred stock holders are paid.

> "Preferred stock is senior to common stock but junior to bonds. Therefore, preferred stockholders are paid only when profits have been generated and all debt holders have been paid (but before common stockholders are paid)."
> ***Financial Markets and Institutions*, 4th edition, by Anthony Saunders and Marcia Millon Cornett, McGraw-Hill Irwin, 2009, p. 228.**

> "A simple preference is that after settlement has been made with creditors, the preferred stockholders are entitled to receive the par, stated, or liquidated value of the preferred before any distribution is made to common stock or to any junior preferred issue."
> **"Nonconvertible Preferred Stock," by Richard S. Wilson, in *The Handbook of Fixed Income Securities*, 6th edition, edited by Frank J. Fabozzi, McGraw Hill 2001, p. 347.**

Case 1:14-cv-09662-JSR Document 287-13 Filed 10/01/15 Page 6 of 37 PageID #: 19006

168. On account of its fixed dividend, preferred stock tends to be less sensitive to company information than is common stock.

> "Unlike common stockholders, preferred stockholders do not share in the increased profits that come from good years for the firm, nor in the decreased profits that come from bad years, unless earnings drop far enough to prohibit the preferred dividend payment."
> ***Investments*, 2nd edition, by Nancy L. Jacob and R. Richardson Pettit, Irwin, 1988, p. 335.**

169. Moreover, because dividend payments to preferred stock investors can be reduced only after dividends to common stock investors are reduced to zero, and because the face value principal is in jeopardy generally only if the value of company assets falls so much as to wipe out the common equity, the common equity creates a value buffer for the preferred stock. This value buffer tends to hold up the value of preferred stock, keeping it from falling much in the face of all but the most severe news. When a company undertakes a secondary issue of common equity subsequent to the issue of preferred shares, the additional common stock further augments the buffer protecting the performance and value of preferred stock.

A. *Cammer* and *Krogman* Factor Analysis

1. Trading Volume

170. Petrobras preferred ADR trading data are presented in Exhibit-4b. Throughout the Class Period, Petrobras' preferred ADRs traded regularly and actively. On average, 9.71 million Preferred ADRs changed hands daily.[69]

171. During the Class Period, the average weekly trading volume of Petrobras preferred ADRs was approximately 48.5 million shares, or 6.61% of shares outstanding.[70] This level of trading activity surpasses levels accepted by courts as being indicative of market efficiency for stock.[71] In *Cammer*, the court cited the conclusion of Alan R. Bromberg

[69] Data obtained from FactSet.

[70] Estimated by averaging the ratio of the daily trading volume to the number of shares outstanding, and multiplying by 5 (the number of trading days in a typical week).

[71] *Cammer*, 711 F. Supp. at 1286.

Case 1:14-cv-09662-JSR Document 567-19 Filed 10/31/17 Page 180 of 370 PageID #: 49007

and Lewis D. Lowenfels that "weekly trading of 2% or more of the outstanding shares would justify a strong presumption that the market for the security is an efficient one; 1% would justify a substantial presumption."[72] The trading volume for Petrobras preferred ADRs during the Class Period was more than the threshold for a strong presumption of market efficiency.

172. Both in terms of average daily trading volume and also on the basis of the percentage of outstanding shares traded weekly, the market for Petrobras preferred ADRs was very active. Consistent with the *Cammer* opinion, economic theory, and empirical research, the active trading volume in Petrobras preferred ADRs is strong evidence of the efficiency of the market for Petrobras preferred ADRs over the course of the Class Period.

2. <u>Analyst Coverage and Other Avenues of Information Dissemination</u>

Analyst Coverage

173. With at least fifty different firms covering the Company during the Class Period, Petrobras was the subject of active analyst coverage.

Institutional Ownership and Buy-Side Analysis

174. According to FactSet data on institutional ownership of Petrobras preferred ADRs, a total of 851 major institutions owned Petrobras preferred ADRs during the Class Period.[73]

[72] Id., at 1293.

[73] According to filings that reported holdings, there were 851 institutions that held Petrobras preferred ADR as of the end of each quarter during the class period (31 March 2010, 30 June 2010, 30 September 2010, 31 December 2010, 31 March 2011, 30 June 2011, 30 September 2011, 31 December 2011, 31 March 2012, 30 June 2012, 30 September 2012, 31 December 2012, 31 March 2013, 30 June 2013, 30 September 2013, 31 December 2013, 31 March 2014, 30 June 2014, 30 September 2014, 31 December 2014, 31 March 2015, 30 June 2015). There may have been additional institutions that held Petrobras preferred ADR during the Class Period, though not on the quarterly reporting dates.

Case 1:14-cv-09662-JSR Document 567-19 Filed 10/31/17 Page 50 of 75 PageID #: 19068

News Coverage

175. As explained above, news media coverage of Petrobras is extensive. A Factiva database search established that approximately 20,000 articles were published about the Company during the Class Period.[74] Information also emerged throughout the Class Period in the form of SEC filings, conference calls, and presentations.

176. During the Class Period, therefore, information about Petrobras was readily available to market participants as there was a consistent flow of news provided by news media, analysts, and various other sources. This extensive news coverage is further evidence of the efficiency of the market for the Petrobras preferred ADRs.

177. Petrobras was not an obscure company, escaping the notice of analysts and investors. Rather, the Company was large, well known, widely covered, and widely held. These facts strongly support a finding that the market for Petrobras preferred ADRs was an efficient market during the Class Period.

3. Market Makers and Listing on the New York Stock Exchange

178. The number of market makers is one of the factors the *Cammer* court determined indicates market efficiency.

179. The *Cammer* court's understanding that the market-making infrastructure of a stock market is indicative of its efficiency, or lack thereof, makes the fact that Petrobras preferred ADRs traded on the venerable New York Stock Exchange during the Class Period highly relevant. The NYSE is one of the most renowned, most liquid, and most efficient forums for trading stocks in the world. Stocks on the NYSE are traded under the supervision of a DMM, who is responsible for maintaining a fair and orderly market.[75] In fact, as further discussed in detail in the market maker section above, the *Cammer* court explicitly acknowledged the importance of NYSE listing and the implications of such a listing for market efficiency.

[74] Based on a Factiva search for articles published during the Class Period where "Petroleo Brasileiro SA" was the "Company" search field parameter, the language was "English," and the sources were limited to newswires, the *Wall Street Journal*, or "Major News and Business Sources."

[75] "Organization and Functioning of Securities Markets," by Frank Reilly and Keith Brown, in *Equity and Fixed Income CFA Program Curriculum*, vol. 5, Pearson Custom Publishing, 2008.

Case 1:14-cv-09662-JSR Document 267-10 Filed 10/14/15 Page 51 of 370 PageID #: 419609

180. Furthermore, beginning in April 2005, NASDAQ enabled trading in most NYSE-listed securities, such as Petrobras preferred ADRs, through its market maker structure.[76] This NASDAQ market making activity is in addition to the principal market for listed securities on the NYSE.

181. From February 2010 through June 2015, there were at least 150 market makers for Petrobras preferred ADRs, including such well known firms as: Morgan Stanley, Goldman Sachs, UBS, Barclay's and JP Morgan.[77]

182. The fact that it traded on the NYSE is strong evidence that Petrobras preferred ADRs traded in an efficient market throughout the Class Period. Petrobras' listing on the NYSE gave its securities access to a highly developed network of brokers with its market overseen by the NYSE DMM. These facts are compelling evidence of the efficiency of the market for Petrobras preferred ADRs.

4. F-3 Registration Eligibility

183. As further explained in detail in the S-3 Registration Eligibility section above, a U.S. company is eligible for S-3 registration when, among other things, it has filed Exchange Act reports for a specified length of time and has outstanding float above a certain sizable value. Eligibility for form F-3 registration is the same as for S-3, except that the company must be a foreign private issuer (i.e. not a U.S. company), such as Petrobras.[78] The *Cammer* court noted that S-3 registration eligibility is indicative of market efficiency because the filing requirement ensured that financial data were available to market participants, and the "public float" requirement indicated that many market participants would have examined the information.[79]

184. In 1992, the SEC amended its requirements for S-3 registration eligibility to 12 months of filings and at least $75 million of float. Since 2007, the SEC has allowed domestic companies with less than $75 million of float to file an S-3 registration so long as the

[76] "Nasdaq To Enable Customers To Trade NYSE Stocks," *Reuters*, March 28, 2005.

[77] Market maker data obtained from Bloomberg.

[78] "Eligibility of Smaller Companies to Use Form S-3 or F-3 for Primary Securities Offerings," SEC website, accessed at www.sec.gov/info/smallbus/secg/s3f3-secg htm, 28 January 2008.

[79] *Cammer*, 711 F. Supp. at 1284-85.

company has been filing financial reports for at least a year, has "a class of common equity securities listed and registered on a national securities exchange, and the issuers do not sell more than the equivalent of one-third of their public float in primary offerings over any period of 12 calendar months."[80] However, despite the fact that the $75 million requirement has been loosened, courts continue to focus on this $75 million benchmark when analyzing this *Cammer* factor.[81]

Float

185. Petrobras preferred ADRs' float averaged $15.9 billion during the Class Period, far exceeding the level required for F-3 registration. During the Class Period, float ranged between $3.50 billion and $28.6 billion, always exceeding the minimum requirement for F-3 registration eligibility.

Financial Filings

186. Petrobras regularly filed financial reports with the SEC throughout the Class Period. The financial information in the SEC filings, supplemented by information provided by analysts and news coverage, provided investors with access to financial information about the Company on a continuous basis.

187. As explained above, Petrobras satisfied the filing requirement for F-3 registration throughout the Class Periods, with the exception of one interval when the filing was delayed due specifically to issues related to the allegations in this case

188. To the extent that F-3 registration eligibility indicates company characteristics associated with market efficiency, in particular characteristics of size, transparency, and the availability of relevant financial information. Petrobras possessed those particular characteristics throughout the Class Period. Furthermore, as noted, the Company did file an F-3 Registration Statement during the Class Period, on 30 August 2004.[82]

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[80] "Revisions To The Eligibility Requirements For Primary Securities Offerings On Forms S-3 And F-3," SEC Release No. 33-8878, 19 December 2007.

[81] See, e.g., *Vinh Nguyen v. Radient Pharm. Corp.*, 287 F.R.D. 563, 573 (C.D. Cal. 2012).

[82] Form F-3 Registration Statement, filed by Petróleo Brasileiro S.A. on 30 August 2004.

Case 1:14-cv-09662-JSR Document 863-2 Filed 07/01/18 Page 58 of 370 PageID #: 19011

189. Consistent with the *Cammer* opinion, Petrobras' eligibility to file an F-3 registration is indicative of the efficiency of the market for Petrobras preferred ADRs during the Class Period.

5. Market Capitalization

190. During the Class Period, the market capitalization of Petrobras preferred ADRs averaged $15.9 billion. This value alone, independent of the Company's common stock, would place among the top 1st decile of U.S. companies' market capitalizations – meaning that the aggregate value of the Petrobras preferred ADRs was larger than the respective market capitalizations of at least 90% of all publicly-traded companies in the U.S.[83]

191. Consistent with the *Krogman* court's opinion, the large aggregate value of the Petrobras preferred ADRs throughout the Class Period is further evidence of the efficiency of the market for that security.

6. Float

192. As mentioned above, Petrobras preferred ADR float averaged $15.9 billion during the Class Period. The value of the float of Petrobras' preferred ADRs is the same as that of the market capitalization because, according to the Company's SEC filings, none of the preferred ADRs were held by insiders or affiliated corporate entities.[84] Petrobras' preferred ADR float was larger than the total market capitalization of at least 90% of all publicly-traded companies in the U.S.[85] The size of Petrobras' float indicates it far exceeded the standard of the second *Krogman* factor for market efficiency.

193. On average during the Class Period, there were 741.8 million preferred ADRs in Petrobras' float and the same number of preferred ADRs outstanding, resulting in a float of 100% of preferred ADRs outstanding.

194. The sizeable Petrobras preferred ADR float is indicative of the efficiency of the market for that security throughout the Class Period.

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[83] This calculation is based on averaged month-end data from CRSP for February 2010 through December 2014. I grouped public companies into deciles, so that the 1st decile contains the largest 10% of all public companies listed on the NYSE, Amex, and NASDAQ, while the 10th decile contains the smallest 10%.

[84] Based on a review of Petrobras' annual reports on form 20-F filed with the SEC for the Fiscal Years 2010 – 2014 and the prospectus supplement dated 23 September 2010.

[85] This calculation is based upon averaged month-end data from CRSP for February 2010 through December 2014.

7. Bid-Ask Spread

195. I obtained data on daily closing bid and ask quotes for Petrobras preferred ADRs during the Class Period from FactSet. Exhibit-4b presents Petrobras preferred ADR bid-ask spread data.

196. The average bid-ask spread for Petrobras preferred ADRs over the course of the Class Period was 0.08%. By comparison, the average month-end bid-ask spread over the course of the Class Period for all stocks in the CRSP database was 0.59%, or over 7 times wider.[86] Petrobras preferred ADR bid-ask spreads were therefore substantially narrower than the mean level among all CRSP stocks – which comprises stocks traded on the NYSE, Amex, NASDAQ, and NYSE Arca.

197. In dollar terms, Petrobras' bid-ask spread during the Class Period averaged $0.02 per preferred ADR. For all stocks in the CRSP database, the average bid-ask spread during the Class Period was $0.08 per share.

198. The average bid-ask spread in the market for Petrobras preferred ADRs over the course of the Class Period was well below the typical bid-ask spreads exhibited by other publicly-traded stocks in the United States. Petrobras' narrow bid-ask spread supports a conclusion of market efficiency.

EMPIRICAL EVIDENCE OF MARKET EFFICIENCY FOR PETROBRAS PREFERRED ADRS

199. To test the efficiency of the market for preferred ADRs, I conducted the same four sets of empirical tests that I conducted on the common ADRs. My regression model for the Petrobras preferred ADR included the same variables used in the regression model for the Petrobras common ADR, plus an additional variable consisting of a preferred stock index return to control for any potential preferred stock factor effect.

200. For this purpose, I used the S&P Preferred Stock Index ("Preferred Stock Index"), which is a value-weighted average of a broad selection of preferred stock issues traded on American exchanges. The index level and return of the Preferred Stock Index are presented in Exhibit-11.

[86] This calculation is based upon averaged month-end data from CRSP for February 2010 through December 2014.

201. Therefore, the regression I ran modelled the return of Petrobras preferred ADRs as a function of: 1) a constant term, 2) the returns of the overall U.S. stock market, 3) the Peer Index return net of the Market Index return, 4) the percentage change in the value of the Brazilian real in terms of US dollars, and 5) the Preferred Index return net of the Market Index return.[87]

202. This regression analysis isolates the impact of Company-specific information on Petrobras' Preferred ADRs, removing the various potential market-wide effects.

203. Petrobras' preferred ADRs' prices, trading volume, and returns are shown in Exhibit-4b.

A. Collective Event Study Results

204. As shown in Exhibit-8b, there were a total of 1,388 days during the Class Period on which the Petrobras preferred ADR traded. Of the 1,388 days, 70 days had statistically significant residual returns.

Z-test on corruption-related 6-K days vs. all other days

205. As shown in Exhibit-13d, the preferred ADR exhibited 3 statistically significant residual returns on corruption-related 6-K event days out of the total of 6 corruption-related 6-K days. Of the 1,382 non-event days, 67 had statistically significant returns. Accordingly, the proportion of statistically significant days in the corruption-related 6-K days is 50% and the proportion of statistically significant days among all other days is 4.85%.

206. This difference in proportions is associated with a z-score of 5.04, which is greater than the critical z-statistic threshold of 1.65 for significance at the 95% confidence level, and indicates that the difference between the two samples is too severe to accept the notion that the price of the Petrobras preferred ADR behaves no differently on days with a greater flow of information than all other days. The likelihood of obtaining a proportional difference of this magnitude and associated z-score given that particular explanation is only 0.00002%. Therefore, this proportional difference is deemed statistically significant.

[87] The net returns I refer to are the result of orthogonalizing the Peer Index returns and the Preferred Index returns relative to the Market Index returns. Orthogonalizing is a statistical procedure for removing the component of one explanatory variable that is redundantly captured by another explanatory variable, so as to better observe the relationships between the dependent variable and the unique information content of the explanatory variables.

207. This statistical test result indicates that there was a cause and effect relationship between the release of new, Company-specific information and reactions in the Petrobras preferred ADR price, which therefore establishes that the Petrobras preferred ADR traded in an efficient market during the Class Period.

Z-test on earnings announcement days vs. all other days

208. As shown in Exhibit-13d, there were 6 statistically significant earnings announcement days out of the total of 23 earnings announcement days for the Petrobras preferred ADR. Therefore, there were 64 statistically significant other days out of a total of 1365 other days. Accordingly, the proportion of statistically significant days in the earnings announcement days is 26.09% and the proportion of statistically significant days among all other days is 4.69%.

209. This difference in proportions is associated with a z-score of 4.65, which is greater than the critical z-statistic threshold of 1.65 for significance at the 95% confidence level, and indicates that the difference between the two samples is too severe to accept the notion that the price of the Petrobras preferred ADR behaves no differently on days with a greater flow of information than on all other days. The likelihood of obtaining a proportional difference of this magnitude and associated z-score given that particular explanation is only 0.00017%. Therefore, this proportional difference is deemed statistically significant.

210. This statistical test result indicates that there was a cause and effect relationship between the release of new, Company-specific information and reactions in the Petrobras preferred ADR price, which therefore establishes that the Petrobras preferred ADR traded in an efficient market during the Class Period.

Z-test on corruption-related 6-K days and earnings announcement days vs. all other days

211. As shown in Exhibit-13d, there were 9 statistically significant corruption-related 6-K and earnings announcement days out of the combined total of 28 corruption-related 6-K and earnings announcement days for the Petrobras preferred ADR. Therefore, there were 61 statistically significant other days out of a total of 1,360 other days. Accordingly, the proportion of statistically significant days in the corruption-related 6-K and earnings

announcement days is 32.14% and the proportion of statistically significant days in all other days is 4.49%.

212. This difference in proportions is associated with a z-score of 6.62, which is greater than the critical z-statistic threshold of 1.65 for significance at the 95% confidence level, and indicates that the difference between the two samples is too severe to accept the notion that the price of the Petrobras preferred ADR behaves no differently on days with a greater flow of information than on all other days. The likelihood of obtaining a proportional difference of this magnitude and associated z-score given that particular explanation is virtually nil. Therefore, this proportional difference is deemed statistically significant.

213. This statistical test result indicates that there was a cause and effect relationship between the release of new, Company-specific information and reactions in the Petrobras preferred ADR price, which therefore establishes that the Petrobras preferred ADR traded in an efficient market during the Class Period.

Z-test on all 6-K days vs. all other days

214. As shown in Exhibit-13d, there were 33 statistically significant 6-K days out of the total of 503 6-K days for the Petrobras preferred ADR. Therefore, there were 37 statistically significant non-event days out of a total of 885 non-event days. Accordingly, the proportion of statistically significant days in the 6-K days is 6.56% and the proportion of statistically significant days in all other days is 4.18%.

215. This proportional difference is associated with a z-score of 1.95, which is greater than the critical z-statistic threshold of 1.65 for significance at the 95% confidence level, and indicates that the difference between the two samples is too severe to accept the notion that the price of the Petrobras preferred ADR behaves no differently on days with a greater flow of information than on all other days. The likelihood of obtaining a proportional difference of this magnitude and associated z-score given that particular explanation is only 2.57%. Therefore, this proportional difference is deemed statistically significant.

216. This statistical test result indicates that there was a cause and effect relationship between the release of new, Company-specific information and reactions in the Petrobras preferred ADR price, which therefore establishes that the Petrobras preferred ADR traded in an efficient market during the Class Period.

1. <u>Collective Event Study Summary</u>

217. The event study results show that for all three sets of events identified as appropriate candidates for inclusion in a market efficiency event study for the Petrobras preferred ADR, there was a statistically significantly greater frequency of statistically significant price reactions on the news event dates as compared to the frequency on non-event dates.

218. All four of the empirical proportion tests I conducted demonstrated a cause and effect relationship between the release of new, Company-specific information and reactions in the Petrobras preferred ADR price. These results are compelling evidence in support of the efficiency of the market for the Petrobras preferred ADR during the Class Period.

B. Petrobras Preferred ADR Market Efficiency Summary and Conclusion

219. The Petrobras preferred ADR traded on the NYSE. Many market makers facilitated trading in the security. The Company was widely covered by analysts and the news media. Institutional ownership of the Petrobras preferred ADR was widespread. Trading was very active. The outstanding issue was exceedingly large, larger than the entire market capitalizations of most public companies. Current and historical financial information about the Company was readily available to investors and analysts. The security's bid-ask spread was narrow. The Company qualified for F-3 registration throughout the Class Period, with the exception of one interval when the Company's financial filing was delayed on account of the alleged misdeeds at issue in this case.

220. Not only did the Petrobras preferred ADR satisfy the *Cammer* and *Krogman* factors that indicate market efficiency, but it also satisfied the empirical *Cammer* factor, which demonstrates the essence of market efficiency.

221. Given these facts, I conclude that Petrobras Preferred ADR traded in an efficient market throughout the Class Period.

EFFICIENCY OF THE MARKET FOR PETROBRAS BONDS

222. Most of the Company factors that indicate the market efficiency of the market for the Common ADRs and Preferred ADRs examined above also apply to all other Petrobras securities, including the Company's bonds.

223. For each of the Petrobras Bonds, I obtained historical trading data from the FINRA Trade Reporting and Compliance Engine ("TRACE") using CRSP, and from Bloomberg.

224. Appendix-1 provides descriptions of each of the Petrobras bonds analyzed.

A. *Cammer* and *Krogman* Factor Analysis

225. To investigate whether the market for the Petrobras Bonds was an efficient market, I considered factors that are generally accepted by both financial economists and the courts to be indicative of market efficiency for publicly-traded securities, taking note of distinctive features of bond investing and the bond market that distinguish bonds from common stock.[88]

226. I also tested the efficiency of the market for the Petrobras Bonds empirically.

227. As established above, Petrobras was not a small, obscure company, which escaped the notice of market participants. On the contrary, Petrobras has been a Fortune 500 company since 2005, and was widely followed by investors, analysts, commentators, and news media. Just as the Company's renown fostered the efficiency of the market for the Company's common and preferred ADRs, it similarly fostered the efficiency of the market for Petrobras's debt securities.

1. Analyst Coverage and Other Avenues of Information Dissemination

228. As noted above, based on a review of available analyst reports and conference call transcripts, at least 50 analysts covered Petrobras during the Class Period. The information and analysis these analysts provided is relevant to the efficient pricing of the Petrobras Bonds as well as the Company's common and preferred ADRs.

[88] For example, in *In re Healthsouth Corporation Securities Litigation*, 261 F.R.D. 633 (N.D. Ala. 2009), the court found that the *Cammer* and *Krogman* factors were appropriate for evaluating the efficiency of the market for bonds, "while taking into account the differences in the trading of bonds from stocks."

Case: 1:14-cv-09662-JSR Document 687-10 Filed 10/01/16 Page 56 of 75 PageID #: 19018

229. Consequently, the widespread analyst coverage of the Company over the course of the Class Period promoted the efficiency of the market for the Petrobras Bonds.

230. As noted above, at least 20,000 news articles about Petrobras were published in the press over the Class Period. Thus, the media facilitated the flow of information about the Company to the marketplace. Bond investors had easy access to news about the Company, which promoted the efficiency of the market for the Petrobras Bonds.

2. Credit Rating Agencies

231. Another feature that contributed to the efficiency of the market for the Petrobras Bonds was the coverage provided by the major bond rating agencies (i.e. Fitch, Moody's, and S&P). These agencies performed analysis, rating, and surveillance on the Petrobras Bonds throughout the Class Period, and commented in the financial press about these bonds. Consequently, not only was the Company well covered throughout the Class Period, but so too were the Petrobras Bonds specfically.

232. Throughout the Class Period, these three rating agencies published detailed research and credit analyses on each class of the Petrobras Bonds, including the senior unsecured Bonds.

233. To arrive at bond ratings, these ratings agencies perform detailed analyses of bond issuers, taking into account a vast array of business and financial information. According to S&P literature, S&P will "assign a rating only when adequate information is available."[89]

234. The rating agencies monitor and periodically update company ratings. As described in the following quote from the S&P website, S&P collects and disseminates material information about the companies it rates:

> "Once a rating is assigned, we maintain on-going review of material factors that could affect the rating, such as changes in the capital structure, an acquisition or other major economic developments. Generally, an issuer credit rating is reviewed formally at least once a year at the time of a meeting with the issuer's management. We expect management to provide to us prompt notice of material financial and operational changes that could affect the rating."

[89] http://www2.standardandpoors.com/spf/pdf/media/credit_ratings_fact_sheet_020507.pdf

Credit Ratings Fact Sheet, on www.standardandpoors.com, 2 February 2007.

235. Coverage and reports by Fitch, Moody's, and S&P are channels through which information and analysis about Petrobras reach investors and are subsequently incorporated into the prices of the Petrobras Bonds. This infrastructure for disseminating information and analysis fosters the efficiency of the market for the Petrobras Bonds. Table-2 presents the Moody's and S&P ratings for Petrobras Senior Unsecured Notes throughout the Class Period.

Table-2: Moody's Credit Rating History

Moody's Credit Rating History

Class Period: 22 January 2010 through 28 July 2015

Senior Unsecured

Date	Rating	Watch
11/5/2007	Baa1	
6/21/2011	A3	
10/3/2013	Baa1	
10/21/2014	Baa2	
12/23/2014	Baa2	*-
1/29/2015	Baa3	*-
2/24/2015	Ba2	*-
4/27/2015	Ba2	

Note:

*- Rating under review negative implication

Source: Bloomberg

3. <u>Institutional Ownership</u>

236. During the Class Period, each of the Petrobras Bonds was held by numerous sophisticated institutional investors according to the FactSet Ownership database. While bonds are not classified by the SEC as 13-F securities, and thus are not included in the reports required by the SEC for all large institutions, it is the case that mutual funds registered under the Investment Company Act of 1940 ("40 Act Funds") are required by the SEC to file quarterly Form 4-30D reports detailing their holdings. These 40-Act Fund holdings

represent only a portion of the total institutional holdings of bonds, and provide a conservative estimate, or lower limit, of total institutional holdings.

237. A total of 214 distinct mutual funds held one or more Petrobras Bonds at some point during the Class Period. Table-3 shows how many mutual funds owned each of the Petrobras Bonds at some point during the Class Period.

Table-3: Number of Institutions that Held Petrobras Bonds During the Class Period

Petrobras Bonds	Number of Institutions Holding Petrobras Bonds
71645WAT8	47
71645WAR2	82
71645WAS0	26
71645WAM3	27
71645WAP6	48
71645WAQ4	19
71645WAU5	33
71647NAC3	28
71647NAA7	20
71647NAE9	17
71647NAB5	50
71647NAF6	57
71647NAJ8	12
71647NAH2	34
71647NAM1	49
71645WAH4	11
71645WAN1	32
71645WAL5	11
71647NAL3	14
71647NAD1	12
71647NAK5	25
71647NAG4	35

Source: FactSet.

238. Ownership by institutional investors, of which mutual funds represent but a fraction, indicates market efficiency because, among other reasons, investment institutions generally employ financial analysts and portfolio managers who conduct their own research on securities and who make investment decisions based on that research.

4. F-3 Registration Eligibility

239. As stated above, the F-3 registration eligibility that indicated the efficiency of the market for Petrobras common and preferred ADRs also indicates the efficiency of the markets for the Petrobras Bonds. F-3 registration eligibility establishes that a company is large enough to receive attention from the market and that financial data is available for investors to analyze. These characteristics foster the efficiency of the market for the Petrobras Bonds, just as they did for the common stock.

5. Prominent Underwriters and Market Makers

240. The Petrobras Bonds were not obscure securities placed into the financial marketplace by little-known underwriters. Rather, the Petrobras Bonds were underwritten by some of Wall Street's largest and most prominent investment banks.

241. According to SEC filings, there were at least 20 underwriters of the Petrobras Bonds, including the following Wall Street investment banks: BTG Pactual, Citigroup, HSBC, Itau, JPM, Santander, Credit Agricole CIB, MUFG, BB Securities, Morgan Stanley, Banco Votorantim, Societe Generale, BofAML, Standard Chartered Bank, Bank of China, Bradesco BBI, Banca IMI, Scotiabank, Credit Suisse First Boston, Lehman Bros, and, UBS.

242. Typically, investment banks that underwrite bonds subsequently serve as market makers in the issues.

> "It would be a mistake to think that once the bonds are all sold, the investment banking firm's ties with the deal are ended. Those who bought the bonds will look to the investment banking firm to make a market in the issue. This means that the investment banking firm must be willing to take a principal position in secondary market transactions."
> **"The Primary and Secondary Bond Markets," by Frank J. Fabozzi and Frank J. Jones, chapter 3, in The Handbook of Fixed Income Securities, 7th edition, edited by Frank J. Fabozzi and Steven V. Mann, McGraw-Hill, 2005, p. 33.**

243. Additionally, many investment banks that published analyst reports covering the Petrobras common ADR, stated in those reports that they also make a market in Petrobras Bonds. HSBC and Credit Suisse published reports specifically addressing the Petrobras Bonds during the Class Period.

> "At the time of publication of this report, HSBC Securities (USA) Inc. is a Market Maker in securities issued by this company."
> **"Credit Comment – 4Q2009 earnings release," by John Kollar and Catherine Yim, HSBC, credit analyst report, 24 March 2010, p. 5.**

> "As at the date of this report, Credit Suisse acts as a market maker or liquidity provider in the debt securities of the subject issuer(s) mentioned in this report."
> **"Petrobras Credit Update – can new management deliver?" by Jamie Nicholson, et al., Credit Suisse, fixed income research report, 10 February 2015, p. 13.**

244. Based on the number of firms that underwrote the Petrobras Bonds, and the disclosures that underwriting firms subsequently made markets in the securities they underwrote, it appears that there were numerous market makers for the Petrobras Bonds. Additional evidence confirming a developed market-making infrastructure is the considerable trading volume in these debt securities.

6. Outstanding Par Value and Float

245. The *Cammer* and *Krogman* opinions cite a Company's large market capitalization as being indicative of market efficiency for the common stock. This factor, which fosters market efficiency for the common and preferred ADRs, would similarly foster efficiency of the markets for the Company's bonds. Petrobras' large market capitalization and renown among securities analysts and the media would indicate market efficiency for all of the Company's bonds.

246. Beyond Petrobras' large stock market capitalization, the total par value of the Company's outstanding bonds was larger than the market capitalizations of most publicly-traded companies. The aggregate par value of the Petrobras Bonds totaled $41.4 billion and was larger than the market capitalizations of 90% of all public companies listed on the NYSE, Amex, and NASDAQ during the Class Period.[90]

[90] This calculation is based on averaged month-end data from CRSP between 30 January 2010 and 30 June 2014. I performed my calculation by grouping public companies into deciles, so that the 1st decile contains the largest 10% of all public companies listed on the NYSE, Amex, and NASDAQ exchanges, while the 10th decile contains the smallest 10%.

247. Indeed, as shown in Table-4 for each of the Petrobras Bonds, the outstanding par value of each issue individually was comparable to the market capitalizations of publicly-traded companies, and was far above the market capitalization threshold necessary for S-3 registration.[91]

Table-4: Petrobras Bonds Par Amounts

Bond	Issue Date	Issued Amount (Millions)	Follow-Up Offering Date	Follow-Up Offering Amount	Aggregate Amount (Millions)
December 2018 Notes	12/3/2003	$750			$750
March 2018 Notes	9/29/2006	$1,000	7/1/2009	$750	$1,750
October 2016 Notes	9/29/2006	$500	2/2/2007	$399	$899
March 2019 Notes	2/4/2009	$1,500	7/1/2009	$1,250	$2,750
January 2020 Notes	10/23/2009	$2,500			$2,500
Janaury 2040 Notes	10/23/2009	$1,500			$1,500
January 2016 Notes	1/20/2011	$2,500			$2,500
January 2021 Notes	1/20/2011	$2,500	2/1/2012	$2,750	$5,250
January 2041 Notes	1/20/2011	$1,000	2/1/2012	$1,250	$2,250
February 2017 Notes	2/1/2012	$1,750			$1,750
May 2016 Floating Rate Notes	5/13/2013	$1,000			$1,000
January 2019 Floating Rate Notes	5/13/2013	$1,500			$1,500
May 2016 Notes	5/13/2013	$1,250			$1,250
May 2023 Notes	5/13/2013	$3,500			$3,500
May 2043 Notes	5/13/2013	$1,750			$1,750
January 2019 Notes	5/13/2013	$2,000			$2,000
March 2017 Notes	3/10/2014	$1,600			$1,600
March 2044 Notes	3/10/2014	$1,000			$1,000
March 2017 Floating Rate Notes	3/10/2014	$1,400			$1,400
March 2020 Floating Rate Notes	3/10/2014	$500			$500
March 2020 Notes	3/10/2014	$1,500			$1,500
March 2024 Notes	3/10/2014	$2,500			$2,500
Total Amount Issued					**$41,399**

248. Petrobras' public filings did not indicate that any substantial portion of the Company's outstanding bonds was held by insiders. Consequently, the Petrobras Bonds' float equaled the amount outstanding. Just as the outstanding par values were very large, so too were the floats. These facts are evidence of the efficiency of the market for the Petrobras Bonds during the Class Period.

[91] In the court in the In re Enron Corp. Sec., Derivative & "ERISA" Litig., 529 F. Supp. 2d 644 (S.D. Tex. 2006), found the large aggregate outstanding par amount was indicative of market efficiency.

7. Trading Volume

249. As discussed above, securities that are more senior in a company's capital structure, such as bonds, have more predictable cash flows and generally have more certain and stable valuations than do common stock. As a result, bonds will typically exhibit less price volatility and trade less frequently than a company's common stock. Nonetheless, the Petrobras Bonds traded actively during the Class Period.

250. I examined the trading volume for each of the Petrobras Bonds.[92] For each of the issues, I analyzed the weekly trading volume during the portion of the Class Period following each offerings respective issuance. For each of the Bonds, I accounted appropriately for transactions that required removal, e.g., canceled or reversed trades. In addition, I considered only transactions that occurred between 9:30 a.m. and 4:00 p.m so that the volume and associated prices corresponded to regular trading days. Appendix-3 presents my treatment of the TRACE data.

251. Table-5 presents the weekly average trading volume for the Petrobras Bonds.

[92] I obtained the TRACE volume data from CRSP. Many trades are reported with volumes estimated as $5 million "plus" or $1 million "plus," indicating that the actual par value traded was greater than these reported estimated volumes. In calculating trading volumes I used these estimated lower bound volumes, and it follows that the aggregated volume is also a lower bound for the true aggregate volume. Weekly trading volume for the bonds is computed by summing the volume, dividing by the number of trading days in the interval, and then multiplying by five.

Table-5: Weekly Average Trading Volume For The Petrobras Bonds During the Class Period

Bond	Average Weekly Trading Volume
March 2024 Notes	8.85%
March 2044 Notes	8.63%
May 2043 Notes	4.86%
March 2020 Floating Rate Notes	4.63%
March 2020 Notes	4.33%
May 2023 Notes	4.10%
January 2041 Notes	3.15%
January 2019 Notes	3.10%
March 2019 Notes	2.86%
January 2021 Notes	2.78%
February 2017 Notes	2.33%
January 2019 Floating Rate Notes	2.24%
March 2017 Notes	2.20%
March 2017 Floating Rate Notes	2.14%
Janaury 2040 Notes	2.04%
May 2016 Floating Rate Notes	1.98%
May 2016 Notes	1.94%
January 2020 Notes	1.84%
January 2016 Notes	1.39%
March 2018 Notes	1.37%
October 2016 Notes	1.32%
December 2018 Notes	1.10%

252. Even using the thresholds the *Cammer* court established in the context of judging a market for common stock, the Class Period average weekly volumes of the March 2024 Notes, March 2044 Notes, May 2043 Notes, March 2020 Floating Rate Notes, March 2020 Notes, May 2023 Notes, January 2041 Notes, January 2019 Notes, March 2019 Notes, January 2021 Notes, February 2017 Notes, January 2019 Floating Rate Notes, March 2017 Notes, March 2017 Floating Rate Notes, and January 2040 Notes all met or surpassed the 2% threshold that establishes a ***strong*** presumption that these securities traded in an efficient market.

253. Using the thresholds applicable to common stock, which are understood to trade more frequently than bonds, the Class Period average weekly volumes of the May 2016 Floating Rate Notes, May 2016 Notes, January 2020 Notes, January 2016 Notes, March 2018 Notes, October 2016 Notes, and December 2018 Notes exceed the 1% threshold that establishes a substantial presumption of market efficiency.

254. The market for Petrobras Bonds was very active. Trading volume was high, even relative to the standard for common stock. Consistent with the *Cammer* opinion, economic theory, and published research, this fact is evidence of market efficiency.

8. Trade Frequency

255. The trading frequency of the Petrobras Bonds, in addition to their trading volume, is further evidence of their market efficiency. A published, peer-reviewed, study by Mahanti et al. [2008] observes that relatively few corporate bonds trade more frequently than on 200 days in a year.[93] According to the study, the bonds examined typically trade every 12-14 days.[94] I measured the frequency of the trading in the Petrobras bonds in order to compare the behavior of the Petrobras bonds with that of the bonds in the Mahanti et al. study.

256. I calculated the average number of days between successive trades for each of the Petrobras Bonds. The results are presented in Exhibit-12.[95]

257. The average number of days between successive trades ranges between 0.015 and 0.333 for the Petrobras Bonds over the Class Period. The average number of days between trades ranges between 0.006 and 0.372 for the Petrobras Bonds across all periods, meaning that the Petrobras Bonds typically trade numerous times per day rather than once every few days. As shown in the exhibit, all of the Petrobras Bonds traded more frequently than even the first decile of frequently traded bonds in the Mahanti paper. The

93 "Latent Liquidity: A new measure of liquidity, with an application to corporate bonds," by Sriketan Mahanti, Amrut Nashikkar, Marti G. Subrahmanyam, George Chacko, and Gaurav Mallik, *Journal of Financial Economics*, 2008, p. 278.

94 "Latent Liquidity: A new measure of liquidity, with an application to corporate bonds," by Sriketan Mahanti, Amrut Nashikkar, Marti G. Subrahmanyam, George Chacko, and Gaurav Mallik, *Journal of Financial Economics*, 2008, p. 282.

95 For comparability with custodial records, which show only trades by investors, I exclude interdealer trades reported by TRACE and calculate the time elapsed between successive trades, assuming 9 hours per trading day.

trading frequency of the Petrobras Bonds is evidence that the bonds trade in a well-developed market and is compelling evidence of market efficiency.

9. The Market for the Petrobras Bonds was Well Developed and Transparent

258. Not only do the characteristics of the Petrobras Bonds indicate market efficiency, but the Petrobras Bonds enjoyed the benefits of the United States bond market in general – a large, active market, with well-developed infrastructure for trading and the dissemination of information.

259. According to information provided by the Securities Industry and Financial Markets Association, at the end of 2013, for example, the outstanding principal of bonds in the United States was approximately $37.7 trillion. The corporate sector accounted for over $7.4 trillion of this total. The size of the American bond market is actually greater than that of the stock market. The corporate sector alone, while not as big as the stock market, is definitely sizable.

260. The size, activity, and infrastructure of the United States corporate bond market facilitated and enhanced the market efficiency of the Petrobras Bonds.

261. The features of the U.S. corporate bond market, in addition to the features already mentioned above, include regulatory oversight by the SEC, easy availability to investors of required filings, availability of pricing benchmarks, and access for investors to firm-specific and bond-specific information on systems such as Bloomberg. Each of these factors contributed to the efficiency of the market for the Petrobras Bonds.

262. TRACE was launched on 1 July 2002 after being developed by the National Association of Securities Dealers (now the Financial Industry Regulatory Authority, or “FINRA”) to facilitate reporting of bond transactions. All broker/dealers who are members of FINRA are required to report over-the-counter corporate bond transactions for eligible bonds to TRACE.[96]

[96] TRACE Fact Book – 2014, accessed on 17 July 2015, (http://www.finra.org/sites/default/files/2014-TRACE-Fact-Book.pdf)

Case: 1:14-cv-09662-JSR Document 697-2 Filed 10/07/16 Page 65 of 370 PageID #: 419028

263. The introduction of TRACE has increased the transparency in the corporate debt market by making trading data available to participants. Academic research has demonstrated that trade execution costs fell by 50% for bonds that were eligible for TRACE, and that execution costs also declined for similar non-TRACE-eligible bonds.[97] These empirical facts indicate that TRACE has improved the efficiency of the market for corporate debt as a whole and Petrobras debt in particular, as pricing for Petrobras debt instruments is (and was) available through TRACE.

264. The efficiency of the market for the Petrobras Bonds is further supported by the widespread availability of pricing and volume data for the Petrobras Bonds. Pricing and volume data for the Petrobras Bonds are (and were) available on Bloomberg, FT Interactive, and the electronic trading platform MarketAxess.

B. Empirical Evidence of Petrobras Bond Market Efficiency

265. To address the empirical factor and determine whether the prices of the Petrobras Bonds reacted appropriately to new information, I performed event study tests and also tested whether the Petrobras Bonds moved significantly with market interest rates.

266. Because of their senior status, bond values are substantially insulated from all but the most extreme news by a valuation cushion provided by the common stock. As a result, bonds are the least sensitive of all securities to firm specific news while being the most sensitive to a change in a firm's probability of default.[98] When new common stock is issued, the cushion is increased and the valuation sensitivity of bonds to Company news is further reduced.

267. As such, the 503 6-K days I used to test the efficiency of the market for the Petrobras ADRs would not, collectively, constitute a good candidate for a market efficiency study on the Petrobras Bonds. The information reported in most of the 6-Ks was of only moderate importance, and reasonably would have only slight impact on the valuation of the bonds, rather than eliciting statistically significant price changes. Consequently, I performed the collective event study on the set of corruption-related 6-K days, and on the

[97] "Market Transparency, Liquidity Externalities, and Institutional Trading Costs in Corporate Bonds," by Hendrik Bessembinder, William Maxwell and Kumar Venkataraman, *Journal of Financial Economics*, 2006.

[98] See, "Understanding the Efficiency of the Market for Preferred Stock," by Michael Hartzmark and H. Nejat Seyhun, *Virgina Law and Business Review*, Spring 2014, pp. 9-10.

combined set of corruption-related 6-K and earnings announcement days. These sets of events comprise days on which new information was released that could reasonably have significantly altered the markets' expectations regarding Petrobras' ability to repay its Bonds.

1. <u>Bond Regression Analysis</u>

268. My regression model for the Petrobras Bonds included the same Market Index, Peer Index (net of market returns), and the foreign exchange factor used in my regression models for the Petrobras ADRs. In addition to these factors, I also included an interest rate variable as explained below.

Controlling for the Overall Bond Market Factor for Fixed-rate Bonds

269. As shown in Appendix-1, of the 22 Petrobras Bonds, 18 were fixed-rate bonds and 4 were floating rate bonds. To compute Petrobras bond residual returns and control for changing interest rates, for each bond, I ran a regression modeling the daily return of the bond as a function of: 1) a constant term, 2) the returns of the overall U.S. stock market, 3) the Peer Index return net of the Market Index return, 4) the percentage change in the value of the Brazilian real in terms of US dollars, and 5) the return of a market benchmark bond ("Benchmark Bond").

270. The daily returns of the Benchmark Bond were computed by pricing each day a hypothetical bond with the same coupon and maturity as the respective Petrobras bond, applying the yield-to-maturity from a market index matched to the Company's credit rating. The market index yield was drawn from the BofA Merrill Lynch US Corporate A Index, the BofA Merrill Lynch US Corporate BBB Index, and the BofA Merrill Lynch US Corporate BB Index ("the Merrill Indices") with the same rating as the rating assigned by Moody's to Petrobras at each point in time. When the Company's credit rating was between the ratings of the Merrill Indices, I interpolated between the yields of the two Merrill Indices whose ratings straddled that of the Company.[99]

[99] Data for the Merrill Indices obtained from BofA Merrill Lynch via Bloomberg.

271. For example, on 22 January 2010, the start of the Class Period, the Company carried a Moody's A3 rating. The yield from the BofA Merrill Lynch US Corporate A Index was 4.334% that day and the yield from the BofA Merrill Lynch US Corporate BBB Index was 5.145% that day. Therefore, the yield of the Benchmark Bond for the Senior Unsecured Notes was 4.875%, in between the respective yields of the A and BBB indices. This yield was then used to price benchmark bonds with the same coupon and maturities of each of the Petrobras fixed-rate bonds. The next trading day, 25 January 2010, the A-rated index yield rose to 4.368% and the BBB rated index yield rose to 5.193%, resulting in a new yield for the Benchmark Bond of 4.918%. This yield was then used to reprice the benchmark bonds. For this sequence of yields, the logarithmic return of the Benchmark Bond for the January 2016 Notes on 25 January 2010 was -0.22%. In this manner, daily returns for the Benchmark Bond for each of the Senior Unsecured Notes were computed.

272. On days when the rating of Petrobras changed, the Benchmark Bond return was computed based on the prior day's rating, and on subsequent days, Benchmark Bond returns were computed based on the new rating. Exhibit-9 presents the Benchmark Bond yields, and Table-6 shows the Moody's ratings for Petrobras Senior Unsecured Notes.

Table-6: Moody's and S&P Credit Rating History

Moody's Credit Rating History

Class Period: 22 January 2010 through 19 March 2015

Senior Unsecured

Date	Rating	Watch
11/5/2007	Baa1	
6/21/2011	A3	
10/3/2013	Baa1	
10/21/2014	Baa2	
12/23/2014	Baa2	*-
1/29/2015	Baa3	*-
2/24/2015	Ba2	*-

Note:

*- Rating under review negative implication

Source: Bloomberg

273. The Benchmark Bond return variable in the regression model controls for the effect of changes in the market interest rate.

Controlling for the Overall Bond Market Factor for Floating Rate Bonds

274. As shown in Appendix-1, of the 22 Petrobras Bonds, 4 were floating rate bonds whose coupons were linked to 3-month LIBOR. Floating rate bonds are designed to be relatively insensitive to changing market interest rates. The typical negative impact of a rise in interest rates, for example, is counteracted by the increase in the coupon rate. When rates fall, the lower coupon countervails against the positive impact of the lower discount rate. Consequently, for the floating rate bonds the Benchmark Bond return variable would not appropriately capture the interest rate effect. As such, instead of constructing a Benchmark Bond, I used the change in the ICE 3-month U.S. LIBOR rate as the explanatory variable for interest rates on the four floating rate bonds. The series of daily changes in the LIBOR rate are presented in Exhibit-10.[100]

Running the Regression

275. Using Petrobras Bond trading data from TRACE, I performed a regression analysis for each of the Petrobras Bonds on daily returns covering the entire portion of the Class Period when each respective security was traded.

276. As is common in the bond market, for each bond there were some days on which no trading took place. For each bond, the regression data series included only days on which there was a trading price for two consecutive trading days, so that a one-day return could be computed. All bond returns, for the Petrobras Bonds and Benchmark Bonds, are one-day logarithmic returns.

277. Exhibit-7c presents the regression results.

[100] Data obtained from Bloomberg.

Collective Event Study Results

278. As shown in Exhibit-8ca through Exhibit-8cg, the total number of days for which a return could be calculated for the Petrobras Bonds ranged from 228 to 1,345 days per bond and the total number of statistically significant days ranged from 13 to 81 days.[101]

(1) Z-test on corruption-related 6-K days vs. all other days

279. As shown in Exhibit-13a, there was a range of 2 to 5 statistically significant corruption-related 6-K days per bond out of a total of 4 to 6 corruption-related 6-K days per bond. There was a range of 11 to 76 statistically significant non-event days per bond out of a total of 224 to 1,339 non-event days per bond. Accordingly, the proportion of statistically significant days in the corruption-related 6-K days ranged from 33.33% to 83.33% per bond and the proportion of statistically significant days among all non-event days ranged from 3.50% to 6.17% per bond.

280. The differences in the proportions of days is associated with z-scores ranging from 2.96 to 8.38, all of which are greater than the critical z-statistic threshold of 1.65 for significance at the 95% confidence level. These results indicate that all of the bonds exhibited statistically significant returns significantly more frequently within the event day samples than within the non-event day samples. These results prove that all of the bonds demonstrated that they react to Company-specific information with statistically significant frequency. This conclusion is confirmed for each of the bonds at a confidence level exceeding 99%.

281. This statistical test result indicates that there was a cause and effect relationship between the release of new, Company-specific information and reactions in the Petrobras Bond prices for all of the bonds. This finding establishes that the Petrobras Bonds traded in an efficient market during the Class Period.

[101] Because the Petrobras Bonds were issued at different times and some traded less frequently than others, the sample of total days and total significant days reflects the actual number of days on which a particular bond traded.

(2) Z-test on corruption-related 6-K days and earnings announcement days vs. all other days

282. As shown in Exhibit-13b, for the combined set of corruption-related 6-K events and earnings announcements, there was a range of 2 to 9 statistically significant days per bond out of a total of 7 to 28 such dates. There was a range of 11 to 72 statistically significant non-event days per bond out of a total of 228 to 1,345 non-event days per bond. Accordingly, the proportion of statistically significant days in the combined event day sample ranged from 16% to 44% per bond, while the proportion of statistically significant days among the non-event days ranged between 3.38% and 5.66% per bond.

283. These differences in frequencies are associated with z-scores ranging from 2.51 to 6.17, all of which are greater than the critical z-statistic threshold of 1.65 for significance at the 95% confidence level. These results indicate that all of the bonds exhibited statistically significant returns significantly more frequently within the event day samples than within the non-event day samples. These results prove that all of the bonds demonstrated that they react to Company-specific information with statistically significant frequency. This conclusion is confirmed for each of the bonds at a confidence level exceeding 99%.

284. This statistical test results indicate that there was a cause and effect relationship between the release of new, Company-specific information and reactions in the Petrobras Bond prices. This finding establishes that the Petrobras Bonds traded in an efficient market during the Class Period.

2. Collective Event Study Summary

285. The event study results show that there was a statistically significantly greater frequency of statistically significant price reactions on the news event dates as compared to the frequency on non-event dates for all of the bonds.

286. These test results demonstrate a cause and effect relationship between the release of new, Company-specific information and reactions in the Petrobras Bond prices for all of the bonds. These results are compelling evidence in support of the efficiency of the market for the Petrobras Bonds during the Class Period.

C. Further Empirical Evidence of Bond Market Efficiency

287. In addition to the bond event study results, the bond regressions provide additional compelling evidence of the efficiency of the market for the Petrobras Bonds. The fundamental value of a bond changes with Company information (e.g. credit risk), and also on a day-to-day basis in response to changes in the market interest rate. A higher market interest rate would make a fixed coupon bond less attractive to investors, and should therefore depress its price, all else equal. Alternatively, a lower market interest rate should boost the bond price, all else equal.

> "We have seen already that an inverse relationship exists between bond prices and yields, and we know that interest rates can fluctuate substantially. As interest rates rise and fall, bondholders experience capital losses and gains."
> **Investments, 8th edition, by Zvi Bodie, Alex Kane, and Alan J. Marcus, McGraw-Hill Irwin, 2009, p. 513.**

288. In an efficient market, investors would take note of market interest rate changes and such changes would be incorporated into the trading prices of the bonds.

289. The Benchmark Bond return variable in the regression models reflects changes in market interest rates. As noted above, the regression results show that all of the fixed-rate bonds moved significantly with the Benchmark Bond return. The Benchmark Bond return is a significant explanatory variable of Petrobras bond returns at an extremely high level of statistical significance. This finding is compelling proof of the efficiency of the market for the fixed-rate bonds throughout the Class Period.

290. Floating rate notes, unlike fixed-rate notes, are largely protected from the valuation impact of changing market interest rates. The value of a floating rate note is mostly insulated from market interest rate movements, as changes in the coupon payments offset changes in the discount rate. Consequently, floating rate notes generally carry far less interest rate risk and market price risk than do fixed-rate notes. Instead, it is credit risk that tends to be the major risk faced by investors in floating rate notes.

> "The major risk involved in floaters has to do with changes in the firm's financial strength. The yield spread is fixed over the life of the security, which may be many years. If the financial health of the firm deteriorates, then investors will demand a greater yield premium than is offered by the security. In this case, the price of the bond will fall."
> ***Investments*, 8th edition, by Zvi Bodie, Alex Kane, and Alan J. Marcus, 2009, p. 449.**

291. The day-to-day cause-and-effect relationship thusly demonstrated between changes in market interest rates and the prices of the fixed-rate Petrobras Bonds, proves that market participants did not ignore valuation relevant information when pricing and trading Petrobras bonds. This fact proves that the market for Petrobras Bonds was efficient throughout the Class Period.

D. Bond Efficiency Summary and Conclusion

292. The following factors indicate that the Petrobras Bonds traded in an efficient market: coverage by analysts and ratings agencies; widespread news coverage; F-3 registration eligibility (with the noted exception); prominent and numerous underwriters; the large size of the issues, individually and in the aggregate; active trading volume; frequent trading; large trade size; transparency provided by TRACE; and the general size and activity of the U.S. bond market.

293. Not only did the Petrobras Bonds exhibit the above mentioned factors that indicate market efficiency, but they also satisfied the empirical *Cammer* factor, which established that the Petrobras Bonds traded in an efficient market. The event studies proved that there was a cause and effect relationship between the release of information and movements in Petrobras's bond prices. The regression analysis proved that the bond prices moved appropriately with interest rates over the course of the Class Period.

294. Given these facts, I conclude that the Petrobras Bonds traded in an efficient market over the course of the Class Period.

COMMON DAMAGE METHODOLOGY

295. Counsel for the Lead Plaintiff asked me to opine on whether per share damages could be measured for each Class member under Section 10(b) of the Exchange Act using a common methodology for all Class members.

A. Section 10(b) Per Share Damage Methodology

296. Assuming a Plaintiff verdict on the allegations of fraud, Section 10(b) per share damages can be measured as follows:

i. First, valuation tools, which would include event study analysis such as that described herein, and potentially other empirical analyses if necessary, would be used to establish that the disclosure(s), correcting the alleged misrepresentations and omissions, caused the price of Petrobras securities to fall. This analysis, after controlling for potentially non-fraud-related information, would establish that the alleged misrepresentations and omissions had caused the security price to be artificially inflated, and that the corrective disclosure(s) caused the inflation to dissipate, in turn causing investor losses. This analysis would apply on a class-wide basis for all securities.

ii. Second, an inflation ribbon would be constructed, using generally empirical analysis and valuation tools, indicating how much artificial inflation caused by the alleged misrepresentations and omissions was in the price of the Petrobras securities on each day during the Class Period. An inflation ribbon is a time series of the difference between the security's actual price observed in the marketplace, and the estimated price that the security would have traded at each day had there been full disclosure from the outset of the Class Period. Construction of the inflation ribbon generally employs event study analysis, combined with widely used and generally accepted valuation tools and models. The inflation ribbon is often constructed by working chronologically backwards from the final corrective disclosure to the start of the Class Period, accounting for alleged fraud-related residual price declines as they occurred. Inflation prior to a corrective disclosure that dissipated inflation is greater than the inflation afterward by the amount of inflation that dissipated. This analysis would also apply on a class-wide basis.

iii. Third, the measure of per share damages generally applied in 10b-5 cases is the reduction in the inflation ribbon over an investor's holding period (the

economic/inflation loss). That is, for each Class member, per share damages would be calculated as the difference between the inflation on the date shares were purchased and the inflation on the date those same shares were subsequently sold. Per share damages are limited, however, to be no greater than the decline in share price over the holding period, which is the investment loss actually sustained. Pursuant to the Private Securities Litigation Reform Act of 1995 (the "PSLRA") (15 U.S.C. § 78u-4(e)), for any shares sold during the 90-day period after the end of the Class Period, per share damages would be calculated as the lesser of the reduction in the dollar inflation over the investor's holding period (the economic/inflation loss), or the decline in the security price (the investment loss), where the terminal security price is deemed to be the average price from the final corrective disclosure date to the sale date. Also pursuant to the PSLRA, for any shares held 90 days or more beyond the final corrective disclosure, damages would equal the lesser of the reduction in the dollar inflation over the investor's holding period (the economic/inflation loss) or the decline in the security price (the investment loss), where the terminal security price is deemed to be the average price over the 90 days following the final corrective disclosure. The calculation of each Class member's damages would be a mechanical arithmetical exercise, conducted the same way for all Class members, applying the results of the Class-wide analyses described above to each Class member's trading data.

297. Consequently, each Class member's per share damages under Section 10(b) can be computed in the same way, common to all Class members, using readily available daily pricing information, in accordance with widely used and generally accepted methodologies and the PSLRA.

298. I have not yet been asked to calculate damages for any of the claims alleged on behalf of the class, and such calculations will likely depend, in part, on the completion of discovery, which I understand is currently stayed pursuant to the PSLRA. However, the methodology described above is generally accepted and widely used for calculating

damages under Section 10(b) consistently on a Class-wide basis in securities class actions.

LIMITING FACTORS AND OTHER ASSUMPTIONS

299. This report is furnished solely for the purpose of court proceedings in the above referenced matter and may not be used or referred to for any other purpose. The analysis and opinions contained in this report are based on information available as of the date of this report. I reserve the right to supplement or amend this report, including in the event additional information becomes available.

Steven P. Feinstein, Ph.D., CFA

Exhibit-7a

Petrobras Common ADR Regression Summary

Regression Period	Intercept	Market Index Coefficient	Market Index *t*-Stat	Peer Index Coefficient	Peer Index *t*-Stat	Foreign Exchange Coefficient	Foreign Exchange *t*-Stat	Residual Standard Error	R-Squared	Adjusted R-Squared
1/22/2010 - 7/28/2011	-0.001	0.904	10.28	0.668	5.84	0.480	3.90	1.35%	0.495	0.491
7/29/2011 - 7/27/2012	-0.002	1.278	15.32	0.417	2.25	0.313	2.59	1.63%	0.648	0.644
7/30/2012 - 7/26/2013	-0.001	1.208	7.04	1.410	4.68	0.929	4.18	1.91%	0.324	0.315
7/29/2013 - 7/28/2014	0.000	0.733	3.48	0.748	2.55	1.165	6.59	2.11%	0.272	0.263
7/29/2014 - 7/28/2015	0.001	1.332	4.69	1.613	7.08	1.576	7.82	3.34%	0.438	0.431

Exhibit-7b

Petrobras Preferred ADR Regression Summary

Regression Period	Intercept	Market Index Coefficient	Market Index *t*-Stat	Peer Index Coefficient	Peer Index *t*-Stat	Foreign Exchange Coefficient	Foreign Exchange *t*-Stat	Preferred Index Coefficient	Preferred Index *t*-Stat	Residual Standard Error	R-Squared	Adjusted R-Squared
1/22/2010 - 7/28/2011	-0 001	0 909	9 75	0 670	5 59	0 625	4 71	0 237	1 30	1 41%	0 512	0 507
7/29/2011 - 7/27/2012	-0 001	1 266	14 45	0 449	2 29	0 306	2 41	-0 018	-0 10	1 71%	0 622	0 616
7/30/2012 - 7/26/2013	-0 001	1 080	7 43	1 163	4 55	1 153	6 12	0 948	2 45	1 62%	0 400	0 391
7/29/2013 - 7/28/2014	0 000	0 630	2 87	0 648	2 12	1 453	7 69	-0 813	-1 37	2 20%	0 298	0 286
7/29/2014 - 7/28/2015	0 000	1 182	4 07	1 537	6 59	1 738	8 37	0 164	0 15	3 42%	0 435	0 426

Exhibit-7c
Petrobras Bonds Regression Summary

Petrobras Bonds	Regression Period	Market Index Coefficient	Market Index *t*-Stat	Peer Index Coefficient	Peer Index *t*-Stat	Foreign Exchange Coefficient	Foreign Exchange *t*-Stat	Interest Rate Coefficient	Interest Rate *t*-Stat	Residual Standard Error	Adjusted R-Squared
May 2043 Notes	5/6/2013 - 7/28/2014	0.199	2.69	0.255	2.34	0.122	1.84	0.831	9.53	0.008	0.321
May 2043 Notes	7/29/2014 - 6/29/2015	0.523	4.11	0.061	0.60	0.258	2.85	0.566	2.85	0.014	0.153
January 2019 Notes	5/6/2013 - 7/28/2014	0.126	4.25	0.059	1.35	-0.028	-1.07	1.541	12.64	0.003	0.390
January 2019 Notes	7/29/2014 - 6/29/2015	0.235	3.62	-0.006	-0.12	0.118	2.56	1.287	3.01	0.007	0.128
May 2016 Notes	5/6/2013 - 7/28/2014	0.104	4.66	0.061	1.85	-0.042	-2.10	0.784	4.48	0.003	0.122
May 2016 Notes	7/29/2014 - 6/29/2015	0.087	1.88	0.043	1.18	0.027	0.84	2.199	2.54	0.005	0.040
May 2016 Floating Rate Notes	5/6/2013 - 7/28/2014	0.058	1.83	0.024	0.51	-0.048	-1.80	-0.049	-0.31	0.003	0.005
May 2016 Floating Rate Notes	7/29/2014 - 6/29/2015	0.141	2.18	-0.054	-1.07	0.100	2.17	0.443	1.56	0.007	0.065
January 2019 Floating Rate Notes	5/7/2013 - 7/28/2014	0.054	1.51	0.073	1.38	-0.006	-0.21	-0.470	-2.64	0.004	0.024
January 2019 Floating Rate Notes	7/29/2014 - 6/29/2015	0.173	2.25	0.010	0.17	0.107	2.03	0.275	0.83	0.008	0.046
May 2023 Notes	5/6/2013 - 7/28/2014	0.153	3.35	0.057	0.85	0.046	1.14	1.448	12.62	0.005	0.409
May 2023 Notes	7/29/2014 - 6/29/2015	0.325	3.91	0.038	0.58	0.206	3.49	0.987	3.38	0.009	0.176
March 2017 Notes	1/22/2010 - 6/29/2015	0.130	3.80	-0.006	-0.21	0.078	3.04	0.985	2.64	0.004	0.100
March 2020 Notes	1/22/2010 - 6/29/2015	0.219	4.21	0.051	1.19	0.114	2.93	1.098	4.07	0.006	0.137
March 2017 Floating Rate Notes	1/22/2010 - 6/29/2015	0.136	2.23	-0.029	-0.57	0.091	1.98	-0.011	-0.04	0.007	0.033
March 2044 Notes	1/22/2010 - 6/29/2015	0.324	3.03	0.208	2.14	0.314	3.93	0.446	2.64	0.013	0.131
March 2020 Floating Rate Notes	1/22/2010 - 6/29/2015	0.182	2.66	-0.052	-0.92	0.161	3.15	-0.011	-0.03	0.008	0.075
March 2024 Notes	1/22/2010 - 6/29/2015	0.257	3.85	0.118	2.13	0.223	4.48	0.875	3.94	0.008	0.180
December 2018 Notes	1/22/2010 - 7/28/2011	0.105	1.00	-0.131	-1.04	0.128	0.93	0.542	1.77	0.014	0.014
December 2018 Notes	7/29/2011 - 7/27/2012	0.057	1.09	0.010	0.10	-0.006	-0.09	0.808	2.82	0.008	0.019
December 2018 Notes	7/30/2012 - 7/26/2013	0.206	2.72	0.019	0.14	-0.035	-0.36	1.882	5.23	0.008	0.125
December 2018 Notes	7/29/2013 - 7/28/2014	-0.024	-0.50	0.008	0.12	0.008	0.19	1.430	5.75	0.005	0.138
December 2018 Notes	7/29/2014 - 6/29/2015	0.161	2.03	0.105	1.70	0.107	1.94	1.861	3.27	0.009	0.090
October 2016 Notes	1/22/2010 - 7/28/2011	0.025	0.42	-0.036	-0.50	0.047	0.61	0.064	0.32	0.008	-0.006
October 2016 Notes	7/29/2011 - 7/27/2012	0.056	1.54	0.027	0.36	-0.085	-1.68	0.380	1.45	0.006	0.003
October 2016 Notes	7/30/2012 - 7/26/2013	0.082	2.12	0.003	0.04	-0.025	-0.49	1.433	5.32	0.004	0.105
October 2016 Notes	7/29/2013 - 7/28/2014	0.004	0.11	-0.008	-0.16	0.003	0.09	0.547	1.84	0.003	0.000
October 2016 Notes	7/29/2014 - 6/29/2015	0.076	1.84	0.010	0.29	0.088	3.04	1.264	2.14	0.005	0.082
March 2018 Notes	1/22/2010 - 7/28/2011	0.034	0.99	-0.041	-0.97	0.048	1.05	0.585	5.91	0.005	0.084
March 2018 Notes	7/29/2011 - 7/27/2012	0.053	1.81	0.062	1.05	-0.004	-0.09	1.077	6.33	0.005	0.132
March 2018 Notes	7/30/2012 - 7/26/2013	0.163	4.25	0.024	0.36	-0.074	-1.50	1.231	5.97	0.004	0.154
March 2018 Notes	7/29/2013 - 7/28/2014	0.011	0.35	0.017	0.40	-0.013	-0.47	1.299	7.02	0.003	0.173
March 2018 Notes	7/29/2014 - 6/29/2015	0.161	3.00	-0.012	-0.29	0.106	2.80	1.071	2.33	0.006	7.017
March 2019 Notes	1/22/2010 - 7/28/2011	0.113	3.44	-0.141	-3.39	0.136	3.09	0.484	5.32	0.005	0.178
March 2019 Notes	7/29/2011 - 7/27/2012	0.086	2.69	0.044	0.69	-0.004	-0.09	1.134	6.83	0.005	0.164
March 2019 Notes	7/30/2012 - 7/26/2013	0.135	3.44	0.055	0.80	-0.009	-0.18	1.209	6.47	0.004	0.170

Exhibit-7c
Petrobras Bonds Regression Summary

Petrobras Bonds	Regression Period	Market Index Coefficient	Market Index *t*-Stat	Peer Index Coefficient	Peer Index *t*-Stat	Foreign Exchange Coefficient	Foreign Exchange *t*-Stat	Interest Rate Coefficient	Interest Rate *t*-Stat	Residual Standard Error	Adjusted R-Squared
March 2019 Notes	7/29/2013 - 7/28/2014	0.046	1.46	0.011	0.27	0.032	1.17	1.272	8.22	0.003	0.256
March 2019 Notes	7/29/2014 - 6/29/2015	0.217	3.46	0.016	0.31	0.146	3.29	0.984	2.32	0.007	0.135
January 2020 Notes	1/22/2010 - 7/28/2011	0.092	2.55	-0.057	-1.25	0.080	1.66	0.201	2.29	0.005	0.061
January 2020 Notes	7/29/2011 - 7/27/2012	0.032	0.95	0.082	1.18	0.009	0.19	0.882	5.55	0.006	0.108
January 2020 Notes	7/30/2012 - 7/26/2013	0.122	3.06	0.004	0.06	-0.063	-1.23	1.603	9.89	0.004	0.282
January 2020 Notes	7/29/2013 - 7/28/2014	0.038	1.01	0.032	0.64	-0.013	-0.40	1.162	7.36	0.004	0.189
January 2020 Notes	7/29/2014 - 6/29/2015	0.275	4.12	0.047	0.89	0.109	2.30	0.872	2.33	0.007	0.128
January 2040 Notes	1/22/2010 - 7/28/2011	0.170	3.33	0.005	0.08	-0.010	-0.14	0.208	3.52	0.007	0.053
January 2040 Notes	7/29/2011 - 7/27/2012	0.154	2.91	0.092	0.83	-0.004	-0.06	0.978	9.11	0.009	0.264
January 2040 Notes	7/30/2012 - 7/26/2013	0.222	2.88	0.225	1.71	0.034	0.34	1.292	11.21	0.008	0.388
January 2040 Notes	7/29/2013 - 7/28/2014	0.065	0.84	0.107	1.03	0.116	1.70	0.826	7.38	0.007	0.250
January 2040 Notes	7/29/2014 - 6/29/2015	0.465	3.26	0.020	0.17	0.223	2.19	0.759	3.05	0.016	0.112
January 2021 Notes	1/13/2011 - 7/27/2012	0.029	1.22	0.079	1.93	-0.004	-0.14	0.856	9.46	0.005	0.199
January 2021 Notes	7/30/2012 - 7/26/2013	0.163	4.39	0.038	0.59	-0.023	-0.48	1.615	12.12	0.004	0.388
January 2021 Notes	7/29/2013 - 7/28/2014	0.047	1.36	0.053	1.15	0.000	-0.01	1.110	8.86	0.003	0.267
January 2021 Notes	7/29/2013 - 6/29/2015	0.250	3.50	0.038	0.68	0.167	3.29	1.124	3.34	0.008	0.156
January 2041 Notes	1/13/2011 - 7/27/2012	-0.026	-0.55	0.169	2.07	0.065	0.99	0.700	8.02	0.009	0.183
January 2041 Notes	7/30/2012 - 7/26/2013	0.231	3.22	0.023	0.19	0.049	0.52	1.147	10.97	0.008	0.356
January 2041 Notes	7/29/2013 - 7/28/2014	0.107	1.27	-0.024	-0.21	0.113	1.55	0.764	6.41	0.008	0.188
January 2041 Notes	7/29/2013 - 6/29/2015	0.544	4.15	0.126	1.21	0.280	0.28	3.002	0.69	3.120	0.169
January 2016 Notes	1/13/2011 - 7/27/2012	0.008	0.49	0.071	2.57	-0.005	-0.23	0.867	7.64	0.003	0.151
January 2016 Notes	7/30/2012 - 7/26/2013	0.023	1.02	-0.043	-1.11	0.008	0.27	0.868	4.48	0.002	0.069
January 2016 Notes	7/29/2013 - 7/28/2014	0.004	0.24	-0.020	-0.93	0.009	0.68	0.489	2.86	0.002	0.030
January 2016 Notes	7/29/2013 - 6/29/2015	0.052	2.17	0.031	1.61	0.049	2.91	0.879	1.53	0.003	0.086
February 2017 Notes	1/25/2012 - 7/26/2013	0.092	5.31	0.034	1.20	-0.047	-2.25	1.232	10.57	0.002	0.240
February 2017 Notes	7/29/2013 - 7/28/2014	0.024	1.13	-0.022	-0.75	0.021	1.10	1.018	6.44	0.002	0.171
February 2017 Notes	7/29/2014 - 6/29/2015	0.120	2.78	0.015	0.43	0.090	2.95	1.483	2.75	0.005	0.111

Exhibit II

Professor Steven P. Feinstein's Report on Market Efficiency

IN RE: ELETROBRAS SECURITIES LITIGATION (No. 15-cv-5754-JGK)

UNITED STATES DISTRICT COURT
SOUTHERN DISTRICT OF NEW YORK

IN RE: ELETROBRAS SECURITIES LITIGATION	No. 15-cv-5754-JGK

REPORT ON MARKET EFFICIENCY

PROFESSOR STEVEN P. FEINSTEIN, PH.D., CFA

June 30, 2017

TABLE OF CONTENTS

I. SCOPE OF PROJECT AND REPORT

1. I was asked by Kahn Swick & Foti LLP and Kaplan Fox & Kilsheimer LLP, co-Lead Counsel for the Plaintiffs, to determine whether certain securities of Centrais Elétricas Brasileiras S.A. Eletrobras (“Eletrobras” or the “Company”) traded in efficient markets during the period from 17 August 2010 to 24 June 2015 (the “Class Period”). These securities (collectively, “Eletrobras securities”) are: 1) common American Depositary Shares (the “common ADS” or the “Eletrobras common ADS”); 2) Class B Preferred American Depositary Shares (the “preferred ADS” or the “Eletrobras preferred ADS”); and 3) two U.S. Dollar (“USD”) denominated debt securities[1] (the “Notes” or “Eletrobras Notes”).
2. In addition, I have been asked to opine on whether damages under Section 10(b) of the Exchange Act of 1934 and SEC Rule 10(b)-5 adopted thereunder (collectively, “Section 10(b)”) can be computed in this matter for all Class members using a common class-wide methodology that is consistent with the Plaintiffs’ allegations of liability.
3. Toward these ends, I analyzed the markets for Eletrobras’ securities, the price behavior of the securities, and the factors that are generally accepted to be indicative of market efficiency for publicly-traded securities. I examined Company press releases, conference call transcripts, security analyst reports, news articles, SEC filings, the security prices, trading volume, the performance of the overall U.S. and Brazilian stock markets, and the performance of Eletrobras’ peer group, as well as other pertinent data and documents. I read the Consolidated Second Amended Complaint for Violations of Federal Securities Laws (“Complaint”) dated 26 February 2016, and considered the allegations therein. Exhibit-1 lists the documents I considered in preparing this report and arriving at the opinions expressed herein.
4. This report presents my methodology, findings, and conclusions.
5. I understand that discovery is ongoing in this matter. I reserve the right to amend, refine, or modify my opinion and report, including in the event any additional information or analyses become available.

[1] The two bonds at issue are ISINs USP22854AF3 and USP22854AG1.

II. CREDENTIALS

6. I am an Associate Professor of Finance at Babson College, and the founder and president of Crowninshield Financial Research, Inc., a financial economics consulting firm.

7. I hold a Ph.D. in Economics from Yale University, a Master of Philosophy degree in Economics from Yale University, a Master of Arts in Economics from Yale University, and a Bachelor of Arts degree in Economics from Pomona College. I also hold the Chartered Financial Analyst ("CFA") designation, granted by the CFA Institute.

8. At Babson College, I have taught undergraduate and MBA level courses in Capital Markets, Investments, Equity Analysis, Fixed Income Analysis, Financial Management, Risk Management, Quantitative Methods, and Security Valuation. I have also taught executive courses on investments and corporate financial management for numerous corporations. Other courses I have taught are listed in my curriculum vitae, which is attached as Exhibit-2.

9. At Babson College, I have held the Chair in Applied Investments and served as the Director of the Stephen D. Cutler Investment Management Center, a research and education center dedicated to the study and teaching of investments and capital markets.

10. Prior to my joining the faculty at Babson College, I taught finance at Boston University. Previously, I was an Economist at the Federal Reserve Bank of Atlanta where my primary responsibilities were to monitor financial markets, analyze proposed regulation, and advise the Bank President in preparation for his participation in meetings of the Federal Open Market Committee – the government body responsible for monetary policy in the United States.

11. I have published in the field of finance. My finance articles have appeared in the *Atlanta Federal Reserve Bank Economic Review*, *Derivatives Quarterly*, *Derivatives Weekly*, *The Engineering Economist*, *The Journal of Risk*, *The American Bankruptcy Institute Journal*, *The Journal of Financial Planning*, *The Journal of Forensic Economics*, *Managerial Finance*, *Risk Management*, and *Primus*. I am the author of *Finance and Accounting for Project Management*, published by the American Management Association. I wrote two chapters in the book *The Portable MBA in Finance and Accounting* – one on corporate financial planning and the other on risk management. I have presented research at the annual conventions of the American Finance Association, the Academy of Financial

Services, the Multinational Finance Society, the Financial Management Association, the Taxpayers Against Fraud Education Fund Conference, and the International Conference on Applied Business Research. Co-authored papers of mine have been presented at the Eastern Finance Association meetings and the Midwestern Finance Association meetings. A list of all the publications I authored in the previous ten years can be found in my curriculum vitae, which is attached as Exhibit-2.

12. I have been selected to review papers for numerous finance journals and conferences, and I have reviewed finance textbook manuscripts for Prentice-Hall, Elsevier, Blackwell, and Southwestern Publishing. I have been quoted on matters relating to finance and investments in *The Wall Street Journal*, *The Washington Post*, *The New York Times*, *The Financial Times*, *The Boston Globe*, and *Bloomberg News*, and my research relating to financial analysis and valuation has been discussed in *The Wall Street Journal*, *Bond Buyer*, and *Grant's Municipal Bond Observer*.

13. I am a member of the American Finance Association, the Financial Management Association, the North American Case Research Association, the National Association of Forensic Economics, the CFA Institute, and the CFA Society Boston, where I have served as a member of the education committee and ethics subcommittee. I served on the Fixed Income Specialization Examination Committee of the CFA Institute.

14. The CFA designation is the premier credential for financial analysts worldwide. In order to receive this credential, applicants must pass a series of three exams covering such topics as economics, equity analysis, financial valuation, business analysis, quantitative methods, investment analysis, portfolio management, risk management, financial accounting, and ethical and professional standards. For over ten years I taught in the Boston University CFA Review Program and the CFA Society Boston Review Program – two of the leading review programs that prepared candidates for the CFA exams. In both of these programs I taught candidates at the most advanced level.

15. In addition to my teaching, research, CFA, and academic community responsibilities, I practice extensively as a financial consultant. Past clients include the United States Securities and Exchange Commission, the Internal Revenue Service, the Attorney General of the State of Illinois, and the National Association of Securities Dealers. As a financial consultant, I have conducted analyses and presented opinions related to markets,

valuation, and damages in over 70 cases. Exhibit-3 lists my prior testimony appearances over the past four years.

16. I am the sole owner of the consulting firm Crowninshield Financial Research, which receives compensation for the work performed by me and the analysts who assist me on this case. My firm is being compensated at a rate of $875 per hour for my work. My compensation is neither contingent on my findings nor on the outcome of this matter.

III. <u>CONCLUSIONS</u>

17. The Eletrobras securities traded in efficient markets over the course of the Class Period. The Eletrobras securities satisfied the factors set forth in *Cammer v. Bloom*, 711 F. Supp. 1264, 1273 (D.N.J. 1989) and *Krogman v. Sterritt*, 202 F.R.D. 467 (N.D. Tex. 2001), which, consistent with financial economic principles and empirical research, indicate market efficiency.

18. The Eletrobras common ADS satisfied all of the *Cammer* and *Krogman* factors by wide margins. In particular, with a high degree of statistical certainty, event study analysis proves that there was a cause and effect relationship between the release of new, Company-specific information and movements in the Eletrobras common ADS price. Additional statistical tests examining the behavior of the Eletrobras common ADS on important announcement dates collectively further indicate that the Eletrobras common ADS responded to new Company-specific information throughout the Class Period. The empirical analyses not only indicate market efficiency, but prove that the common ADS demonstrated market efficiency.

19. The Eletrobras preferred ADS traded in an efficient market over the course of the Class Period. The Eletrobras preferred ADS satisfied all *Cammer* and *Krogman* factors. Statistical tests prove that there was a cause and effect relationship between the release of new Company-specific information and movements in the Eletrobras preferred ADS prices, which is the essence of market efficiency.

20. The Eletrobras Notes traded in an efficient market over the portion of the Class Period during which they traded. The Notes satisfied all of the *Cammer* and *Krogman* factors. Moreover, statistical tests prove that there was a cause and effect relationship between the release of new information expected to be material to the Notes and movements in the

Case 1:15-cv-05754-JGK Document 63-2 Filed 10/06/17 Page 10 of 37 PageID #: 19052

prices of the Notes. The prices of both Notes moved appropriately in response to new, Company-specific information and in response to changes in market interest rates continuously over the course of the Class Period.

21. Based on the foregoing, I conclude that the markets for the Eletrobras common ADS, Eletrobras preferred ADS, and the Eletrobras Notes markets were efficient over the course of the Class Period.

22. Damages in this matter can be computed on a class-wide basis for all securities and for all Class Members using a common methodology that is consistent with the Plaintiffs allegations of liability.

IV. BACKGROUND

A. About the Company

23. Eletrobras is not only the largest electric utility company in Brazil, but is the largest electric utility company in all of Latin America. Throughout the Class Period, Eletrobras through its subsidiaries was "involved in the generation, transmission, and distribution of electricity in Brazil."[2] According to the Company, its revenues derived mainly from: 1) "the generation of electricity and its sale to electricity distribution companies and free consumers;" 2) "the transmission of electricity on behalf of other electricity concessionaires;" and 3) "the distribution of electricity to end consumers."[3]

24. For the fiscal year ended 31 December 2014, net operating revenue from the Company's generation segment accounted for 62.8% of the Company's total revenue.[4] The transmission segment accounted for 16.5%,[5] and distribution segment accounted for 22.1%.[6] As of 31 December 2014, the Company conducted its business in Brazil through

[2] Centrais Elétricas Brasileiras S.A. Eletrobras, Form 20-F for the fiscal year ended 31 December 2015, filed 11 October 2016, p. 42.

[3] Id.

[4] Centrais Elétricas Brasileiras S.A. Eletrobras, Form 20-F for the fiscal year ended 31 December 2014, filed 11 October 2016, p. 47.

[5] Id., p. 61.

[6] Id., p. 65.

Case 1:15-cv-05754-JGK Document 63-10 Filed 10/06/17 Page 17 of 37 PageID #: 19053

"fourteen regional subsidiaries and Itaipu [a hydroelectric plant jointly owned by the Paraguayan government], 179 SPEs [special purpose entities], and non-controlling interests in 25 companies."[7]

25. Throughout the Class Period, the Brazilian federal government was the Company's controlling shareholder.[8] For fiscal years 2010 through 2015, the Brazilian federal government owned between 51% and 54% of the Company's voting shares.[9]

26. For fiscal years 2010, 2011, 2012, 2013, 2014, and 2015, Eletrobras reported net operating revenue of R$26.8 billion, R$29.5 billion, R$28.0 billion, R$23.8 billion, R$30.1 billion, and R$32.6 billion, respectively.[10] The Company reported total assets for fiscal years 2010, 2011, 2012, 2013, 2014, and 2015 of R$146.9 billion, R$139.4 billion, R$142.7 billion, R$138.6 billion, R$144.5 billion, and R$149.5 billion respectively.[11]

27. As an electric utility company operating in Brazil, the Company is required by Brazilian law to apply for, and be granted, concession(s) from the Brazilian government for the right to supply electrical energy services (generation, transmission, distribution).[12] As of 31 December 2011, approximately 31% of Eletrobras' outstanding concessions for electric power generation were due to expire by the end of 2015.[13] According to the Company,

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[7] Id., p. 102.

[8] Centrais Elétricas Brasileiras S.A. Eletrobras, Form 20-F for the fiscal year ended 31 December 2015, filed 11 October 2016, p. 26.

[9] Centrais Elétricas Brasileiras S.A. Eletrobras, Form 20-F for the fiscal year ended 31 December 2011, filed 22 May 2012, pp. 93-94; Centrais Elétricas Brasileiras S.A. Eletrobras, Form 20-F for the fiscal year ended 31 December 2012, filed 1 May 2013, p. 98; Centrais Elétricas Brasileiras S.A. Eletrobras, Form 20-F for the fiscal year ended 31 December 2013, filed 30 April 2014, p. 98; Centrais Elétricas Brasileiras S.A. Eletrobras, Form 20-F for the fiscal year ended 31 December 2014, filed 11 October 2016, p. 149; Centrais Elétricas Brasileiras S.A. Eletrobras, Form 20-F for the fiscal year ended 31 December 2015, filed 11 October 2016, p. 142.

[10] Centrais Elétricas Brasileiras S.A. Eletrobras, Form 20-F for the fiscal year ended 31 December 2014, filed 11 October 2016, p. 16; and Centrais Elétricas Brasileiras S.A. Eletrobras, Form 20-F for the fiscal year ended 31 December 2015, filed 11 October 2016, p. 15.

[11] Centrais Elétricas Brasileiras S.A. Eletrobras, Form 20-F for the fiscal year ended 31 December 2014, filed 11 October 2016, p. 15; and Centrais Elétricas Brasileiras S.A. Eletrobras, Form 20-F for the fiscal year ended 31 December 2015, filed 11 October 2016, p. 15.

[12] Centrais Elétricas Brasileiras S.A. Eletrobras, Form 20-F for the Fiscal Year Ended 31 December 2012, filed 1 May 2013, p. 54; Centrais Elétricas Brasileiras S.A. Eletrobras, Form 20-F for the fiscal year ended 31 December 2014, filed 11 October 2016, p. 9.

[13] Centrais Elétricas Brasileiras S.A. Eletrobras, Form 20-F for the Fiscal Year Ended 31 December 2011, filed 22 May 2012, p. 12.

Case 1:15-cv-05754-JGK Document 63-1 Filed 10/06/17 Page 17 of 37 PageID #: 719054

when a "concession expires, all assets, rights and privileges that are materially related to the rendering of the electricity services revert to the Brazilian Government."[14]

28. On 11 September 2012, the Brazilian government passed Provisional Measure 579 ("PM 579") with the intent to lower the cost of electric power to consumers "through the reduction of three tariff components: generating cost, transmission cost and sectoral burden."[15] This legislation hit Eletrobras especially hard, more so than other electric utility companies, because it was understood that the Company would have to accept the new unfavorable terms, whereas other companies had a choice as to how they would respond to this new legislation.[16] Under the rules and regulations promulgated by PM 579, the Company explained that the legislation had "materially changed the Brazilian electricity sector," as the concessions would be renewed "at significantly lower tariff levels," thus decreasing Eletrobras' revenue and earnings.[17] The Company recorded, "a significant charge of R$10.09 billion in its 2012 financial statements, based on the impacts of the renewal of its concessions."[18]

29. In 2009, prior to the start of the Class Period, Brazilian federal police began "Operação Lava Jato" ("operation Car Wash" or "Lava Jato") as an investigation in criminal money laundering activities within Brazil.[19] Lava Jato implicated major state-controlled

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[14] "Following the expiration, the concessionaire is entitled to indemnification for its investments in assets that have not been fully amortized or depreciated at the time of expiration." (Centrais Elétricas Brasileiras S.A. Eletrobras, Form 20-F for the Fiscal Year Ended 31 December 2011, filed 22 May 2012, p. 50.)

[15] Centrais Elétricas Brasileiras S.A. Eletrobras, Form 20-F for the Fiscal Year Ended 31 December 2012, filed 1 May 2013, F-16.

[16] For example, Fitch issued a press release on 7 November 2012 titled: "Fitch: Brazil Concession Renewal Offer Negative for Eletrobras, Marginal for Others," which explained that "early renewal of some expiring electricity concessions will have various impacts for different companies … Fitch considers the offer negative for Centrais Eletricas Brasileiras S.A. (Eletrobras, IDR 'BBB') and neutral to negative for other affected companies. … Fitch expects Eletrobras to accept the government's proposal given the government's ownership interest, which will pressure Eletrobras' credit quality given its weak credit metrics for the assigned rating category on a standalone basis. Under this scenario, the absence of direct financial support from the Federal Government could result in a negative rating action."

[17] Centrais Elétricas Brasileiras S.A. Eletrobras, Form 20-F for the Fiscal Year Ended 31 December 2012, filed 1 May 2013, p. 71.

[18] Id., pp. 71-72.

[19] Centrais Elétricas Brasileiras S.A. Eletrobras, Form 20-F for the fiscal year ended 31 December 2014, filed 11 October 2016, p. 1.

companies, such as Eletrobras, in a "broad payment scheme" for construction contracts.[20] By 31 December 2015, Eletrobras had replaced its entire board of directors, its CEO, and its compliance officer, and created an independent compliance department in response to the findings uncovered by federal law enforcement authorities and the allegations raised by these authorities.[21]

B. About the Eletrobras Common and Preferred ADS

30. While Eletrobras' common and preferred shares trade on the São Paulo Stock Exchange and the Madrid Stock Exchange in Spain, the Company's common ADS and preferred ADS are traded on the New York Stock Exchange.[22]

31. An American Depositary Receipt ("ADR") is also referred to as an American Depositary Share ("ADS"). According to the SEC, "an ADR is a negotiable certificate that evidences an ownership interest in ADS which, in turn, represent an interest in the shares of a non-U.S. company that have been deposited with a U.S. bank. It is similar to a stock certificate representing shares of stock. The terms ADR and ADS are often used interchangeably by market participants."[23] This report refers to the Eletrobras shares listed on the New York Stock Exchange ("NYSE") as ADSs.

32. JPMorgan Chase Bank, N.A. ("JPMorgan") serves as the depositary trust for both the common and preferred ADS.[24] As of 31 December 2013, JPMorgan held 65,800,425 common shares and 21,072,812 preferred shares of Eletrobras' underlying common and class B preferred stock as ADS in the U.S. out of a total of 1,087,050,297 and 265,436,883 shares outstanding, respectively.[25] Both the common and preferred ADS each represent

[20] Id.

[21] Id.

[22] Centrais Elétricas Brasileiras S.A. Eletrobras, Form 20-F for the fiscal year ended 31 December 2015, filed 11 October 2016, p. 44.

[23] "Investor Bulletin: American Depositary Receipts," SEC Office of Investor Education and Advocacy, August 2012, p. 1.

[24] Centrais Elétricas Brasileiras S.A. Eletrobras, Form 20-F for the fiscal year ended 31 December 2014, filed 11 October 2016, p. 190.

[25] Id., p. 149.

one common or preferred share of underlying Eletrobras stock, respectively.[26] Holders of the common and preferred ADS receive dividends from the respective ADS.[27] According to Brazilian Corporate Law and the Company's by-laws, Eletrobras was required to pay shareholders (including ADS holders) at least 25.0% of its adjusted net income from the preceding fiscal year.[28]

33. During the Class Period, the Eletrobras common ADS price peaked at $16.24 per share on 8 April 2011, according to price data obtained from CRSP. On 24 June 2015, the last day of the Class Period, the Eletrobras common ADS price had fallen to $1.97, representing a decline of 87.9% from the Class Period peak.

34. During the Class Period, the Eletrobras preferred ADS price peaked at $19.95 on 05 April 2011, according to price data obtained from Bloomberg. On 24 June 2015, the last day of the Class Period, the preferred ADS price fell to $2.83, representing a decline of 85.8% from the Class Period peak.

C. About the Eletrobras Notes

35. The two Eletrobras Notes at issue in this case are USD denominated debt securities with face values of $1.0 billion and $1.75 billion.[29] Both of the Notes are senior unsecured securities, meaning that they are senior to the Eletrobras common ADS and preferred ADS in the corporate capital structure. Bond interest must be paid before common and preferred stock dividends are distributed, and in the event of a bankruptcy, bond principal must be repaid to investors before common and preferred stock holders receive any asset liquidation proceeds.[30]

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[26] Id.

[27] Id., p. 42-43.

[28] Id., p. 158. The Company was required to give class "B" preferred shares "a priority in the distribution of dividends, at 6% each year over the capital linked to those shares. In addition, preferred shares must receive a dividend 10% over the dividend paid to the common shares." (Id.)

[29] Eletrobras Listing Circular, 31 July 2009 and Eletrobras Listing Circular, 27 October 2011.

[30] See, e.g., Institute, CFA. *2016 CFA Level I Volume 5 Equity and Fixed Income* CFA Institute, July 2015, p. 295 & p. 591: "Payment of interest and repayment of principal (amount borrowed) are a prior claim on the company's earnings and assets compared with the claim of common shareholders." In the event of a bankruptcy, "unsecured creditors have a right to be paid in full before holders of equity interests (common and preferred shareholders) receive value on their interests."

36. The corporate bond market is a highly developed and active market. However, the characteristics of corporate bonds (such as the Eletrobras Notes) differ from those of common stock and preferred shares, which cause their trading behavior and price movements to differ from those of common stock and preferred shares. For example, bonds trade with less frequency than common and preferred shares, and they are by design less sensitive to Company news.

> "Corporate bonds will likely trade less frequently than stocks because outside macro-economic and internal financial factors generally both have smaller effects on bond pricing than on stock pricing. Unlike common stocks, corporate bonds have predictable cash flows, predictable terminal values, fixed upside opportunities—namely, redemption at par value or $100 in our example—and priority on the corporation's assets. As such, many corporate bonds are close substitutes for one another. On the other hand, corporate equity does not have predictable cash flows, predictable terminal values, fixed upside opportunities, or priority on the corporate assets."
> **"Fraud on the Market: Analysis of the Efficiency of the Corporate Bond Market," by Michael L. Hartzmark, Cindy A. Schipani, and H. Nejat Seyhun, *Columbia Business Law Review*, Volume 2011, Number 3, (footnote omitted).**

37. Nonetheless, despite their various distinguishing features, published peer-reviewed empirical research has concluded that the markets for corporate bonds with characteristics similar to those of the Eletrobras Notes are efficient markets:

> "[T]he informational efficiency of corporate bond prices is similar to that of the underlying stocks. We find that stocks do not lead bonds in reflecting firm-specific information. We further examine price behavior around earnings news and find that information is quickly incorporated into both bond and stock prices, even at short return horizons. Finally, we find that measures of market quality are no poorer for the bonds in our sample than for the underlying stocks."
> **"The Informational Efficiency of the Corporate Bond Market: An Intraday Analysis," by Edith Hotchkiss and Tavy Ronen, *The Review of Financial Studies*, 2002.**

1. About the AF3 Notes

38. On 30 July 2009, prior to the start of Class Period, Eletrobras issued $1.0 billion in aggregate principal amount of senior unsecured notes due 30 July 2019, designated with the ISIN USP22854AF3 (the "AF3 Notes"). The AF3 Notes pay coupon interest semi-annually on the 30th of January and July at a rate of 6.875% per annum.[31] The AF3 Notes were issued at a price of $99.112 per $100 of par.[32]

39. In the U.S. the AF3 Notes were issued under Rule 144A. In the listing circular, Eletrobras described the AF3 Notes thusly:

> "The notes have not been and will not be registered under the Securities Act and may not be offered or sold within the United States or to, or for the account or benefit of U.S. persons except pursuant to an exemption from, or in a transaction not subject to, the registration requirements of the Securities Act. Accordingly, the notes are being offered hereby only (a) to 'qualified institutional buyers' (as defined in Rule 144A under the Securities Act), or QIBs, in compliance with Rule 144A under the Securities Act and (b) in offers and sales that occur outside the United States to persons other than U.S. persons ('non-U.S. purchasers,' which term shall include dealers or other professional fiduciaries in the United States acting on a discretionary basis for non-U.S. beneficial owners (other than an estate or trust)), in offshore transactions meeting the requirements of Rule 903 of Regulation S."
> **Eletrobras Listing Circular, 31 July 2009, p. 146.**

2. About the AG1 Notes

40. On 27 October 2011, Eletrobras issued $1.75 billion in aggregate principal amount of senior unsecured notes due 27 October 2021, designated with the ISIN USP22854AG1 (the "AG1 Notes"). The AG1 Notes pay coupon interest semi-annually on the 27th of April and October at a rate of 5.750% per annum.[33] The AG1 Notes were issued at a price of $100 per $100 of par.[34]

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[31] Eletrobras Listing Circular, 31 July 2009.

[32] Id., p. 11.

[33] Eletrobras Listing Circular, 27 October 2011.

[34] Id., p. 10.

41. In the U.S. the AG1 Notes were issued under Rule 144A. In the listing circular, Eletrobras described the AG1 Notes thusly:

> "The notes have not been and will not be registered under the Securities Act and may not be offered or sold within the United States or to, or for the account or benefit of U.S. persons except pursuant to an exemption from, or in a transaction not subject to, the registration requirements of the Securities Act. Accordingly, the notes are being offered hereby only (a) to 'qualified institutional buyers' (as defined in Rule 144A under the Securities Act), or QIBs, in compliance with Rule 144A under the Securities Act and (b) in offers and sales that occur outside the United States to persons other than U.S. persons ('non-U.S. purchasers,' which term shall include dealers or other professional fiduciaries in the United States acting on a discretionary basis for non-U.S. beneficial owners (other than an estate or trust)), in offshore transactions meeting the requirements of Rule 903 of Regulation S."
> **Eletrobras Listing Circular, 27 October 2011, p. 144.**

V. <u>EFFICIENT MARKET DEFINED</u>

42. The definition of market efficiency set forth by Judge Alfred J. Lechner, Jr. in the 1989 *Cammer v. Bloom* decision is often cited as a legal authority on the meaning of market efficiency and is consistent with the definition generally accepted by the academic finance community:

> "As relevant here, courts have permitted a rebuttable presumption of reliance in the case of securities traded in 'efficient markets' (i.e., markets which are so active and followed that material information disclosed by a company is expected to be reflected in the stock price)."
> ***Cammer v. Bloom* Opinion, 711 F. Supp. 1264, 1273 (D.N.J. 1989).**

43. Judge Lechner also cited the definitions offered by commentators Alan R. Bromberg and Lewis D. Lowenfels, and by finance professor Eugene Fama:

> "An efficient market is one which rapidly reflects new information in price."
> **Alan Bromberg & Lewis Lowenfels, *Securities Fraud and Commodities Fraud*, §7.4 (Dec. 2003); see also *Cammer*, 711 F. Supp. at 1276.**

> "A market in which prices always 'fully reflect' available information is called 'efficient.'"
> **"Efficient Capital Markets A Review of Theory and Empirical Work," by Eugene Fama, *Journal of Finance*, 1970, cited in *Cammer*, 711 F. Supp. at 1280.**

44. Professor Fama elaborated and refined his definition in a *Halliburton II* amici curiae that he co-authored:

> "But economists do *not* generally disagree about whether market prices respond to new material information. In particular, there is little doubt that the stock price will increase reasonably promptly after favorable news about a company is released and decline after unfavorable news. Our conclusion that prices generally move reasonably promptly in the predicted direction in response to unexpected material public information (favorable or unfavorable) is perfectly consistent with the view that there are sometimes anomalies in the way markets process information and that bubbles can exist."
> **Brief of Financial Economists as Amici Curiae in Support of Respondents, *Halliburton Co. and David Lesar v., Erica P. John Fund, Inc., FKA Archdiocese of Milwaukee Supporting Fund*, Inc. 5 February 2014, p. 3 (emphasis in original).**

45. The Supreme Court in the *Basic v. Levinson* decision focused on the same important characteristic at the heart of these definitions of market efficiency:

> "The fraud on the market theory is based on the hypothesis that, in an open and developed securities market, the price of a company's stock is determined by the available material information regarding the company and its business … ."
> ***Basic v. Levinson,* 485 U.S. 224, 243, 108 S. Ct. 978, 988-89, 99 L. Ed. 2d 194 (1988); see also *Cammer*, 711 F. Supp. at 1276.**

46. The 2013 *Amgen* decision defined market efficiency similarly:

> "The fraud-on-the market premise is that the price of a security traded in an efficient market will reflect all publicly available information about a company ... ."
> ***Amgen Inc. v. Conn. Ret. Plans & Trust Funds*, 133 S. Ct. 1184, 1190, 185 L. Ed. 2d 308 (2013).**

47. In its 2014 *Halliburton II* decision, the Supreme Court addressed the cause and effect relationship at the center of market efficiency thusly

> "Even the foremost critics of the efficient-capital-markets hypothesis acknowledge that public information generally affects stock prices. … Debates about the precise *degree* to which stock prices accurately reflect public information are thus largely beside the point. 'That the . . . price [of a stock] may be inaccurate does not detract from the fact that false statements affect it, and cause loss,' which is 'all that *Basic* requires.'"
> ***Halliburton Co. v. Erica P. John Fund, Inc.*, 134 S. Ct. 2398, 2410, 189 L. Ed. 339 (2014) (emphasis in original).**

48. An efficient market, as defined by *Cammer, Basic, Amgen,* Bromberg and Lowenfels, and Fama, is a market in which available information is rapidly incorporated into the price of a security such that the trading price reflects all available information. As these cases and academic work recognized, market efficiency is relevant to a securities case as it addresses the question of whether false information (e.g., in the form of an alleged misrepresentation or omission) would likely have impacted the prices at which investors bought and sold securities, and which were relied upon.

A. The *Cammer* Factors

49. The *Cammer* opinion lays out five factors that generally indicate the market for a security is efficient. As described below, economic rationales support each factor as an indicator of market efficiency. The five factors are: 1) trading volume, 2) coverage by securities analysts, 3) number of market makers, 4) eligibility for S-3 registration, and 5) empirical evidence that the security price reacts to new, company-specific information.

50. Empirical research has confirmed that volume, number of market makers, and analyst coverage are indicative of market efficiency:

> "Consistent with the efficiency indicators used recently by the courts, the inefficient firms have lower mean trading volume, fewer market makers, lower analyst following, and lower institutional ownership (number and percentage) than efficient firms."
> **"The Fraud-on-the-Market Theory and the Indicators of Common Stocks' Efficiency," by Brad M. Barber, et al., *Journal of Corporation Law*, 1994, p. 302.**

51. Barber, et al., also found that high institutional ownership was indicative of market efficiency.

52. With respect to the empirical factor, Barber, et al., used empirical tests as the standard for market efficiency by which to judge the significance of the other variables. Consequently, they acknowledge the importance of the empirical factor.

53. Consistent with financial economic theory and empirical research, the language used by the *Cammer* Court describes the factors not as five ***necessary*** factors, but rather as indicative of the degree to which the market for a security is expected to be efficient:

> "There are several different characteristics pertaining to the markets for individual stocks which are probative of the degree to which the purchase price of a stock should reflect material company disclosures."
> ***Cammer*, 711 F. Supp. at 1283.**

54. The *Cammer* opinion describes the nature of the five factors as follows:

> "There are several types of facts which, if alleged, might give rise to an inference that Coated Sales traded in an efficient market. It is useful to set forth an explanation of how the existence of such facts would cause the understanding that disclosed company information (or misinformation) would be reflected in the company's stock price, the underpinning of the fraud on the market theory. *Peil, supra, 806 F.2d at 1160.*"
> **Id. at 1285-86 (footnote omitted).**

> "First, plaintiffs could have alleged there existed an average weekly trading volume during the class period in excess of a certain number of shares."
> **Id. at 1286.**

> "Second, it would be persuasive to allege a significant number of securities analysts followed and reported on a company's stock during the class period."
> **Id.**

> "Third, it could be alleged the stock had numerous market makers."
> **Id.**

> "Fourth, as discussed, it would be helpful to allege the company was entitled to file an S-3 Registration in connection with public offerings…"
> **Id. at 1287.**

Case: 1:15-cv-01046-SO Doc #: 563-2 Filed: 10/03/24 ... PageID #: 19063

> "Finally, it would be helpful to a plaintiff seeking to allege an efficient market to allege empirical facts showing a cause and effect relationship between unexpected corporate events or financial releases and an immediate response in the stock price."
> **Id.**

> "As previously noted, one of the most convincing ways to demonstrate efficiency would be to illustrate, over time, a cause and effect relationship between company disclosures and resulting movements in stock price."
> **Id. at 1291.**

B. The *Unger/Krogman* Factors

55. In addition to the five *Cammer* factors that indicate market efficiency, the Fifth Circuit Court of Appeals in *Unger v. Amedisys,* 401 F.3d 316 (5th Cir. 2005), and the district court in *Krogman v. Sterritt,* 202 F.R.D. 467 (N.D.Tex. 2001), concluded that three additional factors were also indicative of market efficiency.

56. These additional factors, the *Unger/Krogman* factors, are: 1) the company's market capitalization, 2) the stock's float, and 3) the typical bid-ask spread.

57. Market capitalization is the total value of all outstanding shares. It equals the number of shares outstanding times the price per share. Reasonably, the larger a firm's market capitalization, the more prominent and well known the company will be. Larger companies tend to attract more analyst and news media coverage, and gain the attention of greater numbers of investors, including large institutional investors. All of these characteristics, which accompany a large market capitalization, promote market efficiency.

58. The stock's float is the number of shares outstanding, less shares held by insiders and affiliated corporate entities. It is generally the number of shares available for trading by outside investors in the open market. Float is highly correlated with market capitalization, but it focuses on the shares available for trading rather than all outstanding shares. Stocks with large floats tend to trade more actively, attract more analyst and news media coverage, and garner the attention of greater numbers of investors, including large institutional investors. All these characteristics, that accompany a high float level, promote market efficiency.

59. The bid-ask spread is the difference between the price at which market makers are offering to buy a security and the price at which they are offering the security for sale. If a security is actively traded and information about the security is readily available, the bid-ask spread will tend to be narrow. Moreover, a narrow bid-ask spread makes trading in the security less costly for investors, and thereby tends to attract greater interest, greater coverage, and greater volume, which in turn are factors that are generally considered to promote market efficiency.

VI. <u>EFFICIENCY OF THE MARKET FOR THE ELETROBRAS COMMON ADS</u>

60. To assess whether the market for the Eletrobras common ADS was efficient during the Class Period, I analyzed the market for and behavior of the Eletrobras common ADS, focusing on the factors that are generally accepted to be indicative of market efficiency for a publicly-traded security.

A. *Cammer* Factors

1. <u>Trading Volume</u>

61. Throughout the Class Period, Eletrobras common ADS traded regularly and actively. On average, 1.05 million common ADS changed hands daily.[35] Eletrobras' common ADS trading data are presented in Exhibit-4a.

62. In addition to average daily trading volume, another volume metric to consider in determining market efficiency is the percentage of outstanding ADS that turn over each week. During the Class Period, the average weekly trading volume of Eletrobras common ADS was approximately 5.2 million shares, or 7.43% of all common ADS outstanding.[36] This level of trading activity is well above levels accepted by courts as being indicative of market efficiency for common stock.[37] In the case of the common stock of Coated Sales,

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[35] Data obtained from CRSP.

[36] Estimated by averaging the ratio of the daily trading volume to the number of common ADS outstanding, and multiplying by 5 (the number of trading days in a typical week).

[37] *Cammer*, 711 F. Supp. at 1286.

Inc., the *Cammer* Court cited the conclusion of Alan R. Bromberg and Lewis D. Lowenfels that "weekly trading of 2% or more of the outstanding shares would justify a strong presumption that the market for the security is an efficient one; 1% would justify a substantial presumption."[38] The trading volume for Eletrobras common ADS during the Class Period was above the threshold for a strong presumption of market efficiency.

63. Both in terms of average daily trading volume and percentage of outstanding ADS traded weekly, the market for Eletrobras common ADS was active. Consistent with the *Cammer* opinion, economic theory, and empirical research, the active trading volume in Eletrobras common ADS is strong evidence of the efficiency of the market for Eletrobras common ADS over the course of the Class Period.

2. <u>Analyst Coverage and Other Avenues of Information Dissemination</u>

a) Analyst Coverage

64. Securities analysts disseminate and interpret information about the companies they cover. They conduct research and provide valuation opinions, helping market participants acquire relevant information and understand the implications of that information for valuation and investment decisions. Consequently, securities analysts facilitate the flow of information and the digestion of information within the marketplace. These functions promote market efficiency.

65. From Capital IQ, Thomson Eikon, and counsel, I obtained analyst reports on Eletrobras published during the Class Period by 13 firms: Barclays, Bradesco, BTG Pactual, Credit Suisse, Deutsche Bank, Factor Corretora, HSBC, Itau, J. Safra, JPMorgan, Morgan Stanley, Santander, and UBS.[39]

66. Transcripts of Eletrobras' conference calls conducted during the Class Period reveal that at least five additional firms also followed Eletrobras: 3G Radar, Banco de Brasil, Bank of America Merrill Lynch, BNP Paribas, and Citigroup.

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[38] Id., at 1293.

[39] The complete list of analyst reports I obtained is presented in Exhibit-1.

67. Consequently, analysts affiliated with at least 18 firms covered Eletrobras during the Class Period.

68. Barber, et al., [1994] found that coverage by one or two analysts strengthened the presumption of efficiency for a publicly traded stock.[40] Therefore, consistent with the *Cammer* opinion, financial economic principles, and published research, the coverage of Eletrobras by professional securities analysts is evidence of the efficiency of the market for Eletrobras common ADS during the Class Period.

b) Institutional Ownership and Buy-Side Analysis

69. Thomson Eikon compiles and provides institutional ownership data derived from SEC Form 13-F filings. The data show the holdings of Eletrobras common ADS by major investment institutions as of the end of each quarter. Major institutions are defined as firms or individuals that exercise investment discretion over the assets of others in excess of $100 million. Large investment firms often employ financial analysts who conduct their own research on the securities they buy. According to the SEC filings compiled and reported by Thomson Eikon data, at least 335 major institutions owned Eletrobras common ADS during the Class Period.[41] This fact further supports a finding that the market for the Eletrobras common ADS was efficient during the Class Period.

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[40] Brad M. Barber et al., "The Fraud-on-the-Market Theory and the Indicators of Common Stocks' Efficiency," *Journal of Corporation Law*, 1994.

[41] According to the SEC filings compiled and reported by Thomson Eikon, 335 unique institutions held Eletrobras common ADS on at least one of the following reporting dates: 30 September 2010, 31 December 2010, 31 March 2011, 30 June 2011, 30 September 2011, 31 December 2011, 31 March 2012, 30 June 2012, 30 September 2012, 31 December 2012, 31 March 2013, 30 June 2013, 30 September 2013, 31 December 2013, 31 March 2014, 30 June 2014, 30 September 2014, 31 December 2014, and 31 March 2015. There may have been additional institutions that held Eletrobras common ADS during the Class Period, though not on the quarterly reporting dates.

c) News Coverage

70. The news media also facilitate the flow of information to the marketplace, thereby promoting market efficiency. In the case of Eletrobras, such coverage was extensive. A Factiva database search established that over 3,700 articles were published, in English, about the Company during the Class Period.[42]

71. The articles obtained from Factiva include published news articles and press releases. Information about Eletrobras also emerged throughout the Class Period in the form of SEC filings, conference calls, and presentations.

72. Therefore, during the Class Period, information about Eletrobras was readily available to market participants, provided by news media, analysts, and various other sources. This news coverage is further evidence of the efficiency of the market for Eletrobras common ADS.[43]

3. Market Makers and Listing on the New York Stock Exchange

73. The number of market makers is one of the factors the *Cammer* Court determined indicates market efficiency. Market makers are financial intermediaries who trade in a particular security, standing ready to buy and sell with individual investors, institutions, and other market makers. A large number of market makers implies that many market participants are trading that particular security, which generally results in a high degree of liquidity and a narrower bid-ask spread. With a large number of market makers, it is generally easy for investors to execute trades in a timely fashion and with reasonable transaction costs.

74. The subject company in the *Cammer* case, Coated Sales, Inc., was listed on the NASDAQ, an over-the-counter market consisting of multiple competing market makers, using electronic systems to make quotes and effect trades.

[42] Based on a Factiva search for articles published during the Class Period where "Centrais Elétricas Brasileiras SA" was the "Company" search field parameter, the language was "English," and the sources were "All."

[43] This factor was deemed particularly important in *Cheney v. CyberGuard Corp.*, 213 F.R.D. 484 (S.D. Fla. 2003).

75. The *Cammer* Court's understanding that the market-making infrastructure of a stock market is indicative of its efficiency, or lack thereof, makes the fact that Eletrobras common ADS traded on the NYSE during the Class Period highly relevant. The NYSE is one of the most renowned, most liquid, and most efficient forums for trading stocks in the world. Securities on the NYSE are traded under the supervision of a lead market maker or "Designated Market Maker" ("DMM"), formerly known as a specialist.[44] DMMs are responsible for maintaining a fair and orderly market for each security to which they are assigned.[45]

76. In fact, citing Bromberg and Lowenfels, the *Cammer* Court explicitly acknowledged the importance of an NYSE listing and the implications of such a listing for market efficiency.

> "We think that, at a minimum, there should be a presumption – probably conditional for class determination – that certain markets are developed and efficient for virtually all the securities traded there: the New York and American Stock Exchanges, the Chicago Board Options Exchange and the NASDAQ National Market System."
> ***Cammer*, 711 F. Supp. at 1292 (quoting Bromberg & Lowenfels, *Securities Fraud and Commodities Fraud*, §8.6 (1988)).**

77. At the time of the *Cammer* opinion the NYSE and NASDAQ were distinctly separate exchanges. NASDAQ market makers did not make markets for NYSE-listed securities. However, since that time, the stock markets have evolved, and beginning in April 2005, NASDAQ enabled trading in most NYSE-listed securities through its market making platform.[46] This NASDAQ market making activity is in addition to the principal market for listed securities on the NYSE.

78. According to Bloomberg, there were at least 183 market makers for Eletrobras common ADS during the Class Period, including such well known firms such as Morgan Stanley, Merrill Lynch, JPMorgan, and Goldman Sachs.[47]

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[44] "Fact Sheet; Designated Market Makers," NYSE Euronext, 2012.

[45] "Organization and Functioning of Securities Markets," by Frank Reilly and Keith Brown, in *Equity and Fixed Income CFA Program Curriculum*, vol. 5, Pearson Custom Publishing, 2008.

[46] "Nasdaq to Enable Customers to Trade NYSE Stocks," *Reuters*, March 28, 2005.

[47] Market maker data obtained from Bloomberg.

79. The facts that it traded on the NYSE and had a large number of market makers are strong evidence that the Eletrobras common ADS traded in an efficient market throughout the Class Period. Eletrobras' listing on the NYSE gave its ADS access to a highly developed network of brokers with its market overseen by the NYSE DMM. These facts are compelling evidence of the efficiency of the market for Eletrobras common ADS.

4. F-3 Registration Eligibility

80. F-3 registration is to foreign companies listed on U.S. exchanges what S-3 registration is to domestic companies. A U.S. company is eligible for S-3 registration, and a foreign company is eligible for F-3 registration, when, among other things, it has filed Exchange Act reports for a specified length of time and has outstanding float above a certain sizable value. At the time of the *Cammer* opinion, the conditions for S-3 registration were that a company had filed financial reports with the SEC for 36 months, and had outstanding float over $150 million held by non-affiliates, or $100 million of such float coupled with annual trading volume exceeding 3 million shares. Eligibility for Form F-3 registration is the same as for S-3, except that the company must be a foreign private issuer, i.e. not a U.S. company.[48]

81. In 1992, the SEC amended its requirements for S-3 registration eligibility to 12 months of filings and at least $75 million of float. Since 2007, the SEC has allowed domestic companies with less than $75 million of float to file an S-3 registration so long as the company has been filing financial reports for at least a year, has "a class of common equity securities listed and registered on a national securities exchange, and the issuers do not sell more than the equivalent of one-third of their public float in primary offerings over any period of 12 calendar months."[49] Despite the fact that the $75 million requirement has been loosened, courts continue to focus on this $75 million benchmark when analyzing this *Cammer* factor.[50]

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[48] "Eligibility of Smaller Companies to Use Form S-3 or F-3 for Primary Securities Offerings," SEC website, accessed at www.sec.gov/info/smallbus/secg/s3f3-secg htm.

[49] "Revisions to The Eligibility Requirements for Primary Securities Offerings On Forms S-3 And F-3," SEC Release No. 33-8878, 19 December 2007.

[50] See, e.g., *Vinh Nguyen v. Radient Pharm. Corp.*, 287 F.R.D. 563, 573 (C.D. Cal. 2012).

Case 1:15-cv-05754-JGK Document 63 Filed 10/24/16 Page 28 of 57 PageID #: 19070

82. The *Cammer* Court noted that S-3 registration eligibility is indicative of market efficiency because the filing requirement ensured that financial data were available to market participants, and the "public float" requirement indicated that many market participants would have examined the information.[51]

> "Proposed Form S-3 recognizes the applicability of the efficient market theory to the registration statement framework with respect to those registrants which usually provide high quality corporate reports, including Exchange Act reports, and whose corporate information is broadly disseminated, because such companies are widely followed by professional analysts and investors in the market place. Because of the foregoing observations made by the SEC, the existence of Form S-3 status is an important factor weighing in favor of a finding that a market is efficient."
> ***Cammer*, 711 F. Supp. at 1284-85.**

> "The 'public float' aspect of the Form S-3 requirements ensures that enough investors have in fact read the previously filed document."
> **Id. at 1285.**

> "Again, it is the number of shares traded and value of shares outstanding that involve the facts which imply efficiency."
> **Id. at 1287.**

a) Float

83. A company's float is the number or value of its shares that can potentially trade freely in the marketplace. It is generally defined as the number or value of outstanding shares, minus insider holdings and shares owned by affiliated corporate entities.[52]

84. Eletrobras is the largest public utility in Latin America. It is not a small or obscure company with thinly traded securities. On the contrary, its market capitalization and float placed it among the tier of largest companies in the world. Even restricting the quantification of float to just the common ADS, the value of freely trading ADS by itself was far above the threshold for F-3 registration. I computed Eletrobras' common ADS

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[51] *Cammer*, 711 F. Supp. at 1284-85.

[52] For a discussion of the generally accepted definitions of shares outstanding and float, see "Float Adjustment Methodology," *S&P Dow Jones Indices*, July 2012.

float using data on ADS outstanding and insider holdings presented in the Company's 20-F filings and ADS price data obtained from CRSP.

85. The average float of just the Eletrobras common ADS, not including the shares that traded in Brazil and Spain, was $454.7 million during the Class Period, far exceeding the level required for F-3 registration. During the Class Period, float ranged between $118.0 million and $1.17 billion, always exceeding the minimum requirement for F-3 registration eligibility.

b) Financial Filings

86. Eletrobras regularly reported financial results and filed financial reports with the SEC throughout the Class Period. The financial information in the SEC filings, supplemented by information provided by analysts, news coverage, Brazilian financial filings, and the Company itself provided investors with access to financial information about the Company on a continuous basis.

87. At the start of the Class Period, the Company was ineligible for F-3 registration on account of its two delayed 20-F filings for the fiscal years 2009 and 2010.[53] Similarly, in the latter part of the Class Period, Eletrobras delayed its Form 20-F filing for the fiscal year 2014.[54]

88. Despite the delayed SEC 20-F filings, the Company did regularly report financial results during these periods to analysts and investors. Nonetheless, the Company was eligible for F-3 registration for more than half the Class Period.

89. To the extent that F-3 registration eligibility indicates company characteristics associated with market efficiency, the Company clearly possessed those particular characteristics throughout the Class Period.

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[53] Centrais Electricas Brasileiras S.A. Eletrobras, Form 12b-25, filed 1 July 2010; and Centrais Electricas Brasileiras S.A. Eletrobras, Form 12b-25, filed 30 June 2011.

[54] Centrais Electricas Brasileiras S.A. Eletrobras, Form 12b-25, filed 30 April 2015.

B. *Unger/Krogman* Factors

90. In addition to evaluating market efficiency using the *Cammer* factors, I also examined Eletrobras common ADS and its market with respect to the three additional *Unger/Krogman* factors.

1. Market Capitalization

91. Over the course of the Class Period, the publicly issued common equity of the Company ranged between $1.7 billion and $17.6 billion, placing Eletrobras among the tier of largest companies in the world.

92. Focusing only on the common ADS, the portion of the common equity listed and traded in the United States was also sizeable. During the Class Period, the market capitalization of Eletrobras common ADS alone averaged $454.7 million, putting Eletrobras' common ADS in the 5th decile of U.S. companies by size – meaning that the value of the Eletrobras common ADS alone was larger than at least 50% of all publicly-traded companies in the U.S.[55]

93. Consistent with the *Krogman* Court's opinion, the sizeable market capitalization of Eletrobras and its common ADS is evidence of efficiency of the market for the common ADS.

2. Float

94. Eletrobras' common ADS float averaged $454.7 million during the Class Period. While float excludes shares held by insiders and affiliated corporate entities, according to the Company's SEC filings, none of the Eletrobras common ADS were held by insiders or affiliated corporate entities.[56] The size of Eletrobras common ADS float indicated it satisfied the second *Unger/Krogman* factor for market efficiency.

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[55] Using averaged month-end data from CRSP for August 2010 to May 2015. I grouped public companies into deciles, so that the 1st decile contains the largest 10% of all public companies listed on the NYSE, AMEX, NASDAQ, and ARCA while the 10th decile contains the smallest 10%.

[56] Eletrobras 20-F filings for the Fiscal Years 2010 – 2015.

95. Float can also be analyzed as a percentage of total common ADS outstanding, as well as in share and value terms. On average during the Class Period, there were 70.5 million common ADS in Eletrobras' float and the same number of common ADS outstanding, resulting in a float of 100% of common ADS outstanding.

96. The magnitude of the Eletrobras' common ADS float indicates efficiency.

3. Bid-Ask Spread

97. I obtained data on daily closing bid and ask quotes for Eletrobras common ADS during the Class Period from CRSP.

98. I measured the percentage bid-ask spread as the difference between the bid and ask quotes, divided by the average of the bid and ask quotes, which is the standard way of measuring percentage bid-ask spreads in the finance literature.[57] Exhibit-4a presents Eletrobras' common ADS bid-ask spread data.

99. The average bid-ask spread for Eletrobras common ADS over the course of the Class Period was 0.26%. By comparison, the average month-end bid-ask spread over the course of the Class Period for all stocks in the CRSP database was 0.65%.[58] Eletrobras' common ADS average bid-ask spread was substantially narrower than the mean level among all other CRSP stocks – which are the stocks traded on the NYSE, AMEX, NASDAQ, and ARCA.

100. In dollar terms, Eletrobras' common ADS bid-ask spread during the Class Period averaged $0.01 per common ADS. For all stocks in the CRSP database, the average bid-ask spread during the Class Period was $0.10 per share.[59]

101. The average bid-ask spread in the market for Eletrobras common ADS over the course of the Class Period was well below the typical bid-ask spreads exhibited by other publicly-traded stocks in the United States. Eletrobras' narrow bid-ask spread strongly supports a conclusion of market efficiency.

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[57] "Price Reversals, Bid-Ask Spreads, and Market Efficiency," by Allen B. Atkins and Edward A. Dyl, *Journal of Financial and Quantitative Analysis*, Vol. 25, No. 4, 1990, pp. 535-547.

[58] This calculation is based upon averaged month-end data from CRSP for August 2010 through May 2015.

[59] Id.

VII. EMPIRICAL EVIDENCE OF MARKET EFFICIENCY FOR THE ELETROBRAS COMMON ADS

102. Of the five *Cammer* factors, the empirical factor was cited by the *Cammer* Court as "one of the most convincing ways to demonstrate efficiency:"

> "As previously noted, one of the most convincing ways to demonstrate efficiency would be to illustrate over time, a cause and effect relationship between company disclosures and resulting movements in stock price."
> ***Cammer*, 711 F. Supp. at 1291.**

103. The special importance the *Cammer* Court placed on the empirical factor is justified by economic principles, as the empirical factor focuses on the essence of market efficiency whereas the other four factors are indicators that generally signal market efficiency.

104. I conducted two sets of empirical tests of the efficiency of the market for Eletrobras common ADS.

105. The first empirical test was an event study that investigates whether Eletrobras common ADS reacted to major news events during the Class Period. The events tested were announcement of: 1) delayed financial filings due to the developments in Lava Jato; and 2) major developments affecting Eletrobras' concession terms pursuant to Provisional Measure 579 ("PM 579"). Consistent with financial valuation principles and the academic literature, the events in these categories would generally have a substantial impact on a company's stock price. Significant stock price reactions to these events, therefore would indicate market efficiency.

106. The second empirical analysis was a set of tests that collectively examine a broad set of events that occurred over the course of the Class Period. The events tested during this period were all earnings announcements. These tests address whether Eletrobras common ADS exhibited market efficiency by examining whether the common ADS responded to the increased flow of information that generally transpires on earnings announcement dates.

107. According to finance literature, the flow of company-specific information is elevated on earnings announcement dates. More movement on days with more information would indicate a cause and effect relationship between information and movements in the Eletrobras ADS price, which is the essence of informational market efficiency. I

conducted this collective event analysis using a Z-test, an F-test, and an Ansari-Bradley volatility test. These statistical tests compare Eletrobras common ADS price dynamics on earnings dates to the behavior of the ADS price on all other dates in the Class Period. A pattern of greater dispersion and larger stock price movements on earnings announcement dates indicates market efficiency.

A. Event Study Test of Market Efficiency

108. The event study is the paramount tool for testing market efficiency, as renowned financial economist and Nobel laureate Eugene Fama attests:

> "The cleanest evidence on market-efficiency comes from event studies, especially event studies on daily returns. When an information event can be dated precisely and the event has a large effect on prices, the way one abstracts from expected returns to measure abnormal daily returns is a second-order consideration. As a result, event studies give a clear picture of the speed of adjustment of prices to information."
> **"Efficient Capital Markets II," by Eugene F. Fama, *Journal of Finance*, 1991, p. 1607.**

109. Event study analysis is one of the most commonly used analytic methodologies employed by finance researchers. Campbell, Lo, and MacKinlay [1997] present an excellent description and examples of the methodology and write about how it is generally accepted and widely used in academic research.[60] Crew, et al., [2012] write about how the methodology is generally accepted and widely used in forensic applications.[61]

110. An event study measures how much a security price rises or falls in response to new, company-specific information. One component of an event study is statistical regression analysis that determines how much of a security price change is explained by market and sector factors, rather than company-specific information, so that those influences can be statistically factored out. The portion of a security price change that cannot be attributable to market or sector factors is called the residual security price movement or "residual

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[60] Chapter 4 of *The Econometrics of Financial Markets*, by John Y. Campbell, Andrew W. Lo, and A. Craig MacKinlay, Princeton University Press, 1997.

[61] "Federal Securities Acts and Areas of Expert Analysis," by Nicholas I. Crew, et al., in Chapter 24 of the *Litigation Services Handbook*: *The Role of the Financial Expert*, 5th ed., edited by Roman L. Weil, Daniel G. Lentz, and David P. Hoffman, John Wiley & Sons, Inc., 2012.

return." The event study isolates the residual return and also tests whether the residual return can reasonably be explained as merely a random fluctuation.

111. If a security's return over an event period is statistically significant, it indicates that the security price movement cannot be attributed to market factors, sector factors, or to random volatility, but rather was caused by new, company-specific information. Such proof of a cause and effect relationship between the release of information and movement in the security price establishes market efficiency.

1. A Caveat about Non-Significant Security Price Movements

112. It is important to note that an event study tests the joint hypothesis that the security trades in an efficient market and that the valuation impact of the information disseminated on an event date is of such a large magnitude as to exceed the threshold for statistical significance. A finding of statistical significance indicates market efficiency, but a finding of non-significance does not necessarily establish inefficiency because a modest non-significant stock price reaction may be the appropriate and efficient stock price reaction to a particular disclosure.[62]

113. For example, if a company reports earnings that are in line with the expectations of analysts and investors, even though the announcement would be important, the mix of information may not have changed sufficiently on that date to elicit a statistically significant security price reaction. Similarly, if a misrepresentation is made alongside countervailing confounding news that impacts the stock price in the opposite direction, one might not reasonably expect this mix of new information to cause a statistically significant stock price reaction. In these examples, a modest stock price movement, or even no movement at all, may be the appropriate stock price reaction. In such cases, the event study finding that the stock return was non-significant would not indicate inefficiency. In fact, in such cases, a non-significant stock price movement would indicate that the stock is behaving as it should in an efficient market.

[62] "Event Studies in Securities Litigation: Low Power, Confounding Effects, And Bias," by Alon Brav and J.B. Heaton, *Washington University Law Review*, 30 March 2015, p. 602.

114. Similarly, when a company deceives analysts and investors by concealing important information, the effect of the concealment would generally not be a significant stock price movement at the time of the concealment and over its duration. The concealment would maintain the mix of information as it previously was, so the appropriate price reaction would be a maintenance of the price level where it previously was.

115. Therefore, ideal candidate events for inclusion in a market efficiency event study are events on which company-specific information was released that is new, unexpected, not confounded by major countervailing news, and may potentially be of such import as to reasonably be expected to elicit a stock price reaction over the threshold for statistical significance. When there are few ideal candidate events in the period being tested, one may run a collective event study, which is a methodology that examines a broader collection of news events collectively. By comparing a broad collection of news days to typical non- or lesser-news days, the methodology can assess whether information flow affects the security price, thereby demonstrating market efficiency.

2. Event Selection

a) Financial Filing Delays Due to the Lava Jato Investigation

116. The academic literature recognizes that a company's ability to timely file its financial statements is an important consideration for investors assessing the value of its securities.

> "Timely disclosures are important to the securities market … Given that timeliness is an important characteristic of relevant information and is one of the determinants of security prices, we expect the market to respond negatively to NT filings."
> **"Analyzing Late SEC Filings for Differential Impacts of IS and Accounting Issues," by Jian Cao, et al., *International Journal of Accounting Information Systems*, Vol. 11, 2010, pp. 189-207 (internal citations omitted).**

117. In particular, researchers have noted that the specific reason provided by a company for delay in its financial filings is an important determinant of the magnitude of the expected market reaction. For example, Cao, et al. [2010] finds:

> "[T]hat the reasons stated for delayed filings on Form 12b-25 explain differences in market response. More negative market returns are associated with filing delays caused by IS [information systems] issues, SOX implementations, and SEC investigations. Accounting and administrative reasons are less significant. These results suggest that late filers bear different costs of noncompliance with SEC filing requirements depending on whether the underlying reasons are systems-related or accounting-related."
> **Id. (internal citations omitted).**

118. Based on these principles, for the purposes of a market efficiency event study, I selected event dates during the Class Period on which Eletrobras announced that it would be unable to timely file its financial statements due to issues stemming from the Lava Jato investigation. Given the alleged scale, details, and implications of Lava Jato, delays in the Company's financial filings attributed to Lava Jato would be of high importance to Eletrobras investors.

119. There were two such events during the Class Period: 30 April 2015 and 14 May 2015. The announcements on these dates include the following information:

i) **30 April 2015** – on this day, Eletrobras filed a Form 12b-25 notifying the SEC that it was unable to file its financial statements for the fiscal year ended 31 December 2014.[63] According to the Company, it "recently became aware of press reports stating that the former CEO of Camargo Corrêa allegedly stated in his testimony in relation to *Operação Lava-Jato* (Operation Car Wash) that the consortium of companies bidding for the mechanical assembly of the Angra 3 power plant allegedly made illegal payments to the CEO of our wholly owned subsidiary, Eletrobras Thermonuclear S.A. Eletronuclear ('Eletronuclear'). The CEO of Eletronuclear has requested leave of absence. The internal committee established by the Company to investigate any allegations made in the press in relation to *Operação Lava-Jato* (Operation Car Wash) has not yet concluded its internal investigation and the Company's Board of Directors, to which such

[63] Centrais Elétricas Brasileiras S.A. Eletrobras, Form 12b-25, filed 30 April 2015.

internal committee reports, has provided authorization for the Company to take all necessary measures to engage a specialized independent firm to conduct an external investigation into these allegations. Accordingly, the Company is unable to file its Form 20-F as of and for the year ended December 31, 2014 by April 30, 2015."[64] In accordance with the SEC's rules, the Company had until 15 May 2015, or fifteen days, to file its Form 20-F.[65]

ii) **14 May 2015** – on this day, Eletrobras issued a press release notifying shareholders that it was unable to file its Form 20-F by 15 May 2015.[66] The Company noted that "[a]t this time, we are unable to provide a specific date by which we will file our 2014 20-F."[67] The Company attributed this second and indefinite delay to the fact that it was still "in the process of engaging a specialized independent firm to conduct an external investigation into these allegations."[68]

[64] Id.

[65] SEC's rules regarding notifications of late filings was obtained from: https://www.law.cornell.edu/cfr/text/17/240.12b-25, which cites: "With respect to any report or portion of any report described in paragraph (a) of this section which is not timely filed because the registrant is unable to do so without unreasonable effort or expense, such report shall be deemed to be filed on the prescribed due date for such report if… [t]he subject annual report, semi-annual report or transition report on Form 10-K, 20-F, 11-K, N-SAR, or N-CSR, or portion thereof, will be filed no later than the fifteenth calendar day following the prescribed due date; or the subject quarterly report or transition report on Form 10-Q or distribution report on Form 10-D, or portion thereof, will be filed no later than the fifth calendar day following the prescribed due date."

[66] Centrais Elétricas Brasileiras S.A. Eletrobras, Form 6-K, filed 15 May 2015.

[67] Id.

[68] Id.

Case 1:15-cv-05035-... Document ... Filed 10/... Page ... of 37... PageID #: ...19080

b) Selection of PM 579 Related Events

120. To operate in Brazil as an electric utility company, a company must apply for, and be granted, operating concessions from the Brazilian government.[69] As of 31 December 2011, approximately 31% of Eletrobras' existing operating concessions for electric power generation were due to expire by the end of 2015.[70] According to the Company, when a "concession expires, all assets, rights and privileges that are materially related to the rendering of the electricity services revert to the Brazilian Government."[71]

121. On 11 September 2012, the Brazilian government passed PM 579 with an aim to lower the cost of electric power to consumers "through the reduction of three tariff components: generating cost, transmission cost and sectoral burden."[72] The Company explained that the new rules and regulations had "materially changed the Brazilian electricity sector," as concessions would be renewed "at significantly lower tariff levels," thus decreasing Eletrobras' revenue and earnings.[73] The Company recorded, "a significant charge of R$10.09 billion in its 2012 financial statements, based on the impacts of the renewal of its concessions."[74] The Company further explained that:

> "its revenues from concessions that have been renewed under the new law will be significantly lower, and the Company may incur in net losses in the 2013 fiscal year and in subsequent periods. … Eletrobras was party to a number of contracts for the sale of electricity in connection with concessions that were scheduled to expire in the years 2015 through 2017. With the implementation of Law No. 12,783, many of these contracts have

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[69] Centrais Elétricas Brasileiras S.A. Eletrobras, Form 20-F for the Fiscal Year Ended 31 December 2012, filed 1 May 2013, p. 54; Centrais Elétricas Brasileiras S.A. Eletrobras, Form 20-F for the fiscal year ended 31 December 2014, filed 11 October 2016, p. 9.

[70] Centrais Elétricas Brasileiras S.A. Eletrobras, Form 20-F for the Fiscal Year Ended 31 December 2011, filed 22 May 2012, p. 12.

[71] "Following the expiration, the concessionaire is entitled to indemnification for its investments in assets that have not been fully amortized or depreciated at the time of expiration." (Id., p. 50.)

[72] Centrais Elétricas Brasileiras S.A. Eletrobras, Form 20-F for the Fiscal Year Ended 31 December 2012, filed 1 May 2013, F-16.

[73] Id., p. 71.

[74] Id.

become 'onerous' to Eletrobras, as they were based on old tariff levels. At the prior tariff levels, these contracts were expected to be profitable, but based on the new reduced tariff levels, the contracts will result in further losses for Eletrobras."
Centrais Elétricas Brasileiras S.A., Form 20-F for the Fiscal Year Ended 31 December 2012, filed 1 May 2013, pp. 71.

122. Given the breadth and magnitude of the regulatory changes to the Brazilian electrical utilities industry and the Company's expectation of the significant adverse effects of those changes on its financial performance, Eletrobras investors would have considered new developments stemming from PM 579 to be important.

123. I identified five dates during the period 11 September 2012 to 3 December 2012, on which new, important valuation-relevant information was released by either the Brazilian government or the Company regarding developments related to PM 579. The events and information were as follows.

i) **12 September 2012** – on 11 September 2012, after the close of trading, the Brazilian government promulgated PM 579.[75] The news media considered this a decidedly negative development for the electric utilities industry in Brazil.[76]

ii) **2 November 2012** – on 1 November 2012, after the close of trading, the Brazilian government announced that it would offer up to R$20 billion of reimbursements for the unamortized assets of the electric utilities companies that choose to renew their concessions under the new rules of PM 579. Eletrobras was allocated R$14 billion.[77]

[75] Provisional Act No. 579, The Presidency of the Republic, dated 11 September 2012.

[76] See, e.g., "Brazil electric company stocks tumbled Wednesday after the government demanded that utilities charge lower prices starting next year, and as it signaled Brazil wouldn't pay out a very high reimbursement value for past investments" ("Brazil Utilities Tumble on Price Cuts, Meager Reimbursement," *Dow Jones*, 12 September 2012, 12:34 PM); and ("Eletrobras CEO: Government Measures to Have 'Significant' Impact," *Dow Jones*, 12 September 2012, 3:24 PM.)

[77] "Eletrobras To Get USD 6.892bn Reimbursement for Unamortized Investments," *SeeNews Latin America*, 2 November 2012.

According to media reports, Eletrobras had expected a total reimbursement of approximately R$30 billion.[78]

iii) **19 November 2012** – on 16 November 2012 (Friday), after the close of trading, Eletrobras released a study analyzing the financial impact of PM 579 on its business.[79] Itau analysts highlighted two takeaways: 1) "the Eletrobras study did make one point very clear to us: there will be no more dividends for ELET6, at least in the short-to-medium term," and 2) "Eletrobras will implement a massive capital increase in the short term (2013) amounting to something between BRL 15 and 20 billion."[80]

iv) **30 November 2012** – on this day, the Brazilian government announced that it would increase the total pool of reimbursements to electric utilities companies from R$20 billion to R$29.87 billion.[81] Eletrobras would receive a constant proportional share of the pool, therefore it would receive a significantly larger reimbursement.

v) **3 December 2012** – on this day, Eletrobras shareholders voted in favor of renewing the concession contracts in accordance with the new industry rules set forth in PM 579.[82]

124. While the news on 12 September 2012 was clearly negative for Eletrobras, it was similarly so for the Brazilian electricity sector as a whole. Other Brazilian electric utility companies would also be adversely impacted by this development. As a result, in an efficient market, such an effect would be captured by the sector effect and reflected in the explained return

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[78] "Eletrobras ADRs Suffer From Disappointing Concessions Reimbursement," *SeeNews Latin America*, 5 November 2012.

[79] Obtained from Eletrobras RSS Feed: http://eletrobras.com/pt/Lists/noticias/ExibeNoticias.aspx?ID=502; I verified the timing of study through *Valor Economico's* (a major Brazilian news source) online search archive which indicated that it released the Eletrobras study at 6:43 PM on 16 November 2012.

[80] "Blood, Toil, Tears and Sweat," by Marcos Severine and Mariana Coelho, Itau, analyst report, 18 November 2012, p. 1.

[81] "Energy Showdown: Brazilian Utilities vs. Brazilian Government," *GlassLewis*, 3 December 2012; "Eletrobras preferred stock jumps 23.56% on Friday on revised MP 579 conditions," *SeeNews Latin America*, 3 December 2012.

[82] "Eletrobras Shareholders Approve Renewal of Power Licenses," *Dow Jones*, 3 December 2012, 8:37 AM.

of Eletrobras common ADS. Therefore, the Eletrobras common ADS residual return may not be over the threshold for statistical significance. Thus, a nonsignificant common ADS price movement following this announcement would be consistent with market efficiency.

125. The other developments stemming from PM 579 were particularly adverse to Eletrobras, more so than for other electric utility companies operating in Brazil, because Eletrobras was compelled to accept the new discounted terms, whereas other companies had a choice of how to respond to this new legislation, among which choices included rebidding, renewing, or exiting. [83]

3. Isolating the Impact of Company-Specific Information

126. Event study analysis determines how much of the Company's security return following each of the events was driven by Company-specific information as opposed to market and peer group factors.

127. This method, which is generally accepted and widely used in econometric modeling, involves running a regression to determine how the price of Eletrobras common ADS typically behaved in relation to the overall U.S. stock market, the Brazilian stock market, its peer group, and in this case, the value of the Brazilian currency. The regression results are then used to determine how much of each event day's actual return is explained by the U.S and Brazilian markets, peer group, and currency factors. The portion of the security return attributable to the U.S. and Brazilian markets, peer group, and currency factors is called the "explained return."

128. The explained return is then subtracted from the actual return, to isolate the "residual return," which is the security's return after controlling for the explanatory factors.

[83] For example, Fitch issued a press release on 7 November 2012 titled: "Fitch: Brazil Concession Renewal Offer Negative for Eletrobras, Marginal for Others," which explained that "early renewal of some expiring electricity concessions will have various impacts for different companies … Fitch considers the offer negative for Centrais Eletricas Brasileiras S.A. (Eletrobras, IDR 'BBB') and neutral to negative for other affected companies. … Fitch expects Eletrobras to accept the government's proposal given the government's ownership interest, which will pressure Eletrobras' credit quality given its weak credit metrics for the assigned rating category on a standalone basis. Under this scenario, the absence of direct financial support from the Federal Government could result in a negative rating action."

129. For the overall U.S. stock market factor I used the CRSP NYSE/AMEX/NASDAQ/ARCA Market Index ("CRSP Market Index"), which is a generally accepted and widely used measure of the overall stock market performance for securities traded in the United States. The CRSP Market Index appropriately incorporates payment of dividends by the constituent companies.

130. For the Brazilian stock market factor I constructed a value-weighted index from the market capitalizations and returns of the constituent companies in the Bovespa Index ("Ibovespa" or "IBOV"), which gauges the performance of the Brazilian stock market.[84] I excluded Eletrobras in my reconstruction of the index, (the "Brazilian Market Index").[85] The Brazilian Market Index appropriately incorporates payment of dividends by the constituent companies.

131. For the sector factor, I constructed an equal-weighted index from the returns of the constituent companies in the BM&FBOVESPA Electric Utilities Index ("IEE"), which is the index that Eletrobras identified as representative of its peers in its annual market letters issued during the Class Period.[86] I excluded Eletrobras in my reconstruction of the index (the "Peer Index").[87] I constructed the Peer Index as an equal-weighted index, like the original IEE. The Peer Index appropriately incorporates payments of dividends by the constituent companies.

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[84] "Bovespa Index (Ibovespa)," *BM&FBOVESPA*, http://www.bmfbovespa.com.br/en_us/ products/indices/broad-indices/bovespa-index-ibovespa.htm.

[85] Because Eletrobras' ELET3 and ELET6 were components of the IBOV, the Brazilian Market Index was reconstructed using the pro-rated weights of the remaining constituents which were rebalanced every four months (e.g., January-April; May-August; and September-December). ("BM&FBOVESPA Indices: Concepts And Practices Manual," *BM&FBOVESPA*, dated June 2014, p. 6). Constituent companies and weights were obtained from Bloomberg.

[86] See, e.g., Centrais Elétricas Brasileiras S.A. Eletrobras, Form 6-K, filed 16 April 2015, p. 23.

[87] Because Eletrobras' ELET3 and ELET6 (at separate times) were components of the BM&FBOVESPA Electric Utilities Index, the Peer Index was reconstructed using the equal weights of the remaining constituents which were reset every quarter (e.g., January-March; April-June; July-September, and October-December). ("BM&FBOVESPA Electric Utilities Index (IEE) Methodology," *BM&FBOVESPA*, dated February 2014, p. 4). Constituents obtained from Bloomberg.

132. For the currency factor, I used the daily logarithmic returns computed from the daily spot foreign exchange rate of the Brazilian real (Real/$) as of the close of trading in New York, or 4:00 PM eastern time.[88]

133. All returns used in the regressions are logarithmic returns – that is, the natural logarithm of the ratio of the current day's closing price plus dividends to the previous day's closing price.[89] Logarithmic returns are commonly used in event studies and equity analysis. Analysts and researchers generally use logarithmic returns instead of percent price changes because of various computational advantages.[90]

134. Eletrobras' common ADS prices, trading volume, and returns are shown in Exhibit-4a. CRSP Market Index, Brazilian Market Index, Peer Index, and Foreign Exchange returns are presented in Exhibit-5.

135. I ran the regression on daily returns covering the entire Class Period, 17 August 2010 through 24 July 2015. The choice of using the Class Period for the regression estimation period is a widely used and generally accepted methodology in event study analysis.

> "Three general choices for the placement of an estimation window are before the event window, surrounding the event window, and after the event window."
>
> **"Materiality and Magnitude: Event Studies in the Courtroom," by David I. Tabak and Frederick C. Dunbar in *Litigation Services Handbook: The Role of the Financial Expert*, 3rd ed., edited by Roman L. Weil, Michael J. Wagner, and Peter B. Frank, John Wiley & Sons, Inc., 2001, p. 19.5.**

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[88] If the 4:00 PM spot price was not available, I used the 3:30 PM spot price to compute daily returns. If the 3:30 PM spot price was not available, I used the 3:00 PM spot price. If still unavailable, I continued to search for available spot prices in earlier 30 minute intervals until 9:30 AM. Bloomberg LP functions: USDBRL Curncy HP ["SOURCE" field: F160] to ["SOURCE" field: F093].

[89] Because the Brazilian Market Index and Peer Index returns were computed using data as of the close of trading in in Sao Paulo, which is at times earlier than the close in New York, I repeated all tests using the adjustment for nonsynchronous data presented in "Estimating Betas From Nonsynchronous Data," by Myron Scholes and Joseph Williams, *Journal of Financial Economics*, 1977. This analysis showed that the qualitative conclusions drawn from my event study are robust to this adjustment.

[90] Appendix-1 presents the mathematical formula for the logarithmic return and a discussion of the measure.

136. I also ran regressions on two sub-periods. I performed a Chow test to determine whether the relationship between the price of Eletrobras common ADS and the explanatory variables changed following 11 September 2012, when PM 579 was announced, an event that according to the Company "materially changed the Brazilian electricity sector."[91] The Chow test indicated that there was indeed a structural change in the regression relationship at that juncture.[92]

137. Therefore, I ran regressions on daily returns covering, respectively, the entire Class Period, Interval-1 (17 August 2010 through 11 September 2012), and Interval-2 (12 September 2012 through 24 June 2015). I used dummy variables to control for potentially abnormal returns on 1) filing delay event days; 2) PM 579 event days; and 3) earnings announcement dates. Using dummy variables to control for potentially atypical events in the estimation period, especially when those dates are the subject of the event study analysis, so that the model parameters properly reflect typical stock price movements, is a widely used and generally accepted methodology, as noted in the academic and finance literature.[93]

138. The regression results for the Class Period are presented in Exhibit-6aa. The regression results for Interval-1 and Interval-2 are presented in Exhibit-6ab and Exhibit-6ac, respectively.

139. I computed the explained return of Eletrobras common ADS on each event date by adding: 1) the estimated regression intercept term, 2) the respective day's CRSP Market Index return multiplied by the CRSP Market Index coefficient estimated by the regression, 3) the respective day's Brazilian Market Index return multiplied by the Brazilian Market Index coefficient estimated by the regression, 4) the respective day's Peer Index return multiplied by the regression's Peer Index coefficient; and 5) the foreign exchange return multiplied by the regression's foreign exchange factor coefficient.

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[91] Centrais Elétricas Brasileiras S.A. Eletrobras, Form 20-F for the Fiscal Year Ended 31 December 2012, filed 1 May 2013, p. 71.

[92] See, e.g., *A Guide to Econometrics*, by Peter Kennedy, chapter 8, 6th edition, Blackwell Publishing, 2008.

[93] See, Aktas, et al. [2007]; Binder [1985]; Box and Tiao [1975]; Larcker, et al. [1980]; Malatesta [1986]; and Thompson [1985].

140. I then computed the residual returns for each event date by subtracting the explained return from the actual return.

4. *t*-Test

141. For each event, a statistical test called a *t*-test was conducted to determine whether the residual return of the Eletrobras common ADS was statistically significant. Statistical significance means that the event return, after controlling for the market, peer group, and foreign exchange effects was of such magnitude that it cannot be attributed to random volatility, but alternatively must have been caused by Company-specific information. A *t*-test compares the residual return on an event date to the typical residual return exhibited over the corresponding regression estimation sub-period. If the event date residual return is far greater (positively or negatively) than the typical residual return, the *t*-test indicates that the residual return is statistically significant.[94]

142. The daily event study results for the Eletrobras common ADS using the regression results from Interval-1 and Interval-2 are presented in Exhibit-7a.

B. Event Study Results: Financial Filing Delays due to Lava Jato Investigation

1. 30 April 2015

143. Eletrobras filed a Form 12b-25 notifying the SEC that it was unable to timely file its financial statements for the fiscal year ended 31 December 2014.[95] The Company explained that it:

> "[R]ecently became aware of press reports stating that the former CEO of Camargo Corrêa allegedly stated in his testimony in relation to *Operação Lava-Jato* (Operation Car Wash) that the consortium of companies bidding for the mechanical assembly of the Angra 3 power plant allegedly made illegal payments to the CEO of our wholly owned subsidiary, Eletrobras Thermonuclear S.A. Eletronuclear ('Eletronuclear'). The CEO of Eletronuclear has requested leave of absence. The internal committee

[94] The test is called the *t*-test because it involves the computation of a *t*-statistic, which is the event day residual return divided by the standard deviation of residual returns from the control period, i.e., the regression estimation data comprising all other days. If the absolute value of the *t*-statistic is greater than the critical *t*-statistic value (1.96 for large samples), the likelihood that the residual return could have been caused by random volatility alone is less than 5%, which is generally accepted to be so unlikely that the random volatility explanation can be rejected, and the security return for that day is deemed statistically significant.

[95] Centrais Elétricas Brasileiras S.A. Eletrobras, Form 12b-25, filed 30 April 2015.

Case: 1:15-cv-05754-JGK Doc #: 66-2 Filed: 10/02/17 Page 46 of 76 PageID #: 7190

> established by the Company to investigate any allegations made in the press in relation to *Operação Lava-Jato* (Operation Car Wash) has not yet concluded its internal investigation and the Company's Board of Directors, to which such internal committee reports, has provided authorization for the Company to take all necessary measures to engage a specialized independent firm to conduct an external investigation into these allegations. Accordingly, the Company is unable to file its Form 20-F as of and for the year ended December 31, 2014 by April 30, 2015."
> **Centrais Elétricas Brasileiras S.A., Form 12b-25, filed 30 April 2015.**

144. On 30 April 2015, Eletrobras common ADS fell 8.60% (on a logarithmic return basis). The U.S. Market Index return was -1.08%, the Brazilian Market Index return was 1.57%, the Peer Index return was -0.15%, and the currency factor return was 1.86%. According to the regression model, on 30 April 2015, the explained return on Eletrobras common ADS was -1.50%. The difference between the actual return of -8.60% and the explained return of -1.50% is a residual return of -7.10%.

145. A residual return of -7.10% is an unusually large one-day decline for Eletrobras common ADS. That residual return is associated with a *t*-statistic of -3.02, which indicates that the residual return was too large to have been a random fluctuation. The likelihood of obtaining a residual return of this magnitude and associated *t*-statistic given that particular explanation (a random fluctuation unrelated to news) is less than 1 in 380. Therefore, the stock return is deemed statistically significant.

2. <u>14 May 2015</u>

146. On 14 May 2015, Eletrobras issued a press release notifying shareholders that it was unable to file its Form 20-F by 15 May 2015.[96] The Company noted that "[a]t this time, we are unable to provide a specific date by which we will file our 2014 20-F."[97] The Company explained that this second, and indefinite, delay was in part due to the fact that it was still "in the process of engaging a specialized independent firm to conduct an external investigation into these allegations."[98]

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[96] Centrais Elétricas Brasileiras S.A. Eletrobras, Form 6-K, filed 15 May 2015.

[97] Id.

[98] Id.

147. On 14 May 2015, Eletrobras common ADS fell 2.53% (on a logarithmic return basis). The U.S. Market Index return was 0.97%, the Brazilian Market Index return was 0.51%, the Peer Index return was 1.54%, and the currency factor return was -1.49%. According to the regression model, on 14 May 2015, the explained return on Eletrobras common ADS was 3.51%. The difference between the actual return of -2.53% and the explained return of 3.51% is a residual return of -6.04%.

148. A residual return of -6.04% is an unusually large one-day decline for Eletrobras common ADS. That residual return is associated with a *t*-statistic of -2.58, which indicates that the residual return was too large to have been a random fluctuation. The likelihood of obtaining a residual return of this magnitude and associated *t*-statistic given that particular explanation (a random fluctuation unrelated to news) is less than 1 in 95. Therefore, the stock return is deemed statistically significant.

C. Event Study Results: PM 579 Related Events

1. <u>12 September 2012</u>

149. On 11 September 2012, after the close of trading, the Brazilian government promulgated PM 579.[99] News media commentary characterized the development as decidedly negative for the whole Brazilian electric utilities industry.

> "SAO PAULO--Brazil electric company stocks tumbled Wednesday after the government demanded that utilities charge lower prices starting next year, and as it signaled Brazil wouldn't pay out a very high reimbursement value for past investments. Transmission company CTEEP (TRPL4.BR) saw the biggest drop, losing 31% in Sao Paulo trading, while the broader Ibovespa index climbed 0.4%. Sao Paulo's state-controlled Cesp (CESP6.BR) plunged 27%, while Minas Gerais utility Cemig (CMIG4.BR) slumped 20%. The drop sent the IEE electric power index 3.3% lower, after a 3.3% decline Tuesday."
> **"Brazil Utilities Tumble on Price Cuts, Meager Reimbursement," *Dow Jones Global Equities News*, 12 September 2012, 12:34 PM.**

[99] Provisional Act No. 579, The Presidency of the Republic, dated 11 September 2012.

150. On 12 September 2012, Eletrobras common ADS fell 8.24% (on a logarithmic return basis). The U.S. Market Index return was 0.26%, the Brazilian Market Index return was 0.80%, the Peer Index return was -8.79%, and the currency return was 0.51%. According to the regression model, the explained return on Eletrobras common ADS was -7.92%. The difference between the actual return of -8.24% and the explained return of -7.92% is a residual return of -0.32%.

151. The steep Eletrobras common ADS decline on 12 September 2012 is explainable according to the regression analysis as Eletrobras common ADS participating in the sector's negative response to the negative sector news. The residual return, which measures the common ADS' response to Company-specific news was not significant that day. The non-significant residual common ADS price movement following this event is consistent with market efficiency, though it is not a demonstration of a significant reaction to Company-specific news.

2. <u>2 November 2012</u>

152. On 1 November 2012, after the close of trading, the Brazilian government announced that it would offer up to R$20 billion, of which Eletrobras was allocated R$14 billion, in total reimbursements for the unamortized assets of electric utilities companies that choose to renew their concessions under the new rules of PM 579.[100] The development was disappointing for Eletrobras as the Company had expected a total reimbursement of approximately R$30 billion.[101]

153. News media commentary linked the price decline in Eletrobras common ADS to the development.

> "The American Depository Receipts (ADRs) of Brazilian federal power holding company Eletrobras (NYSE:EBR) on Friday plummeted on the New York stock exchange NYSE after Brazil's government announced a

[100] "Eletrobras To Get USD 6.892bn Reimbursement for Unamortized Investments," *SeeNews Latin America*, 2 November 2012.

[101] "Eletrobras ADRs Suffer From Disappointing Concessions Reimbursement," *SeeNews Latin America*, 5 November 2012.

Case: 1:15-cv-05054-JFK Document 81-10 Filed 10/26/17 Page 19 of 37 PageID #: 19091

> lower-than-expected reimbursement amount for unamortized investments amid the early renewal of utilities' concessions expiring through 2017."
> **"Eletrobras ADRs Suffer From Disappointing Concessions Reimbursement,"** ***SeeNews*, 5 November 2012.**

154. On 2 November 2012, Eletrobras common ADS fell 9.43% (on a logarithmic return basis). The U.S. Market Index return was -0.98%. As the Brazilian market was closed to observe All Souls Day on 2 November 2012. The Brazilian Market Index, the Peer Index, and the currency factor were unchanged. According to the regression model, on 2 November 2012, the explained return on Eletrobras common ADS was 0.06%. The difference between the actual return of -9.43% and the explained return of 0.06% is a residual return of -9.49%.

155. A residual return of -9.49% is an unusually large one-day decline for Eletrobras common ADS. That residual return is associated with a *t*-statistic of -4.04, which indicates that the residual return was too large to have been a random fluctuation. The likelihood of obtaining a residual return of this magnitude and associated *t*-statistic given that particular explanation (a random fluctuation unrelated to news) is less than 1 in 16,900. Therefore, the stock return is deemed statistically significant.

3. <u>19 November 2012</u>

156. On 16 November 2012 (Friday), after the close of trading, Eletrobras released a study analyzing the impact of PM 579 on its future performance.[102]

157. Itau analysts covering the study highlighted two takeaways from the study: 1) "there will be no more dividends for ELET6, at least in the short-to-medium term" and 2) "that Eletrobras will implement a massive capital increase in the short term (2013) amounting to something between BRL 15 and 20 billion." [103]

158. On 19 November 2012, Eletrobras common ADS fell 15.60% (on a logarithmic return basis). The U.S. Market Index return was 1.90%, the Brazilian Market Index return was 2.02%, the Peer Index return was 0.09%, and the currency return was -0.16%. According

[102] Obtained from Eletrobras RSS Feed: http://eletrobras.com/pt/Lists/noticias/ExibeNoticias.aspx?ID=502; I verified the timing of study through *Valor Economico's* (a major Brazilian news source) online search archive which indicated that it released the Eletrobras study at 6:43 PM on 16 November 2012.

[103] "Blood, Toil, Tears and Sweat," by Marcos Severine and Mariana Coelho, Itau, analyst report, 18 November 2012, p. 1.

to the regression model, the explained return on Eletrobras common ADS was 1.38%. The difference between the actual return of -15.60% and the explained return of 1.38% is a residual return of -16.98%.

159. A residual return of -16.98% is an exceptionally large one-day decline for Eletrobras common ADS. That residual return is associated with a *t*-statistic of -7.24, which indicates that the residual return was too large to have been a random fluctuation. The likelihood of obtaining a residual return of this magnitude and associated *t*-statistic given that particular explanation (a random fluctuation unrelated to news) is virtually nil. Therefore, the stock return is deemed statistically significant.

4. 30 November 2012

160. On this day, the Brazilian government announced that it would increase the total pool of reimbursements to electric utilities companies from R$20 billion to R$29.87 billion. [104] Eletrobras would receive a constant proportional share of the pool, therefore it would receive a significantly larger reimbursement than previously expected.

161. News media commentary characterized this development as positive for Eletrobras.

> "Centrais Eletricas Brasileiras (ELET6.BR), the federally controlled utility known as Eletrobras, said Friday that the higher reimbursement it will receive from the government 'assures significantly more favorable conditions' for renewing the company's expiring operating licenses."
> **"Eletrobras Says Higher Reimbursement Value Improves Renewal Terms," *Dow Jones*, 30 November 2012, 8:09 AM.**

> "Last week the government said it would raise the proposed compensations to eight utilities for their early renewal of the concessions of 10 hydroelectric power plants (HPPs). Also, the government said it would compensate local power transmission companies for investments made before 2000, which were not taken into consideration in the previous renegotiation."
> **"Eletrobras Preferred Stock Jumps 23.56% on Friday on Revised MP 579 Conditions," *SeeNews Latin America*, 3 December 2012.**

162. On 30 November 2012, Eletrobras common ADS rose 14.37% (on a logarithmic return basis). The U.S Market Index return was 0.03%, the Brazilian Market Index return

[104] "Energy Showdown: Brazilian Utilities vs. Brazilian Government," *GlassLewis*, 3 December 2012.

was -0.77%, the Peer Index return was 0.21%, and the currency return was 1.76%. According to the regression model, the explained return on Eletrobras common ADS was -2.62%. The difference between the actual return of 14.37% and the explained return of -2.62% is a residual return of 16.99%.

163. A residual return of 16.99% is an unusually large one-day increase for Eletrobras common ADS. That residual return is associated with a *t*-statistic value of 7.24, which indicates that the residual return was too large to have been a random fluctuation. The likelihood of obtaining a residual return of this magnitude and associated *t*-statistic given that particular explanation (a random fluctuation unrelated to news) is virtually nil. Therefore, the stock return is deemed statistically significant.

5. <u>3 December 2012</u>

164. On 3 December 2012, Eletrobras shareholders, majority controlled by the Brazilian government, voted to renew the concession contracts in accordance with the adverse terms dictated by PM 579.[105] *Reuters News* made the following notes regarding the development: 1) "State-led power utility yields to government drive"; and 2) "Minority shareholders protest alleged conflict of interest."[106]

165. News media commentary following the shareholders' vote was decidedly negative.

> "Despite strong opposition from minority shareholders, Brazilian state-owned energy giant Eletrobras has approved a plan to renew its concessions which end between 2015 and 2017. Some shareholders criticized the way the government has implemented its concession renewal strategy, considering it an abuse of power. It is reported that all minority shareholders voted against the renewal of the concessions, with the exception of a fund managed by J. P Morgan Chase Bank."
> **"Brazil: Eletrobras Approves Plan To Renew Concessions,"** ***Valor Economico*****, 4 December 2012.**

> "The shareholders of Brazil's federal power holding company Eletrobras (SAO:ELET3) yesterday cleared the early renewal of the company' concessions expiring between 2015 and 2017, Eletrobras said in a note to the market. The result was guaranteed by the vote of the controlling shareholders, which include Brazil's National Treasury and BNDESPar, the

[105] "Eletrobras Shareholders Approve Renewal of Power Licenses," *Dow Jones*, 3 December 2012, 8:37 AM.

[106] "Update 5-Eletrobras accepts steep rate cuts for renewed licenses," *Reuters News*, 3 December 2012, 7:13 AM.

investment arm of Brazilian National Social and Economic Development Bank (BNDES). Norwegian equity fund Skagen, the main minority shareholder in Eletrobras, voted against the renewal. A number of other minority shareholders rejected the concessions' renewal and may even resort to legal actions in search of reimbursements."
"Eletrobras Shareholders Okay Concessions Renewal," *SeeNews Latin America*, 4 December 2012.

166. On 3 December 2012, Eletrobras common ADS fell 5.62% (on a logarithmic return basis). The U.S. Market Index return was -0.42%, the Brazilian Market Index return was 1.29%, the Peer Index return was 0.61%, and the currency factor return was -0.58%. According to the regression model, the explained return on Eletrobras common ADS was 2.12%. The difference between the actual return of -5.62% and the explained return of 2.12% is a residual return of -7.74%.

167. A residual return of -7.74% is an unusually large one-day decline for Eletrobras common ADS. That residual return is associated with a *t*-statistic of -3.30, which indicates that the residual return was too large to have been a random fluctuation. The likelihood of obtaining a residual return of this magnitude and associated *t*-statistic given that particular explanation (a random fluctuation unrelated to news) is less than 1 in 970. Therefore, the stock return is deemed statistically significant.

D. Collective Event Study: Earnings Announcement Events

168. In addition to assessing market efficiency by observing whether the security price reacted on important news event dates individually, one can also test for market efficiency by assessing collectively: 1) whether the security has a greater frequency of statistically significant price movements on days with greater information flow ("earnings announcement dates") than on more typical days ("non-news dates"); and/or 2) whether the security price generally moves more on days with greater information flow than on typical days with less news. That is, if 1) the security's price movements are more frequently statistically significant among the collection of earnings announcement dates than among all other days; and/or 2) the security's price movements are generally greater among a collection of earnings announcement dates than among all other non-news days, these results would establish that there is a cause and effect relationship between the flow of information and stock price movements, which indicates market efficiency.

Case: 1:15-cv-05754-JGK Document 103-10 Filed 06/23/17 Page 50 of 370 PageID #: 19095

169. I conducted collective empirical tests of market efficiency based on these principles. I focused the analysis on earnings announcements. A company's financial results and forecasts are among the most important considerations to investors assessing the value of its stock. While not every earnings announcement contains new, unexpected, highly impactful valuation information, the finance literature notes that such information more frequently arrives on earnings announcement dates than on ordinary dates.[107]

170. Numerous well-known and highly-regarded academic studies (for example, Beaver [1968], Ball and Brown [1968], Ball [1978], Watts [1978], Patell and Wolfson [1984], and Ball and Kothari [1991]) have specifically examined stock price movements caused by earnings announcements, and concur that earnings announcements are unusually important information events generally.

171. Consequently, a pattern of greater stock price movement on earnings announcement days as compared to all other days is indicative of market efficiency.

172. Table-1 below presents the 20 dates on which Eletrobras announced earnings during the Class Period and the respective event test dates:[108]

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[107] *Financial Reporting: An Accounting Revolution*, 3rd ed., by William H. Beaver, 1998, p. 38; and "Earnings Management to Exceed Thresholds," by Francois Degeorge, Jayendu Patel, and Richard Zeckhauser, *Journal of Business*, 1999, p. 1.

[108] The earnings announcements with numbers [1] through [9] are during Interval-1, while earnings announcements with numbers [10] through [20] are during Interval-2.

Table–1: Earnings Announcement Dates During the Class Period

	Announcement Date	Announcement Time[1]	Effective Event Test Date	Source
[1]	Friday, November 12, 2010	After-Market	Monday, November 15, 2010	*Eletrobras 6-K*
[2]	Friday, May 13, 2011	During Trading	Friday, May 13, 2011	*Eletrobras 6-K*
[3]	Monday, June 20, 2011	After-Market	Tuesday, June 21, 2011	*Eletrobras 6-K*
[4]	Tuesday, August 23, 2011	During Trading	Tuesday, August 23, 2011	*Eletrobras 6-K*
[5]	Friday, November 11, 2011	During Trading	Friday, November 11, 2011	*Eletrobras 6-K*
[6]*	Tuesday, April 17, 2012	During Trading	Tuesday, April 17, 2012	*Eletrobras 6-K*
[7]	Tuesday, April 17, 2012	After-Market	Wednesday, April 18, 2012	*Eletrobras 6-K*
[8]	Tuesday, May 29, 2012	After-Market	Wednesday, May 30, 2012	*Eletrobras 6-K*
[9]	Tuesday, August 14, 2012	After-Market	Wednesday, August 15, 2012	*Eletrobras 6-K*
[10]	Wednesday, November 14, 2012	After-Market	Thursday, November 15, 2012	*Eletrobras 6-K*
[11]	Wednesday, March 27, 2013	After-Market	Thursday, March 28, 2013	*Eletrobras 6-K*
[12]	Wednesday, May 15, 2013	After-Market	Thursday, May 16, 2013	*Eletrobras 6-K*
[13]	Wednesday, August 14, 2013	After-Market	Thursday, August 15, 2013	*Eletrobras 6-K*
[14]	Thursday, November 14, 2013	After-Market	Friday, November 15, 2013	*Eletrobras 6-K*
[15]	Thursday, March 27, 2014	After-Market	Friday, March 28, 2014	*Eletrobras 6-K*
[16]	Thursday, May 15, 2014	After-Market	Friday, May 16, 2014	*Eletrobras 6-K*
[17]	Tuesday, August 12, 2014	After-Market	Wednesday, August 13, 2014	*Eletrobras 6-K*
[18]	Friday, November 14, 2014	After-Market	Monday, November 17, 2014	*Eletrobras 6-K*
[19]	Friday, March 27, 2015	After-Market	Monday, March 30, 2015	*Eletrobras 6-K*
[20]	Friday, May 15, 2015	After-Market	Monday, May 18, 2015	*Eletrobras 6-K*

Note:

* On 17 April 2012, the Company made a preliminary earnings announcement for fiscal year 2011.

[1] Because the press releases contained in the Company's 6-K filings do not provide a timestamp, I determined the timing of the earnings announcements by reviewing market commentary and news published 3 days prior and 3 days after the 6-K filing to identify the effective event test date.

173. There may have been additional valuation-relevant information aside from earnings information disseminated on these event dates to which the stock price reacted. Stock price reactions to such additional information would be further support of market efficiency.

1. Z-Test Analysis of Frequency of Significant Event Returns

174. The Z-test is a commonly used and widely accepted methodology for testing if the difference in the proportion (or incidence) of statistically significant observations for two samples is statistically significant.[109] I conducted a Z-test (also known as an incidence test or proportionality test) comparing earnings announcements during the Class Period against all other lesser or non-news days in the Class Period.

a) Z-Test Over the Class Period

175. As shown in Exhibit-11a, there were a total of 1,215 days during the Class Period on which Eletrobras common ADS traded, and of those days, 73 days had statistically significant residual returns. There were 6 statistically significant earnings announcement days out of the total of 20 earnings announcement days for the Eletrobras common ADS. 67 of the remaining 1,195 non-news days were statistically significant. Accordingly, the proportion of statistically significant days in the news day sample is 30.00% and the proportion of statistically significant days among the non-news sample is 5.61%. The incidence of statistically significant days within the news day group was therefore far greater than within the non-news group.

176. This difference in incidence frequency is associated with a z-score of 4.55, which is greater than the critical z-statistic threshold of 1.65 for significance at the 95% confidence level. This finding indicates that the incidence frequency difference between the two samples is too severe to accept the null hypothesis that the price of the Eletrobras common ADS behaves no differently on news days than on all other days.[110] The likelihood of obtaining a difference in incidence frequency of this magnitude and associated z-score

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[109] "The 'Less Than' Efficient Capital Markets Hypothesis: Requiring More Proof from Plaintiffs in Fraud-on-the-Market Cases," by Paul A. Ferrillo, Frederick C. Dunbar and David Tabak, *78 St. John's L. Rev. 81*, 119-22 (2004); and *In re PolyMedica Corp. Sec. Litig.*, 453 F. Supp. 2d 266 (D. Mass. 2006); *Reference Manual on Scientific Evidence*, 3rd ed. (Washington: The National Academies Press, 2011), pp. 591-597; *Statistics for Business and Economics*, by David R. Anderson, Dennis J. Sweeny, and Thomas A. Williams, 2nd edition, West Publishing, 1984, chapter 10; *Statistical Techniques in Business and Economics*, by Robert D. Mason, Douglas A. Lind, and William G. Marchal, 10th edition, Irwin McGraw-Hill, 1999, chapter 9; *Applied Statistics For Public Policy*, by Brian P. Macfie and Philip M. Nufrio, M.E. Sharpe, 2006, chapter 13; and *Probability and Statistics for Engineering and the Sciences*, by Jay Devore, 9th edition, Cengage Learning, 2016, chapter 9.

[110] The Z-test critical z-score threshold of 1.65 indicates statistical significance at the 95% confidence level for a one-tailed test. Here the critical test statistic is for a one-tailed test because the question at issue is whether news dates have a *greater* frequency of statistically significant returns than "non-news" dates.

given that particular explanation is less than 0.0003%. Therefore, the difference in incidence frequency is deemed statistically significant. This finding proves that the ADS reacts to information, and the market is therefore informationally efficient.

b) Z-Test Over Interval-1

177. As shown in Exhibit-11a, there were a total of 523 days during Interval-1 on which Eletrobras common ADS traded, and of those, 30 days had statistically significant residual returns. There were 4 statistically significant earnings announcement dates among the 9 earnings announcement dates for the Eletrobras common ADS. Of the remaining 514 non-news days, 26 were statistically significant. Accordingly, the proportion of statistically significant days among the earnings announcement dates is 44.44%, and the proportion of statistically significant days in non-news days is 5.06%. There was therefore a greater frequency of statistically significant days within the news day group than within the non-news day group.

178. This difference in proportions is associated with a z-score of 5.04, which is greater than the critical z-statistic threshold of 1.65 for significance at the 95% confidence level, and indicates that the difference between the two samples is too severe to accept the notion that the price of the Eletrobras common ADS behaves no differently on days with a greater flow of information than all other days. The likelihood of obtaining a difference in incidence frequency of this magnitude and associated z-score given that particular explanation is less than 0.00004%. Therefore, the difference in incidence frequency is statistically significant over the course of Interval-1.

c) Z-Test Over Interval-2

179. As shown in Exhibit-11a, there were a total of 692 days during Interval-2 on which Eletrobras common ADS traded, and of those days, 41 days had statistically significant residual returns. There were 3 statistically significant earnings announcement dates out of the 11 earnings announcement dates for the Eletrobras common ADS. Of the remaining 681 non-news days, 38 were statistically significant. Accordingly, the proportion of statistically significant days among the earnings announcement dates is 27.27%, and the proportion of statistically significant days in non-news days is 5.58%. There was therefore

a greater frequency of statistically significant days within the news day group than within the non-news day group.

180. This difference in proportions is associated with a z-score of 3.02, which is greater than the critical z-statistic threshold of 1.65 for significance at the 95% confidence level, and indicates that the difference between the two samples is too severe to accept the notion that the price of the Eletrobras common ADS behaves no differently on days with a greater flow of information than all other days. The likelihood of obtaining a difference in incidence frequency of this magnitude and associated z-score given that particular explanation is only 0.13%. Therefore, the difference in incidence frequency is statistically significant over the course of Interval-2.

2. F-Test and Ansari-Bradley Test Analyses of Event Return Dispersion

181. Announcements of financial results sometimes constitute unexpected good news and sometimes constitute unexpected bad news. In an efficient market, the stock would rise after unexpected good news and fall after unexpected bad news. Therefore, one would expect a wider dispersion of returns on earnings announcement dates, as compared to ordinary days, as long as some of the announcements contained some unexpected good or bad news, and the market is efficient.

182. The dispersion in the distribution of a sample's security price returns is commonly measured by the standard deviation statistic. Comparing the sample standard deviation of returns on earnings announcement dates to the sample standard deviation of returns for all other days, thus tests reactivity to news. A greater sample standard deviation for the earnings announcement returns would indicate market efficiency.

183. The F-test and Ansari-Bradley test are statistical tests that compare standard deviations between two groups. I ran these tests on the complete Class Period, as well as on the two sub-intervals to ascertain whether the dispersion of stock price movements on the earnings announcement dates indicates market efficiency.

184. I ran these tests on the residual returns for Eletrobras common ADS, that is, the computed portion of the stock price returns remaining after controlling for the impact of market, peer group, and currency effects. Running the tests on residual returns focuses the tests more precisely on the effects of Company-specific information on the Company's ADS.

a) *F-Test Over the Class Period*

185. The sample standard deviation of the news days' residual returns was 4.20%. The sample standard deviation of all other returns was 2.00%. Clearly, the news day sample standard deviation was greater than the sample standard deviation for all other days.

186. An F-test assesses whether the difference between the two sample standard deviations is statistically significant, or alternatively, a potentially random result. The F-statistic for the test is 4.4, which is greater than the 95% confidence level critical F-statistic value of 1.60 (with 19 and 1194 degrees of freedom), indicating that the difference in sample standard deviations is statistically significant and meaningful.

187. The F-test finds that the dispersion of news day returns during the Class Period is significantly greater than the dispersion of returns for all other days. This result demonstrates that the price of Eletrobras common ADS moved more on earnings announcement days than on other days. This statistical result indicates that there was a cause and effect relationship between the release of new information and reactions in the Eletrobras common ADS price, which therefore establishes that Eletrobras common ADS traded in an informationally efficient market during the Class Period.

b) *F-Test Over Interval-1*

188. The sample standard deviation of the earnings announcement event day residual returns was 3.46%. The sample standard deviation of all other days' returns was 1.37%. Again, the earnings announcement event day sample standard deviation was greater than the sample standard deviation for all other days.

189. The F-statistic for these two samples is 6.4, which is greater than the 95% confidence level critical F-statistic value of 1.96 (with 8 and 513 degrees of freedom), indicating that the difference in sample standard deviations is statistically significant and meaningful.

190. The F-test finds that the dispersion of earnings announcement event day returns in Interval-1 is significantly greater than the dispersion of returns for all other days. This result demonstrates that the price of the Eletrobras common ADS moved more on earnings announcement event days than on other days. This statistical result indicates that there was a cause and effect relationship between the release of new information and reactions in the

Eletrobras common ADS price, which therefore establishes that Eletrobras common ADS traded in an efficient market during Interval-1.

c) F-Test Over Interval-2

191. The sample standard deviation of the earnings announcement event day residual returns was 4.82%. The sample standard deviation of all other days' returns was 2.34%. Again, the earnings announcement event day sample standard deviation was greater than the sample standard deviation for all other days.

192. The F-statistic for these two samples is 4.3, which is greater than the 95% confidence level critical F-statistic value of 1.84 (with 10 and 680 degrees of freedom), indicating that the difference in sample standard deviations is statistically significant and meaningful.

193. The F-test finds that the dispersion of earnings announcement event days' returns in Interval-2 is significantly greater than the dispersion of returns for all other days. This result demonstrates that the price of the Eletrobras common ADS moved more on earnings announcement event days than on other days. This statistical result indicates that there was a cause and effect relationship between the release of new information and reactions in the Eletrobras common ADS price, which therefore establishes that Eletrobras common ADS traded in an efficient market during Interval-2.

d) Ansari-Bradley Test Over the Class Period

194. The Ansari-Bradley test is another test that determines whether two data samples have significantly different dispersions, which, as discussed above, when applied to a sample of news dates, in comparison to all other dates, would indicate market efficiency. The Ansari-Bradley test is a well-regarded and generally accepted test for comparing sample dispersions and is presented and described in numerous authoritative textbooks.[111]

[111] For example: "Rank-Sum Tests For Dispersions," *The Annals of Mathematical Statistics*, vol. 31, by A.R. Ansari and R. A. Bradley, 1960, pp. 1174-1189; *Applied Nonparametric Statistical Methods*, 4th Edition, by Peter Sprent and Nigel C. Smeeton, 2007, pp. 170-178; *Applied Nonparametric Statistics*, by Wayne W. Daniel, Houghton Mifflin, 1978, pp. 103-107; *Nonparametric Statistical Methods*, by Myles Hollander and Douglas A. Wolfe, John Wiley & Sons, 1973, pp. 142-158; *Beyond ANOVA: Basics of Applied Statistics*, by Rupert G. Miller, Jr., John Wiley & Sons, 1986, pp. 266-278.

195. Applied to the news date returns and the sample of all other returns observed during the Class Period, the Ansari-Bradley test, like the F-test, finds with a high degree of statistical certainty that the dispersion of news day returns was significantly greater than the dispersion of returns on all other days. The Ansari-Bradley C-statistic for the two samples of Eletrobras common ADS residual returns is 3.55, which is greater than the critical C-statistic threshold of 1.65 for significance at the 95% confidence level.[112] This result demonstrates that the price of Eletrobras common ADS moved more on news dates than on other dates during the Class Period.

196. This statistical test result indicates that there was a cause and effect relationship between the release of new, Company-specific information and reactions in the Eletrobras common ADS price, which therefore further supports that Eletrobras common ADS traded in an efficient market during the Class Period.

e) Ansari-Bradley Test Over Interval-1

197. Applied to the earnings announcement event returns and the sample of all other returns observed during Interval-1, the Ansari-Bradley test again finds with a high degree of statistical certainty that the dispersion of earnings announcement event returns was significantly greater than the dispersion of returns on all other days. The Ansari-Bradley C-statistic for the two samples of Eletrobras ADS residual returns is 2.08, which is greater than the critical C-statistic threshold of 1.65 for significance at the 95% confidence level. This result is proof that the price of Eletrobras common ADS moved more on earnings announcement event days than on other days during Interval-1.

198. This statistical test result indicates that there was a cause and effect relationship between the release of new, Company-specific information and reactions in the Eletrobras common ADS price, which therefore establishes that the Eletrobras common ADS traded in an efficient market during Interval-1.

[112] The Ansari-Bradley critical C-statistic threshold of 1.65 indicates statistical significance at the 95% confidence level for a one-tailed test. Here the critical test statistic is for a one-tailed test because the question at issue is whether news dates have *greater* volatility than "non-news" dates.

f) Ansari-Bradley Test Over Interval-2

199. Applied to the earnings announcement event returns and the sample of all other returns observed during Interval-2, the Ansari-Bradley test again finds with a high degree of statistical certainty that the dispersion of earnings announcement event returns was significantly greater than the dispersion of returns on all other days. The Ansari-Bradley C-statistic for the two samples of Eletrobras stock residual returns is 2.55, which is greater than the critical C-statistic threshold of 1.65 for significance at the 95% confidence level. This result is further proof that the price of Eletrobras common ADS moved more on earnings announcement event days than on other days during Interval-2.

200. This statistical test result indicates that there was a cause and effect relationship between the release of new, Company-specific information and reactions in the Eletrobras common ADS price, which therefore establishes that the Eletrobras common ADS traded in an efficient market during Interval-2.

E. Eletrobras Common ADS Market Efficiency Summary and Conclusion

201. Eletrobras common ADS traded on the NYSE and numerous market makers facilitated trading in the security. The Company was widely covered by analysts and news media. Institutional ownership of Eletrobras common ADS was widespread, and trading was active. Market capitalization and float were high (both for the Company as a whole and for the security independently). The security's bid-ask spread was narrow. Current and historical financial information about the Company was readily available to investors and analysts. While at times the Company did not qualify for F-3 registration on account of SEC filing delays, two of those delays were on account of issues related to Plaintiffs' allegations, and nonetheless, throughout the Class Period, information provided by analysts, news coverage, Brazilian financial filings, and the Company itself provided investors with access to financial information about the Company on a continuous basis. To the extent that F-3 registration eligibility indicates company characteristics associated with market efficiency, the Company clearly possessed those particular characteristics throughout the Class Period.

202. Not only did the Eletrobras common ADS satisfy the *Cammer* and *Unger/Krogman* factors that indicate market efficiency (with the F-3 exception noted above), but it also satisfied the empirical *Cammer* factor, which demonstrates the essence of market efficiency.

203. The empirical tests prove that there was a cause and effect relationship between new, important Company information, and movements in the price of Eletrobras common ADS.

204. Given these facts, I conclude that the Eletrobras common ADS traded in an efficient market over the course of the Class Period.

VIII. <u>EFFICIENCY OF THE MARKET FOR THE ELETROBRAS PREFERRED ADS</u>

205. To assess whether the market for the Eletrobras preferred ADS was efficient during the Class Period, I analyzed the market for, and behavior of, Eletrobras preferred ADS, focusing on the factors that are generally accepted to be indicative of market efficiency for a publicly-traded security.

206. Preferred stock is a hybrid security with both bond-like and equity-like characteristics. Like a bond, preferred stock delivers investors fixed periodic payments, called preferred dividends, equal to a specified rate multiplied by a specified face value.

> "Preferred stock has features similar to both equity and debt. Like a bond, it promises to pay to its holders a fixed amount of income each year. In this sense preferred stock is similar to an infinite-maturity bond, that is, a perpetuity. It also resembles a bond in that it does not convey voting power regarding the management of the firm. Preferred stock is an equity investment, however. The firm retains discretion to make the dividend payments to the preferred stockholders; it has no contractual obligation to pay those dividends. Instead, preferred dividends are usually cumulative; that is, unpaid dividends cumulate and must be paid in full before any dividends may be paid to holders of common stock."
> ***Investments*, 8th edition, by Zvi Bodie, et al., McGraw-Hill Irwin, 2009, p. 37.**

207. Preferred shares are a senior claim to common shares, but are generally junior to notes and bonds. Preferred dividends must be paid in full before common dividends can be paid. In the event of a bankruptcy or liquidation, preferred security investors must be repaid the face value of the security before any liquidated asset proceeds are distributed to common

stock investors. On the other hand, obligations to senior bond holders must be satisfied before preferred shareholders are paid.

> "Preferred stock is senior to common stock but junior to bonds. Therefore, preferred stockholders are paid only when profits have been generated and all debt holders have been paid (but before common stockholders are paid)."
> ***Financial Markets and Institutions*, 4th edition, by Anthony Saunders and Marcia Millon Cornett, McGraw-Hill Irwin, 2009, p. 228.**

> "A simple preference is that after settlement has been made with creditors, the preferred stockholders are entitled to receive the par, stated, or liquidated value of the preferred before any distribution is made to common stock or to any junior preferred issue."
> **"Nonconvertible Preferred Stock," by Richard S. Wilson, in *The Handbook of Fixed Income Securities*, 6th edition, edited by Frank J. Fabozzi, McGraw Hill 2001, p. 347.**

208. On account of its seniority and fixed dividends, preferred stock tends to be less sensitive to company information than is common stock.

> "Unlike common stockholders, preferred stockholders do not share in the increased profits that come from good years for the firm, nor in the decreased profits that come from bad years, unless earnings drop far enough to prohibit the preferred dividend payment."
> ***Investments*, 2nd edition, by Nancy L. Jacob and R. Richardson Pettit, Irwin, 1988, p. 335.**

209. Moreover, because dividend payments to preferred security investors can be reduced only after dividends to common security investors are reduced to zero, and because the face value principal is in jeopardy only if the value of company assets falls so much as to wipe out the common equity, the common equity creates a value buffer for the preferred shares. This value buffer tends to hold up the value of preferred stock, keeping it from falling much in the face of all but the most severe news.

A. *Cammer* Factors

210. The Company and market characteristics that indicate efficiency for Eletrobras common ADS also indicate efficiency for the preferred ADS.

1. <u>Trading Volume</u>

211. Throughout the Class Period, Eletrobras preferred ADS traded regularly and actively. On average, 274,005 preferred ADS changed hands daily.[113] Eletrobras preferred ADS trading data are presented in Exhibit-4b.

212. During the Class Period, the average weekly trading volume of Eletrobras preferred ADS was approximately 1.4 million shares, or 4.60% of preferred ADS outstanding.[114] This level of trading activity surpasses levels accepted by courts as being indicative of market efficiency for stock.[115] In *Cammer,* the Court cited the conclusion of Alan R. Bromberg and Lewis D. Lowenfels that "weekly trading of 2% or more of the outstanding shares would justify a strong presumption that the market for the security is an efficient one; 1% would justify a substantial presumption."[116] The trading volume for Eletrobras preferred ADS during the Class Period was above the threshold for a strong presumption of market efficiency.

213. Both in terms of average daily trading volume and on the basis of weekly turnover, the market for Eletrobras preferred ADS was very active. Consistent with the *Cammer* opinion, economic theory, and empirical research, the active trading volume in Eletrobras preferred ADS is strong evidence of the efficiency of the market for Eletrobras preferred ADS over the course of the Class Period.

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[113] Data obtained from Bloomberg.

[114] Estimated by averaging the ratio of the daily trading volume to the number of preferred ADS outstanding, and multiplying by 5 (the number of trading days in a typical week).

[115] *Cammer*, 711 F. Supp. at 1286.

[116] Id., at 1293.

2. Analyst Coverage and Other Avenues of Information Dissemination

214. As explained above, securities analysts facilitate the flow of information and the digestion of information within the marketplace, which promote market efficiency. With at least 18 different firms covering the Company during the Class Period, Eletrobras was the subject of active analyst coverage. Additionally, at least 2 analysts specifically addressed Eletrobras' preferred ADS in their reports.[117]

215. When a security is owned by institutional investors, the institution's financial analysts conduct their own research on the security, which provides additional support for a finding of market efficiency. According to Thomson Eikon data on institutional ownership, at least 195 major institutions owned Eletrobras preferred ADS during the Class Period.[118]

216. News media coverage facilitates the flow of company information to the market place, thereby promoting market efficiency. As explained above, news media coverage of Eletrobras was extensive, and this coverage provided Company information to the preferred ADS investors just as it did for the common ADS investors. A Factiva database search established that over 3,700 articles were published in English about the Company during the Class Period. Company information was also disseminated throughout the Class Period via regulatory filings, conference calls, and presentations.

217. During the Class Period, therefore, information about Eletrobras was readily available to market participants as there was a consistent flow of news provided by news media, analysts, and various other sources. This extensive news coverage is further evidence of the efficiency of the market for the Eletrobras preferred ADS.

[117] See, e.g., "Strong 3Q10 on Genco Subs, but Operating Costs Remain High," by Anderson Frey and Pedro Manfredini, JP Morgan, analyst report, 15 November 2010; and "3Q13 Results: Hurt by Provisions," by Maria Carolina Carneiro, Santander, analyst report, 18 November 2013.

[118] According to the SEC filings complied and reported by Thomson Eikon 196 institutions held Eletrobras preferred ADS on at least one of the following reporting dates: 30 September 2010, 31 December 2010, 31 March 2011, 30 June 2011, 30 September 2011, 31 December 2011, 31 March 2012, 30 June 2012, 30 September 2012, 31 December 2012, 31 March 2013, 30 June 2013, 30 September 2013, 31 December 2013, 31 March 2014, 30 June 2014, 30 September 2014, 31 December 2014, and 31 March 2015. There may have been additional institutions that held Eletrobras preferred ADS during the Class Period, though not on the quarterly reporting dates.

218. Eletrobras was not an obscure company, escaping the notice of analysts and investors. Rather, the Company was large, well known, widely covered, and the preferred ADS were widely held. These facts strongly support a finding that the market for Eletrobras preferred ADS was an efficient market during the Class Period.

3. Market Makers and Listing on the New York Stock Exchange

219. Eletrobras preferred ADS traded on the NYSE during the Class Period. Not only was trading therefore overseen by a designated market maker (DMM), but competing NASDAQ market makers were also able to facilitate preferred ADS trading.

220. From August 2010 through May 2015, there were at least 80 market makers for Eletrobras preferred ADS, including such well known firms as: Barclays, Goldman Sachs, Morgan Stanley, and UBS.[119]

221. At the time of the *Cammer* case opinion, NASDAQ market makers did not participate in the making of markets for NYSE stocks, but nonetheless, an NYSE listing and market making by the NYSE specialist was deemed sufficient to satisfy the market making factor. Over the course of the Class Period, the preferred ADS was listed on the NYSE and its market was also facilitated by numerous NASDAQ market makers.

4. F-3 Registration Eligibility

222. As explained above, a U.S. company is eligible for S-3 registration when, among other things, it has filed Exchange Act reports for a specified length of time and has outstanding float above a certain sizable value. Eligibility for form F-3 registration is the same as for S-3, except that the company must be a foreign private issuer (i.e. not a U.S. company), such as Eletrobras.[120] The *Cammer* Court noted that S-3 registration eligibility is indicative of market efficiency because the filing requirement ensured that financial data were available to market participants, and the "public float" requirement indicated that many market participants would have examined the information.[121]

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[119] Market maker data obtained from Bloomberg.

[120] "Eligibility of Smaller Companies to Use Form S-3 or F-3 for Primary Securities Offerings," SEC website, accessed at www.sec.gov/info/smallbus/secg/s3f3-secg.htm, 28 January 2008.

[121] *Cammer*, 711 F. Supp. at 1284-85.

a) Float

223. Eletrobras preferred ADS' float averaged $283.7 million during the Class Period, far exceeding the level required for F-3 registration. During the Class Period, float ranged between $49.1 million and $657.8 million. Though the Eletrobras preferred ADS float was below $75 million beginning in October 2014 the decline to this level followed the revelation of information related to Lava Jato, which is related to the alleged fraud at the center of this case. Furthermore, Eletrobras' total common equity float always ranged between $1.7 billion and $17.7 billion during the Class Period, far exceeding the threshold required for the Company to be eligible for F-3 registration. Even the size of Eletrobras common ADS float alone ranged from $118.0 million to $1.2 billion during the Class Period, so the Company always met the minimum size requirement for F-3 registration eligibility.

b) Financial Filings

224. As explained above, despite delayed SEC 20-F filings, the Company did regularly report financial results to analysts and investors and the Company was eligible for F-3 registration for more than half the Class Period.

225. To the extent that F-3 registration eligibility indicates company characteristics associated with market efficiency – size and availability of financial data – the Company possessed those characteristics throughout the Class Period, which promoted efficiency in the markets of all of the Eletrobras securities.

B. *Unger/Krogman* Factors

226. In addition to evaluating market efficiency using the *Cammer* factors, I also examined Eletrobras preferred ADS and its market with respect to the three additional *Unger/Krogman* factors.

1. Market Capitalization

227. During the Class Period, the market capitalization of Eletrobras preferred ADS averaged \$283.7 million. This value alone, independent of the Company's common and preferred shares that traded on the São Paulo and Madrid Exchanges, would place Eletrobras' preferred ADS in the 6th decile of U.S. companies by size – meaning that the Eletrobras preferred ADS was larger than at least 40% of all publicly-traded companies in the U.S.[122]

228. Consistent with the *Krogman* Court's opinion, the preferred ADS' sizeable market capitalization is further evidence of the efficiency of the market for Eletrobras preferred ADS.

2. Float

229. As mentioned above, Eletrobras preferred ADS float averaged \$283.7 million during the Class Period. The value of the float of Eletrobras preferred ADS is the same as the value of the entire 29.8 million ADS outstanding, because, according to the Company's SEC filings, none of the preferred ADS were held by insiders or affiliated corporate entities.[123]

230. Eletrobras' preferred ADS float was larger than the total market capitalization of at least 40% of all publicly-traded companies in the U.S.[124]

231. The size of Eletrobras' float indicates it satisfied the second *Unger/Krogman* factor for market efficiency. Eletrobras' preferred ADS float is indicative of the efficiency of the market for that security throughout the Class Period.

3. Bid-Ask Spread

232. I obtained data on daily closing bid and ask quotes for the Eletrobras preferred ADS during the Class Period from Bloomberg.

233. Exhibit-4b presents Eletrobras preferred ADS bid-ask spread data.

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[122] This calculation is based on averaged month-end data from CRSP for August 2010 through May 2015. I grouped public companies into deciles, so that the 1st decile contains the largest 10% of all public companies listed on the NYSE, AMEX, NASDAQ, and ARCA while the 10th decile contains the smallest 10%.

[123] Eletrobras 20-F filings for the Fiscal Years 2010 – 2015.

[124] This calculation is based upon averaged month-end data from CRSP for August 2010 through May 2015.

234. The average bid-ask spread for Eletrobras preferred ADS over the course of the Class Period was 0.29%. By comparison, the average month-end bid-ask spread over the course of the Class Period for all stocks in the CRSP database was 0.65%.[125] Eletrobras' preferred ADS bid-ask spreads were substantially narrower than the mean level among all CRSP stocks – which are stocks traded on the NYSE, AMEX, NASDAQ, and ARCA.

235. In dollar terms, Eletrobras' bid-ask spread during the Class Period averaged $0.02 per preferred ADS. For all stocks in the CRSP database, the average bid-ask spread during the Class Period was $0.10 per share.[126]

236. The average bid-ask spread in the market for the Eletrobras preferred ADS over the course of the Class Period was narrower than the typical bid-ask spreads exhibited by other publicly-traded stocks. The narrow bid-ask spread of Eletrobras' preferred ADS supports a conclusion of market efficiency.

IX. EMPIRICAL EVIDENCE OF MARKET EFFICIENCY FOR ELETROBRAS PREFERRED ADS

237. To test the efficiency of the market for the preferred ADS, I conducted the same two sets of empirical tests that I conducted on the common ADS. My regression model for the Eletrobras preferred ADS included the same variables used in the regression model for the Eletrobras common ADS, plus an additional variable, a preferred stock index variable to control for any potential general preferred stock market effects.

238. As explained above, on account of the equity buffer provided by the common ADS, preferred ADS tend to be less sensitive to company information than is common stock. Typically, only the most dramatic news affects the valuation of preferred equity to a major degree. As such, the financial filing delays I used to test the efficiency of the market for the Eletrobras common ADS may not be reasonable candidates for a study of preferred ADS market efficiency. The information conveyed on these dates would be of relatively moderate importance to Eletrobras preferred ADS investors, and reasonably would have only slight impact on the valuation of the preferred ADS, rather than eliciting statistically

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[125] Id.

[126] Id.

significant price changes. Nonetheless, for completeness, I included these events in the preferred ADS event study. I also tested the PM 579 events, which one would expect to be more valuation relevant for the preferred ADS, as the information released on these dates significantly altered the markets' expectations regarding Eletrobras' ability to pay its preferred dividend.

239. I included in the preferred ADS regression the return on the S&P Preferred Stock Index ("Preferred Stock Index"), which is a value-weighted average of a broad selection of preferred stock issues traded on American exchanges. The index level and return of the Preferred Stock Index are presented in Exhibit-9.

240. The regression I ran modelled the return of Eletrobras preferred ADS as a function of: 1) a constant term, 2) the returns of the CRSP Market Index, 3) the returns of the Brazilian Market Index, 4) the Peer Index return, 5) the daily logarithmic returns computed from the daily spot foreign exchange rate of the Brazilian real (Real/$), and 6) the Preferred Index return.[127]

241. This regression analysis isolates the impact of Company-specific information on Eletrobras' preferred ADS, removing the various potential market-wide, sector, currency, and general preferred equity effects.

242. Eletrobras' preferred ADS prices, trading volume, and returns are shown in Exhibit-4b. The regression results for the Class Period are presented in Exhibit-6ba. The regression results for Interval-1 and Interval-2 are presented in Exhibit-6bb and Exhibit-6bc, respectively.

A. Preferred ADS Event Study Results

243. As shown in Exhibit-7b, the Company's preferred ADS reacted in a statistically significant fashion, and in the same direction, on the same four PM 579 event dates on which the common ADS reacted significantly: 2 November 2012; 19 November 2012; 30 November 2012; and 3 December 2012. For the preferred ADS on 12 September 2012, just as for the common ADS, the nonsignificant stock price movement following this

[127] I repeated the regression and all tests making the adjustment for nonsynchronous data following the methodology in "Estimating Betas from Nonsynchronous Data," by Myron Scholes and Joseph Williams, *Journal of Financial Economics*, 1977. This analysis showed that the qualitative conclusions drawn from my event study are robust to this adjustment.

announcement is consistent with market efficiency. As reasonably expected, while the price of the of the preferred ADS fell substantially on the filing delay event dates (-3.4% and -2.3%, with *t*-statistics of -1.3 and -0.9, respectively), these declines were not so severe as to be over the threshold for statistical significance. The prompt price reactions to the PM 579 events were demonstrations of the preferred ADS trading efficiently. These findings prove that the preferred ADS security price reacted promptly to new information, and therefore the preferred ADS was efficient.

B. Preferred ADS Collective Event Study on Earnings Announcement Days

1. <u>Z-Test Analysis of Frequency of Significant Event Returns</u>

a) Z-Test Over the Class Period

244. As shown in Exhibit-11b, there were a total of 1,215 days during the Class Period on which Eletrobras preferred ADS traded, and of those days, 63 days had statistically significant residual returns. There were 5 statistically significant earnings announcement days among the 20 earnings announcement days for the Eletrobras preferred ADS. 58 of the remaining 1,195 non-news days were statistically significant. Accordingly, the proportion of statistically significant days in the news day sample is 25%, and the proportion of statistically significant days in the non-news sample is 4.85%. The incidence of statistically significant days within the news day group was therefore far greater than within the non-news group.

245. This difference in incidence frequency is associated with a z-score of 4.03, which is greater than the critical z-statistic threshold of 1.65 for significance at the 95% confidence level. This finding indicates that the incidence frequency difference between the two samples is too pronounced to accept the null hypothesis that the price of the Eletrobras preferred ADS behaves no differently on news days than on all other days.[128] The likelihood of obtaining a difference in incidence frequency of this magnitude and associated z-score given that particular explanation is less than 0.003%. Therefore, the difference in incidence frequency is deemed statistically significant. This finding proves

[128] The Z-test critical z-score threshold of 1.65 indicates statistical significance at the 95% confidence level for a one-tailed test. Here the critical test statistic is for a one-tailed test because the question at issue is whether news dates have a *greater* frequency of statistically significant returns than “non-news” dates.

that the preferred ADS reacted to information, and its market is therefore informationally efficient.

b) Z-Test Over Interval-1

246. As shown in Exhibit-11b, there were a total of 523 days during Interval-1 on which Eletrobras preferred ADS traded, and of those, 30 days had statistically significant residual returns. There were 2 statistically significant earnings announcement dates out of the 9 earnings announcement dates for the Eletrobras preferred ADS. Of the remaining 514 non-news days, 28 were statistically significant. Accordingly, the proportion of statistically significant days among the earnings announcement dates is 22.22%, and the proportion of statistically significant days among non-news days is 5.45%. There was a greater frequency of statistically significant days within the news day group than within the non-news day group.

247. This difference in proportions is associated with a z-score of 2.15, which is greater than the critical z-statistic threshold of 1.65 for significance at the 95% confidence level, indicating that the difference between the two samples is too large to accept the notion that the price of the Eletrobras preferred ADS behaved no differently on days with a greater flow of information than on all other days during Interval-1. The likelihood of obtaining a difference in incidence frequency of this magnitude and associated z-score given that particular explanation is only 1.619%. Therefore, the difference in incidence frequency is statistically significant over the course of Interval-1.

c) Z-Test Over Interval-2

248. As shown in Exhibit-11b, there were a total of 692 days during Interval-2 on which Eletrobras preferred ADS traded, and of those days, 40 days had statistically significant residual returns. There were 5 statistically significant earnings announcement dates out of the 11 earnings announcement dates for the Eletrobras preferred ADS. Of the remaining 681 non-news days, 35 were statistically significant. Accordingly, the proportion of statistically significant days among the earnings announcement dates is 45.45%, and the proportion of statistically significant days among non-news days is 5.14%. There was a

greater frequency of statistically significant days within the news day group than within the non-news day group.

249. This difference in incidence frequency is associated with a z-score of 5.68, which is greater than the critical z-statistic threshold of 1.65 for significance at the 95% confidence level, and indicates that the difference between the two samples is too severe to accept the notion that the price of the Eletrobras preferred ADS behaved no differently on days with a greater flow of information than on all other days. The likelihood of obtaining a difference in incidence frequency of this magnitude and associated z-score given that particular explanation is virtually nil. The difference in incidence frequency is statistically significant over the course of Interval-2.

2. <u>Preferred ADS F-Test and Ansari-Bradley Test Analyses of Event Return Dispersion</u>

a) *F-Test Over the Class Period*

250. For the preferred ADS, the sample standard deviation of the news day residual returns was 5.66% over the course of the Class Period. The sample standard deviation of all other returns was 2.13%. The news day sample standard deviation was greater than the sample standard deviation for all other days.

251. The F-statistic for the test is 7.1, which is greater than the 95% confidence level critical F-statistic value of 1.60 (with 19 and 1194 degrees of freedom), indicating that the difference in sample standard deviations is statistically significant and meaningful.

252. The F-test finds that for the preferred ADS the dispersion of news day returns during the Class Period is significantly greater than the dispersion of returns for all other days. This result demonstrates that the price of Eletrobras preferred ADS moved more on earnings announcement days than on other days. This statistical result indicates that there was a cause and effect relationship between the release of new information and reactions in the Eletrobras preferred ADS price, which therefore demonstrates that Eletrobras preferred ADS traded in an informationally efficient market during the Class Period.

b) F-Test Over Interval-1

253. For the preferred ADS, the sample standard deviation of the earnings announcement event day residual returns was 2.03% over the course of Interval-1. The sample standard deviation of all other days' returns was 1.38%. Again, the earnings announcement event day sample standard deviation was greater than the sample standard deviation for all other days.

254. The F-statistic for these two samples is 2.2, which is greater than the 95% confidence level critical F-statistic value of 1.96 (with 8 and 513 degrees of freedom), indicating that the difference in sample standard deviations is statistically significant and meaningful.

255. The F-test finds that for the preferred ADS the dispersion of earnings announcement day returns in Interval-1 is significantly greater than the dispersion of returns for all other days. This result demonstrates that the Eletrobras preferred ADS price moved more on earnings announcement days than on other days. This statistical result indicates that there was a cause and effect relationship between the release of new information and reactions in the Eletrobras preferred ADS price, which therefore demonstrates that Eletrobras preferred ADS traded in an informationally efficient market during Interval-1.

c) F-Test Over Interval-2

256. For the preferred ADS, the sample standard deviation of the earnings announcement event day residual returns was 7.57% over the course Interval-2. The sample standard deviation of all other days' returns was 2.53%. Again, the earnings announcement event day sample standard deviation was greater than the sample standard deviation for all other days.

257. The F-statistic for these two samples is 9.0, which is greater than the 95% confidence level critical F-statistic value of 1.84 (with 10 and 680 degrees of freedom), indicating that the difference in sample standard deviations is statistically significant and meaningful.

258. The F-test finds that for preferred ADS the dispersion of earnings announcement day returns in Interval-2 is significantly greater than the dispersion of returns for all other days. This result demonstrates that the price of the Eletrobras preferred ADS moved more on earnings announcement event days than on other days. This statistical result indicates that there was a cause and effect relationship between the release of new information and

reactions in the Eletrobras preferred ADS price, which therefore demonstrates that Eletrobras preferred ADS traded in an efficient market during Interval-2.

d) Ansari-Bradley Test Over the Class Period

259. The Ansari-Bradley C-statistic for the two samples of Eletrobras preferred ADS residual returns is 1.71, which is greater than the critical C-statistic threshold of 1.65 for significance at the 95% confidence level.[129] This result proves that the price of Eletrobras preferred ADS moved more on news dates than on other dates over the course of the Class Period.

260. This statistical result indicates that there was a cause and effect relationship between the release of new, Company-specific information and reactions in the Eletrobras preferred ADS price, which therefore further supports that Eletrobras preferred ADS traded in an efficient market during the Class Period.

e) Ansari-Bradley Test Over Interval-1

261. Applied to the earnings announcement event returns and the sample of all other returns observed during Interval-1, the Ansari-Bradley test fails to detect a statistically significant difference in the dispersion of earnings announcement event returns compared to the dispersion of returns on all other days. The Ansari-Bradley C-statistic for the two samples of Eletrobras preferred ADS residual returns is 0.2, which is less than the critical C-statistic threshold of 1.65 for significance at the 95% confidence level.

262. In light of the quantifiably greater sample standard deviation of residual returns displayed by the preferred ADS on earnings announcement days in Interval-1 and the significance of the Z-test and F-test results, it appears that the non-significance of the Ansari-Bradley test result is likely attributable to conditions affecting the power of the Ansari-Bradley test in this interval, rather than to potential market inefficiency. Nonetheless, this result by itself in isolation, is not supportive of a conclusion of market efficiency. However, the preponderance of evidence supports a conclusion of market efficiency for the Eletrobras preferred ADS during Interval-1.

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[129] The Ansari-Bradley critical C-statistic threshold of 1.65 indicates statistical significance at the 95% confidence level for a one-tailed test. Here the critical test statistic is for a one-tailed test because the question at issue is whether news dates have *greater* volatility than “non-news” dates.

f) Ansari-Bradley Test Over Interval-2

263. Applied to the earnings announcement event returns and the sample of all other returns observed during Interval-2, the Ansari-Bradley test finds with a high degree of statistical certainty that the dispersion of earnings announcement event returns was significantly greater than the dispersion of returns on all other days. The Ansari-Bradley C-statistic for the two samples of Eletrobras stock residual returns is 2.30, which is greater than the critical C-statistic threshold of 1.65 for significance at the 95% confidence level. This result is further proof that the price of Eletrobras preferred ADS moved more on earnings announcement event days than on other days during Interval-2.

264. This statistical test result indicates that there was a cause and effect relationship between the release of new, Company-specific information and reactions in the Eletrobras preferred ADS price, which therefore demonstrates that the Eletrobras preferred ADS traded in an efficient market during Interval-2.

C. Preferred ADS Market Efficiency Summary and Conclusion

265. The Eletrobras preferred ADS traded on the NYSE. Many market makers facilitated trading in the security. The Company was widely covered by analysts and the news media. Institutional ownership of the Eletrobras preferred ADS was widespread. Trading was very active. The average value of the outstanding issue over the course of the Class Period was large, larger than the entire market capitalizations of 40% of public companies. The security's bid-ask spread was narrow. Current and historical financial information about the Company was readily available to investors and analysts. While at times the Company did not qualify for F-3 registration on account of SEC filing delays, two of those delays were on account of issues related to Plaintiffs' allegations, and nonetheless, throughout the Class Period, information provided by analysts, news coverage, Brazilian financial filings, and the Company itself provided investors with access to financial information about the Company on a continuous basis. To the extent that F-3 registration eligibility indicates company characteristics associated with market efficiency, the Company clearly possessed those particular characteristics throughout the Class Period.

266. Not only did the Eletrobras preferred ADS satisfy the *Cammer* and *Krogman* factors that indicate market efficiency (with the F-3 exception noted above), but it also demonstrated the essence of market efficiency, satisfying the empirical *Cammer* factor.

267. Given these facts, I conclude that the Eletrobras preferred ADS traded in an efficient market throughout the Class Period.

X. EFFICIENCY OF THE MARKET FOR THE ELETROBRAS NOTES

268. As explained above, bonds have characteristics different from common and preferred stock. Therefore, the criteria relied upon by courts in assessing market efficiency for stock markets need to be applied differently to bond markets and courts have acknowledge this.[130] For example, most bonds are traded in an over-the-counter market, as opposed to on a centralized exchange.

269. The trading behavior in bond markets differs markedly from trading in the stock market. Typically, the bond market is made up of large institutional investors, pensions funds, and hedge funds that trade in much larger volumes per transaction than typical trades in the stock market.[131] Bond trades tend to be larger, but less frequent than typical stock trades. However, as the large trades are generally conducted by large institutions, the market participants are informed and trade decisions are backed by research and analysis. While the Eletrobras Notes may not have traded every day during the Class Period, this pattern is typical in the bond market, is not unique to Eletrobras Notes, and does not indicate an inefficient market.

270. Furthermore, as bonds are senior to common and preferred stock in the corporate capital structure, the prices of bonds from a particular company will not always react to company information in the same manner as the prices of common stock from the same company.[132]

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[130] See, e.g., in *In re Healthsouth Corporation Securities Litigation*, 261 F.R.D. 633 (N.D. Ala. 2009), the court found that the *Cammer* and *Krogman* factors were appropriate for evaluating the efficiency of the market for bonds, "while taking into account the differences in the trading of bonds from stocks."

[131] For example, a typical bond transaction is 50 times larger than a typical stock transaction. See, "An Empirical Study of Bond Market Transactions," by G. Hong and A. Warga, *Financial Analysts Journal*, Vol. 56, No. 2, March/April 2000.

[132] See, e.g., "What Does Nasdaq's High-Yield Bond Market Reveal about Bondholder-Shareholder Conflict?" by Gordon J. Alexander, et al., *Financial Management*, Vol. 29(1), Spring 2000, pp. 23–39; "Wealth Redistributions or

That is, when new information is released that causes a price reaction in the common stock of a company, the appropriate price reaction in the company's bonds could sometimes be in the opposite direction or no reaction at all.

271. Though I examine the same factors for the Eletrobras Notes that are relied upon by the courts to indicate market efficiency for common and preferred stock, the application and analysis of these factors is modified in consideration of the characteristics that differentiate the bond markets from the stock markets.

272. For the Eletrobras Notes, I obtained FINRA's historical trading data from counsel. Though the AG1 Notes were not issued until 27 October 2011,[133] according to the trading data, trades were executed as early as 20 October 2011 with settlement on the issue date of 27 October 2011. The tests described herein for the AG1 Notes were performed over the period during the Class Period when trading data was available, 20 October 2011 through 24 June 2015.

A. About Rule 144A Notes

273. According to the SEC website, "under Section 5 of the Securities Act of 1933, all offers and sales of securities must be registered with the SEC or qualify for some exemption from the registration requirements."[134] Notes that are not registered with the SEC and are exempt from the typical registration requirements are commonly referred to as Rule 144A issues. Notes issued under Rule 144A can only be traded by QIBs, which are defined as entities "acting for its own account or the accounts of other qualified institutional buyers, that in the aggregate owns and invests on a discretionary basis at least $100 million in securities of issuers that are not affiliated with the entity."[135]

Changes in Firm Value: An Analysis of Returns to Bondholders and Stockholders Around Dividend Announcements," by George Handjinicolaou, and Avner Kalay, *Journal of Financial Economics* (March 1984) pp. 35–63.

[133] Eletrobras Listing Circular, 27 October 2011.

[134] https://www.sec.gov/answers/rule144 htm.

[135] "Rule 144A–Private Resales of Securities to Institutions," *Securities Lawyer's Deskbook*, The University of Cincinnati College of Law.

274. Companies issue debt under Rule 144A in order to obtain financing swiftly, without the need for a registration statement, which potentially secures more attractive financing terms for the issuer when market conditions are favorable than would otherwise be possible. In an 144A issue, the price of the notes is set by the underwriters after polling QIBs for their level of interest and pricing expectations.[136] In the pricing process, the underwriter and investors will perform the same analyses as analysts do in the secondary market (e.g. comparative companies analysis, earnings potential, expected growth, and macro-economic analyses etc.). Thus, the pricing of an 144A issue is driven by the same valuation considerations as the pricing of a registered issue in the secondary market.

275. Prior to 30 June 2014, transaction data for corporate debt securities trading under Rule 144A was not publicly available through TRACE in real time. Since 30 June 2014, FINRA, through TRACE, began to publicly disseminate Rule 144A transaction data for corporate debt securities.[137]

B. *Cammer* and *Krogman* Factor Analysis

276. To investigate whether the market for the Eletrobras Notes was efficient, I examined the factors that are generally accepted by both financial economists and the courts to be indicative of market efficiency for publicly-traded securities, taking note of the distinctive features of bond investing and the bond market that distinguish bonds from common stock.

277. I also tested the efficiency of the market for the Eletrobras Notes empirically.

278. As established above, Eletrobras was not a small, obscure company, which escaped the notice of market participants. Rather, over the entire time the Eletrobras Notes traded, Eletrobras was a very large, highly visible, and well covered Company – facts that support a conclusion of market efficiency for the Notes, as well as for the common and preferred ADS.

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[136] "Corporate Bond Operational Underwriting Process: Business Practices in 'Plain English,'" *The Bond Market Association*, 9 December 2004, p. 4 & p. 12.

[137] "FINRA Brings 144A Corporate Debt Transactions into the Light," FINRA, press release, 30 June 2014.

1. Analyst Coverage and Other Avenues of Information Dissemination

279. As noted above, based on a review of available analyst reports and conference call transcripts, at least 18 analysts covered Eletrobras during the Class Period, with at least 13 publishing analyst reports. The information and analysis this broad coverage provided promoted the efficiency of the market for the Eletrobras Notes.

280. Over 3,700 news articles about Eletrobras were published in English during the Class Period. The news media facilitated the flow of information about the Company to the marketplace. Bond investors had easy access to news about the Company, which promoted the efficiency of the market for the Eletrobras Notes.

2. Credit Rating Agencies

281. Bond ratings and ongoing surveillance coverage were provided by the three major bond rating agencies, Fitch, Moody's, and Standard & Poor. These agencies performed analysis, rating, and surveillance on the Eletrobras Notes throughout the Class Period, and commented in the financial press about the Notes. Consequently, not only was the Company well covered throughout the Class Period, but so too were the Eletrobras Notes specifically.

282. Throughout the Class Period, the rating agencies published detailed research and credit analyses on the Eletrobras Notes. Of particular note, Moody's and Fitch published detailed research and credit analyses focusing on both of the Eletrobras Notes, in addition to the Company as a whole. For example, Fitch provided the following analysis on 7 November 2012.

> "The Brazilian Government's offer through its Ministry of Mines and Energy (MME) for an early renewal of some expiring electricity concessions will have various impacts for different companies, according to Fitch Ratings. … Fitch considers the offer negative for Centrais Eletricas Brasileiras S.A. (Eletrobras, IDR 'BBB') and neutral to negative for other affected companies. The government's proposal for generation and transmission revenues would significantly reduce Eletrobras' and Companhia de Transmissao de Energia Eletrica Paulista S.A.'s (CTEEP; IDR 'AA+(bra)') cash flow generation. Companhia Paranaense de Energia (COPEL; IDR 'AA+(bra)') could see its EBITDA decline by 10% to 15% beginning in January 2013, if they accept the government's offer given

> these expiring concessions represent only a small proportion of their total cash generating assets. Companhia Energetica de Minas Gerais (CEMIG; IDR 'AA(bra)') will also have a EBITDA reduction of 10% to 15% beginning in January 2013 if they accept the government's offer. This will increase to around 30% after 2015, as Cemig has three important hydroelectric plants with concessions expiring during this period and will not be renewed early under the government's scheme. In general, the government's proposals will limit available funds to reinvest internal cash flow generation back into the electricity sector and reduce their ability to access debt capital markets and bank financing."
> **"Fitch: Brazil Concession Renewal Offer Negative for Eletrobras, Marginal for Others," *Business Wire*, 7 November 2012.**

283. To arrive at bond ratings, these ratings agencies perform detailed analyses of bond issuers, taking into account a vast array of business and financial information. According to S&P literature, S&P will "assign a rating only when adequate information is available."[138]

284. The rating agencies monitor and periodically update company ratings. As described in the following quote from the S&P website, S&P collects and disseminates material information about the companies it rates:

> "Once a rating is assigned, we maintain on-going review of material factors that could affect the rating, such as changes in the capital structure, an acquisition or other major economic developments. Generally, an issuer credit rating is reviewed formally at least once a year at the time of a meeting with the issuer's management. We expect management to provide to us prompt notice of material financial and operational changes that could affect the rating."
> **Credit Ratings Fact Sheet, on www.standardandpoors.com, 2 February 2007.**

285. Coverage and reports by Fitch, Moody's, and S&P are channels through which information and analysis about Eletrobras reached investors, bond investors in particular. This infrastructure for disseminating information and analysis fosters the efficiency of the market for the Eletrobras Notes. Table-2 presents Fitch's ratings for Eletrobras Senior Unsecured Notes throughout the Class Period.[139]

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[138] http://www2.standardandpoors.com/spf/pdf/media/credit_ratings_fact_sheet_020507.pdf.

[139] I focused on Fitch's ratings for the Eletrobras Notes as Fitch was the first of the three credit rating agencies to adjust their ratings for Eletrobras following a major news event during the Class Period.

Table-2: Fitch's Credit Rating History

Class Period: 17 August 2010 through 24 June 2015

Senior Unsecured

Date	Rating
6/29/2009	BBB-
4/6/2011	BBB
12/7/2012	BB

Source: Bloomberg.

3. Institutional Ownership

286. As the Rule 144A notes were held and traded solely by QIBs, the entirety of the issues would have been institutionally owned. In the prospectus filed for the exchange of the 144A notes, the Company stated that the private placement of the AF3 Notes was initially sold to Credit Suisse[140] and the private placement of the AG1 Notes was initially sold to Credit Suisse and Santander.[141] The fact that $2.75 billion (face value) of the 144A notes were issued to, owned by, and traded among institutional investors, is compelling support that they traded in an efficient market.

287. In addition to these 2 banks, at least 133 and 139 institutions owned and traded the AF3 Notes and AG1 Notes, respectively, according to FINRA trading data obtained from counsel. Institutional investors generally employ financial analysts and portfolio managers who conduct their own research on securities and who make investment decisions based on that research.

288. The 100% institutional ownership of the Eletrobras Notes weighs in favor of a finding of market efficiency for the Notes.

[140] Eletrobras Listing Circular, 31 July 2009, p. 138.

[141] Eletrobras Listing Circular, 27 October 2011, p. 138.

4. F-3 Registration Eligibility

289. To the extent that F-3 registration eligibility indicates company characteristics associated with market efficiency, the Company clearly possessed those particular characteristics throughout the Class Period. These characteristics foster the efficiency of the market for the Eletrobras Notes, just as they did for the common and preferred ADS.

5. Prominent Underwriters and Market Makers

290. The Eletrobras Notes were placed into the financial marketplace by prominent underwriters. The AG1 Notes were underwritten by Credit Suisse and the AF3 Notes were underwritten by both Credit Suisse and Santander, which are among the financial markets' largest investment banks.[142] According to the Company's listing circulars for the Eletrobras Notes, Deutsche Bank also acted as the trustee, transfer agent, paying agent, and registrar for the notes.[143]

291. Typically, investment banks that underwrite notes subsequently serve as market makers in the issues.

> "It would be a mistake to think that once the bonds are all sold, the investment banking firm's ties with the deal are ended. Those who bought the bonds will look to the investment banking firm to make a market in the issue. This means that the investment banking firm must be willing to take a principal position in secondary market transactions."
> **"The Primary and Secondary Bond Markets," by Frank J. Fabozzi and Frank J. Jones, chapter 3, in *The Handbook of Fixed Income Securities*, 7th edition, edited by Frank J. Fabozzi and Steven V. Mann, McGraw-Hill, 2005, p. 33.**

292. Additionally, many investment banks that published analyst reports covering the Eletrobras common ADS, stated in those reports that they also make a market in Eletrobras securities. For example, HSBC, Itau, and UBS published reports stating that they made a market in Eletrobras securities during the Class Period.

293. Based on the number of firms that underwrote the Eletrobras Notes, the fact that underwriters typically make markets in the securities they underwrite, and the disclosures that analyst firms made markets in Eletrobras securities, it appears that there were

[142] Eletrobras Listing Circular, 31 July 2009, p. 1; and Eletrobras Listing Circular, 27 October 2011, p. 1.

[143] Eletrobras Listing Circular, 31 July 2009, p. 11; and Eletrobras Listing Circular, 27 October 2011, p. 11.

Case 1:15-cv-05754-JGK Document 83-1 Filed 10/26/17 Page 40 of 74 PageID #: 19126

numerous market makers for the Eletrobras Notes. Additional evidence confirming a developed market-making infrastructure is the considerable trading volume in the Eletrobras Notes over the course of the Class Period.

6. Outstanding Par Value and Float

294. The *Cammer* and *Krogman* opinions cite a company's large market capitalization as being indicative of market efficiency for common stock. This factor, which fosters market efficiency for the common and preferred shares, would similarly foster efficiency in the market for the Eletrobras Notes, as size attracts attention and visibility.

295. Beyond Eletrobras' large common and preferred ADS market capitalizations, the total par value of the Eletrobras Notes was larger than the market capitalizations of most publicly-traded companies. The aggregate par value of the Eletrobras Notes totaled $2.75 billion and was larger than the market capitalizations of at least 80% of all public companies listed on the NYSE, AMEX, NASDAQ, and ARCA during the Class Period.[144]

296. The outstanding par value of the AF3 Notes and AG1 Notes was $1.0 billion and $1.75 billion, respectively, both of them were individually larger than the market capitalizations of most publicly-traded companies.[145]

297. The Company's public filings did not indicate that any substantial portion of the Company's outstanding Notes were held by insiders. Consequently, the float of the Eletrobras Notes equaled the amount outstanding. Just as the outstanding par values were very large, so too were the floats. The substantial outstanding par value and float are indicative of the efficiency of the market for the Eletrobras Notes during the Class Period.

[144] This calculation is based on averaged month-end data from CRSP between October 2011 and May 2015. I performed my calculation by grouping public companies into deciles, so that the 1st decile contains the largest 10% of all public companies listed on the NYSE, AMEX, NASDAQ, and ARCA while the 10th decile contains the smallest 10%. The aggregate par value of the AF3 Notes totaled $1.0 billion and was larger than the market capitalizations of at least 70% of all public companies listed in U.S. during the period August 2010 through September 2011.

[145] The court in the *In re Enron Corp. Sec., Derivative & "ERISA" Litig.*, 529 F. Supp. 2d 644 (S.D. Tex. 2006), found that large aggregate outstanding par amount was indicative of bond market efficiency.

7. Trading Volume

298. As discussed above, securities that are more senior in a company's capital structure, such as bonds, have more predictable cash flows and generally have more certain and stable valuations than do common stock. As a result, bonds will typically exhibit less price volatility and trade less frequently than a company's common stock. Nonetheless, the Eletrobras Notes traded actively during the Class Period.

299. I examined the trading volume for both of the Eletrobras Notes.[146] Appendix-2 describes my organization and treatment of the TRACE data.

300. The weekly average trading volume for the AF3 Notes and AG1 Notes was 1.45% and 3.41%, respectively. Using the thresholds applicable to common stock, which are understood to trade more frequently than bonds, the Class Period average weekly volume of the AF3 Notes exceeded the 1% threshold for a substantial presumption of market efficiency and the average weekly volume of the AG1 Notes exceeded the 2% threshold for a strong presumption of market efficiency.

301. The market for Eletrobras Notes was very active. Trading volume was high, even relative to the standard for common stock. Consistent with the *Cammer* opinion, economic theory, and published research, this fact is compelling evidence of market efficiency.

8. Trade Frequency

302. The trading frequency of the Eletrobras Notes, in addition to their trading volume, is further evidence of their market efficiency. A published, peer-reviewed, study by Mahanti et al. [2008] observes that relatively few corporate bonds trade more frequently than 200 days in a year.[147] According to the study, the bonds examined typically trade every 12-14 days.[148] I measured the frequency of the trading in the Eletrobras Notes in order to compare the trading behavior of the Eletrobras Notes with that of the bonds in the Mahanti et al. study.

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[146] I obtained FINRA trading data for the Eletrobras Notes from counsel. Weekly trading volume for each Note as a percentage of the outstanding issue is computed by summing the face amount traded, dividing by the number of weeks in the Class Period, and then dividing by the size of the outstanding issue.

[147] "Latent Liquidity: A New Measure of Liquidity, With an Application to Corporate Bonds," by Sriketan Mahanti, et al., *Journal of Financial Economics*, 2008, p. 278. The Mahanti et al. analysis is presented in Exhibit-10a.

[148] Id., p. 282.

303. I calculated the average number of days between successive trades for each of the Eletrobras Notes. The results are presented in Exhibit-10b.

304. For the AF3 Notes, the average number of days between successive trades was 0.15 days over the Class Period, meaning that this typically trade numerous times per day rather than once every few days. For the AG1 Notes, the average number of days between successive trades was 0.06 days over the portion of the Class Period that it traded, meaning that it too traded very frequently. As shown in Exhibit-10a and Exhibit-10b, both of the Eletrobras Notes traded more frequently than even the first decile of frequently traded bonds in the Mahanti et al. study. The trading frequency of the Eletrobras Notes is evidence that the Notes traded in a well-developed market and is compelling evidence of market efficiency.

XI. <u>EMPIRICAL TESTS OF THE ELETROBRAS NOTES MARKET EFFICIENCY</u>

305. Because of their senior status, bond values are substantially insulated from all but the most extreme news by a valuation cushion provided by the common and preferred stock. As a result, bonds are the least sensitive of all securities to firm specific news while being the most sensitive to a change in a firm's probability of default.[149] Moreover, the Brazilian Government was the Company's majority shareholder, controlling approximately 51% of Eletrobras outstanding voting shares during the Class Period. While the Brazilian government was not obligated to guarantee any securities or obligations on the Company's behalf, there was a widespread perception of an implied credit guarantee by the Brazilian government. As such, the Eletrobras Notes were even further insulated from volatility and all but the most adverse information.

306. Both of the credit ratings agencies that published reports on the Eletrobras Notes explicitly linked the Company's credit ratings to those of the Brazilian Government. The ratings agencies also stated that the Company's ratings would be impacted by the ratings of the government, and that the Company's standalone rating would be adversely impacted by "any evidence of lack of financial support from the Federal Government."

[149] See, e.g., "Understanding the Efficiency of the Market for Preferred Stock," by Michael Hartzmark and H. Nejat Seyhun, *Virgina Law and Business Review*, Spring 2014, pp. 9-10.

> "Eletrobras' and Furnas' ratings are linked to those of the Federal Republic of Brazil. Therefore, any rating action related to Brazil would impact these ratings. Moreover, given Eletrobras' weak credit metrics for the assigned rating category on a standalone basis, any evidence of lack of financial support from the Federal Government could result in a negative rating action."
> **"Fitch Affirms Eletrobras and Furnas' IDRs at 'BBB'; Outlook Stable," *Business Wire*, 6 September 2012.**

307. It follows that the dates selected for the empirical test performed on the Eletrobras common ADS and preferred ADS would not constitute good event candidates for a market efficiency study on the Eletrobras Notes. As the ratings agencies confirmed, the information reported on most of the event dates tested in those studies was of only moderate importance to Eletrobras Note investors, and reasonably would not elicit statistically significant price changes. Consequently, I performed the event study on 7 December 2012 and 10 December 2012, as these were days on which new information was released that could reasonably have significantly altered the markets' expectations regarding the level of financial support the Brazilian government would be willing to provide to Eletrobras, such that the Company's ability to repay its debts could be affected.

308. The news on these dates include the following information.

i) **7 December 2012** – on this day, Fitch issued a press release announcing that it was downgrading Eletrobras' issuer debt rating, the credit rating on the Company's AF3 Notes, and the credit rating on the Company's AG1 Notes, from BBB to BB.[150] The press release explained: "The downgrade reflects the highly negative impact on Eletrobras' credit quality due to its decision to accept the early renewal of all of its generation and transmission electric concessions that expire between 2015 and 2017. … The new concessions would significantly lower revenues, which will result in zero to negative EBITDA for Eletrobras. … Fitch has also weakened its ratings linkage between the Government and Eletrobras given the Government's

[150] "Fitch Takes Rating Actions on Certain Brazilian Power Companies," *Business Wire*, 7 December 2012, 12:35 PM.

indication that it will not support Eletrobras either through subsidies or equity contributions. While Fitch ultimately believes the Government will be forced to support Eletrobras over the medium term, credit default risks are higher given the government's lack of a clear plan to support the company."[151]

ii) **10 December 2012** – on this day, *Valor Economico* published an article commenting that Eletrobras will require more debt to restructure its subsidiaries following MP 579. The article states: "[o]ne of the main effects of MP 579 on the six federalized distributors Controlled by Eletrobras is that they will need to obtain resources from outside the state for investments and costing in the next two years. Without being able to count on the The Global Reversion Reserve (RGR) as a source of financing, since they will be extinguished after being used to pay the indemnities, Eletrobras will need loans from BNDES and the National Treasury for the restructuring of these Companies."[152]

309. I also considered two other dates when ratings agencies changed Eletrobras' issuer credit rating: 6 April 2011 and 19 December 2012. On 6 April 2011, Fitch upgraded Eletrobras' credit rating from BBB- to BBB and explained that its "rating actions follow the recent upgrade of Brazil's long-term foreign and local currency IDRs to 'BBB' from 'BBB-'."[153] As the Fitch upgrade was not Company-specific, but rather reflected a Brazilian market upgrade, it would not be a good candidate event for a market efficiency event study. On 19 December 2012, Moody's downgraded Eletrobras due to the Company's "decision to accept the federal government's offer contemplated in the federal government's provisional measure #579."[154] The Moody's downgrade followed Fitch's and cited the same reason Fitch provided twelve days earlier. As such, this event would not be expected

[151] Id.

[152] "Brazil: Eletrobras Settles Part of The Debt Owed to Petrobras," *Valor Economico*, 10 December 2012.

[153] "Fitch Upgrades Eletrobras and Furnas to 'BBB'; Outlook Stable," *Business Wire*, 6 April 2011.

[154] "Moody's downgrades Eletrobras' ratings to Baa3; outlook remains negative," *Moody's Investor Service*, press release, 19 December 2012.

to move the price of the Eletrobras Notes by a statistically significant amount and would therefore not be a good candidate event for a market efficiency event study.

A. Note Regression Analysis

310. To compute Eletrobras Notes' residual returns, I ran a regression modeling the daily return of the note as a function of: 1) a constant term, 2) the returns of the CRSP Market Index, 3) the returns of the Brazilian Market Index, 4) the Peer Index return, 5) the daily logarithmic returns computed from the daily spot foreign exchange rate of the Brazilian real (Real/$), and 6) the return of a market benchmark bond ("Benchmark Bond"). The Benchmark Bond return variable in the regression model controls for the effect of changes in the market interest rate.

311. The daily returns of the Benchmark Bond were computed by pricing each day a hypothetical bond with the same coupon and maturity as the respective Eletrobras note, applying the yield-to-maturity from a market index matched to the Company's credit rating. The market index yield was drawn from the BofA Merrill Lynch US Corporate BBB Index, and the BofA Merrill Lynch US Corporate BB Index ("the Merrill Indices") with the same rating as the rating assigned by Fitch to Eletrobras at each point in time. When the Company's credit rat:1ng was between the ratings of the Merrill Indices, I interpolated between the yields of the two Merrill Indices straddling the Company rating.[155]

312. For example, on 17 August 2010, the Company carried a Fitch rating of BBB-. The yield from the BofA Merrill Lynch US Corporate BBB Index was 4.354% that day and the yield from the BofA Merrill Lynch US Corporate BB Index was 6.683% that day. Therefore, the yield of the Benchmark Bond for the Eletrobras Notes was 5.130%, in between the respective yields of the BBB and BB indices. This yield was then used to price benchmark bonds with the same coupon and maturities for both of Eletrobras Notes. The next trading day, 18 August 2010, the BBB-rated index yield fell to 4.343% and the BB rated index yield fell to 6.637%, resulting in a new yield for the Benchmark Bond of 5.108%. This yield was then used to reprice the benchmark bonds. For this sequence of yields, the

[155] Data for the Merrill Indices obtained from BofA Merrill Lynch via Bloomberg.

logarithmic return of the Benchmark Bond for the AF3 Notes on 18 August 2010 was 0.15%. In this manner, daily returns for the Benchmark Bond for both of the Eletrobras Notes were computed.

314. Exhibit-8 presents the yields of the BofA Merrill Lynch US Corporate BBB Index and the BofA Merrill Lynch US Corporate BB Index used to compute the Benchmark Bonds.

1. <u>Running the Regression</u>

315. Using trading data for the Eletrobras Notes, I performed a regression analysis for both of the Notes on daily returns covering the entire portion of the Class Period when each respective security was traded.

316. As is common in the bond market, for each Eletrobras Note there were some days on which no trading took place. For both of the Notes, the regression data series included only days on which there was a trading price for two consecutive trading days, so that a one-day return could be computed. All returns for the Eletrobras Notes and Benchmark Bonds are one-day logarithmic returns.

317. Exhibit-6c and Exhibit-6d present the regression results for the AG1 and AF3 Notes, respectively.

2. <u>Event Study Results</u>

318. As shown in Exhibit-7c, both Eletrobras Notes exhibited statistically significant declines on 7 December 2012 and 10 December 2012. These results demonstrate market efficiency as the Notes reacted promptly to Company-specific news that was important for the valuation of the Notes.

319. The event study shows that for the events tested, there were strongly statistically significant price reactions by the Notes to new Company-specific news. These findings demonstrate a cause and effect relationship between the release of information and movements in the price of the Eletrobras Notes, which is the hallmark of an efficient market.

B. Further Empirical Evidence of Bond Market Efficiency

320. In addition to the bond event study results, the bond regressions provide additional compelling evidence of the efficiency of the market for the Eletrobras Notes. The fundamental value of a bond changes with Company information (e.g. credit risk), and also on a day-to-day basis in response to changes in the market interest rate. A higher market interest rate would make a fixed coupon bond less attractive to investors, and should therefore depress its price, all else equal. Alternatively, a lower market interest rate should boost the bond price, all else equal.

> "We have seen already that an inverse relationship exists between bond prices and yields, and we know that interest rates can fluctuate substantially. As interest rates rise and fall, bondholders experience capital losses and gains."
> ***Investments*, 8th edition, by Zvi Bodie, et al., McGraw-Hill Irwin, 2009, p. 513.**

321. In an efficient market, investors would take note of market interest rate changes and such changes would be incorporated into the trading prices of the Notes.

322. The Benchmark Bond return variable in the regression models reflects changes in market interest rates. The regression results show that both of the Notes moved significantly with the Benchmark Bond return. The Benchmark Bond return is a significant explanatory variable of the Eletrobras Note returns at an extremely high level of statistical significance. This finding is compelling proof of the efficiency of the market for the Eletrobras Notes throughout the Class Period. The day-to-day cause-and-effect relationship thusly demonstrated between changes in market interest rates and the prices of the Eletrobras Notes, proves that market participants did not ignore valuation relevant information when pricing and trading Eletrobras Notes. This fact proves that the market for Eletrobras Notes was efficient throughout the Class Period.

C. Note Efficiency Summary and Conclusion

323. The following factors indicate that the Eletrobras Notes traded in an efficient market: coverage by analysts and ratings agencies; widespread news coverage; Company characteristics associated with F-3 registration eligibility; prominent and numerous underwriters; the large size of the issues, individually and in the aggregate; active trading

volume; frequent trading; large trade size; and the general size and activity of the U.S. bond market.

324. Not only did the Eletrobras Notes exhibit the above-mentioned factors that indicate market efficiency, but they also satisfied the empirical *Cammer* factor, which established that the Eletrobras Notes traded in an efficient market. The event studies proved that there was a cause and effect relationship between the release of allegation-related information and movements in Eletrobras Notes prices. The regression analysis proved that the bond prices moved appropriately with interest rates over the course of the Class Period.

325. Given these facts, I conclude that the Eletrobras Notes traded in an efficient market over the course of the Class Period.

XII. COMMON DAMAGE METHODOLOGY

A. Section 10(b) Damage Methodology

326. Plaintiffs' counsel asked me to opine on whether damages could be measured for each Class member under Section 10(b) of the Exchange Act using a common methodology for all Class members.

327. It should be noted that I have not conducted a loss causation analysis at this time and reserve the right to address such issues at the appropriate stage. The loss causation analysis that will be necessary to actually calculate damages in the current case requires the full development of the record.

328. Nonetheless, the methodology discussed herein allows the calculation of individual and class-wide damages stemming from the various alleged misrepresentations and omissions and therefore will accommodate alternative potential determinations of liability. Economic analysis (including valuation and empirical event study analysis) can be used to estimate the relationship between specific statements or sets of statements and the subsequent effect on prices, in the case of affirmative statements, omissions, and/or corrective disclosures. As such, class-wide damages caused by the specific misrepresentations and omissions ultimately established by the Plaintiffs can be calculated in a straightforward manner common to all Class members. Out-of-pocket damages can be measured as the difference between the amount of security price inflation at purchase and the amount of

inflation in the security price at sale taking into account formulaic prescriptions and caps in relevant case law and statutes.

329. Assuming a Plaintiffs' verdict on the allegations of fraud, Section 10(b) per share (or note) damages, respectively for each investor and security, can be measured as follows:

i) First, valuation tools, which would include event study analysis such as that described herein, and potentially other empirical analyses if necessary, would be used to establish if the disclosure(s), correcting the alleged misrepresentations and omissions, caused the prices of Eletrobras securities to fall. This analysis, after controlling for potentially non-fraud-related information, would establish whether the alleged misrepresentations and omissions had caused the security price to be artificially inflated, and if the corrective disclosure(s) caused the inflation to dissipate, in turn causing investor losses. This analysis would apply on a class-wide basis for all securities.

ii) Second, an inflation ribbon would be constructed for each security, using generally accepted empirical analysis and valuation tools, indicating how much artificial inflation caused by the alleged misrepresentations and omissions was in the price of the Eletrobras securities on each day during the Class Period. An inflation ribbon is a time series of the difference between a security's actual price observed in the marketplace, and the estimated price that the security would have traded at each day had there been full disclosure from the outset of the Class Period. Construction of the inflation ribbon generally employs event study analysis, combined with widely used and generally accepted valuation tools and models. The inflation ribbon is often constructed by working chronologically backwards from the final corrective disclosure to the start of the Class Period, accounting for alleged fraud-related residual price declines as they occurred. Inflation prior to a corrective disclosure that dissipated inflation is greater than the inflation afterward by the amount of inflation that dissipated. The full array of generally accepted and widely used

valuation tools can be applied, if necessary, to calculate the but-for security prices under the assumption of prior full disclosure. This analysis would apply on a class-wide basis.

iii) Third, the measure of per share (or note) damages generally applied in 10(b)-5 cases is the reduction in the inflation ribbon over an investor's holding period (the economic/inflation loss) that was caused by corrective disclosures. That is, for each Class member, per share (or note) damages would be calculated as the difference between the inflation on the date the securities were purchased and the inflation on the date those same securities were subsequently sold, excluding any inflation dissipation caused by factors other than corrective disclosure. Per share (or note) damages are also limited, however, to be no greater than the decline in security price over the holding period, which is the investment loss actually sustained. Pursuant to the Private Securities Litigation Reform Act of 1995 (the "PSLRA") (15 U.S.C. § 78u-4(e)), for any shares (or note) sold during the 90-day period after the end of the Class Period, per share (or note) damages would be calculated as the lesser of the reduction in the dollar inflation over the investor's holding period (the economic/inflation loss), or the decline in the security price (the investment loss), where the terminal security price is deemed to be the average price from the final corrective disclosure date to the sale date. Also pursuant to the PSLRA, for any securities held 90 days or more beyond the final corrective disclosure, damages would equal the lesser of the reduction in the dollar inflation over the investor's holding period (the economic/inflation loss) or the decline in the security price (the investment loss), where the terminal security price is deemed to be the average price over the 90 days following the final corrective disclosure. The calculation of each Class member's damages would be a mechanical arithmetical exercise, conducted the same way for all Class members, applying the results of the

Class-wide analyses described above to each Class member's trading data.

330. Consequently, each Class member's damages under Section 10(b) can be computed in the same way, common to all Class members, using readily available daily pricing information, in accordance with widely used and generally accepted methodologies and the PSLRA.

331. I have not yet been asked to calculate damages for any of the claims alleged on behalf of the Class, and such calculations will likely depend, in part, on the completion of discovery. However, the methodology described above is generally accepted and widely used for calculating damages under Section 10(b) consistently on a Class-wide basis in securities class actions.

XIII. LIMITING FACTORS AND OTHER ASSUMPTIONS

332. This report is furnished solely for the purpose of court proceedings in the above referenced matter and may not be used or referred to for any other purpose. The analysis and opinions contained in this report are based on information available as of the date of this report. I reserve the right to supplement or amend this report, including in the event additional information becomes available.

Steven P. Feinstein, Ph.D., CFA

<u>Exhibit III</u>

Pages from statistics textbook cited by Dr. Feinstein in his report:

Applied Statistics for Public Policy

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 312 of 370. PageID #: 19189

APPLIED STATISTICS
FOR PUBLIC POLICY

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Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 314 of 370. PageID #: 19141

asking if the two groups are different or the same. Again, if we do not reject the null hypothesis, we are probably suggesting that the two samples are actually from the same population.

Understanding Hypothesis Testing for Two Proportions

As an example, the following could be hypothesized statements that investigate the existence of differences between population proportions:

1. The proportion of men versus the proportion of women who oppose abortion.
2. The percent of patients who responded favorably to a new experimental drug versus the percent who responded favorably to a placebo.
3. The proportion of Republicans versus the proportion of Democrats who oppose lowering environmental standards.
4. The percent of adults who smoked cigarettes in 1990 versus the percent who smoked cigarettes in 2000.
5. The percent of trains that arrived on schedule before a new program was put into place versus the percent of trains that arrived on schedule after the new program.
6. The proportion of urban students who failed a statewide proficiency test versus the percent of suburban students who failed a statewide proficiency test.

Similar to what was learned in Chapter 12, it is unlikely that we will ever know *exactly* what any population proportion π truly equals. As a point estimator, we randomly survey a sample from each population and use the sample proportion statistic ρ to predict what π might be. Of course, we still need to recognize that sample proportions are a matter of statistical inference and probability, and will rarely be equal to the true unknown population proportions. Therefore, they may not exactly be the same. We can test, however, whether differences between the hypothesized population proportions are truly different or if they differ due to random chance.

Although the method we will use to make this determination is similar to the six-step process covered in Chapter 12, some minor adjustments need to be made. The first (and obvious) adjustment is that we are comparing two sample proportions as opposed to comparing a single sample proportion to a hypothesized population proportion. The other two adjustments are somewhat theoretical and concern the standard error of proportion and sample size.

The Pooled Standard Error of a Proportion

The primary difference between a hypothesis test of a sample proportion and unknown population proportion (such as that covered in Chapter 12) is the standard error of a proportion. To calculate the test statistic for a single sample proportion, we use Equation (12.1). To calculate a z-score to test for differences between population proportions, we create a *pooled standard error of proportion*. The formula for the two-proportion test statistic is:

$$z = \frac{p_1 - p_2}{\sqrt{\frac{\pi_p\left(1-\pi_p\right)}{n_1} + \frac{\pi_p\left(1-\pi_p\right)}{n_2}}}. \tag{13.1}$$

where p_1 = proportion of first sample,
p_2 = proportion of second sample,
n_1 = size of first sample,
n_2 = size of second sample, and
π_p = weighted pooled proportion.

The weighted pooled proportion π_p is calculated as:

$$\pi_p = \frac{p_1 * n_1 + p_2 * n_2}{n_1 + n_2}. \tag{13.2}$$

Sample Size

Similar to the single population proportion test covered in Chapter 12, when testing for differences between population proportions, we cannot test hypotheses involving proportions from small samples. When testing for differences between population proportions, three sample size criteria need to be satisfied, which are shown below.

In Chapter 12, we saw how to determine adequate sample size in order to test a hypothesis about a population proportion using a sample proportion. When testing for differences between two population proportions with two sample proportions, the necessary criteria are defined a bit differently. The following three conditions must be met for both samples to assure the sample sizes are sufficiently large enough to conduct a hypothesis test for differences:

1. Both samples need to have at least 30 observations each.
2. Both $(n_1 * p_1)$ and $(n_2 * p_2)$ have to be greater than five.
3. Both $[n_1 * (1 - p_1)]$ and $[n_2 * (1 - p_2)]$ have to be greater than five.

As a general rule of thumb, if both samples have at least 100 observations each, you probably do not need to check these three criteria to determine if the samples are of sufficient size. Otherwise, you should check that the three criteria are satisfied. Actually, since POLYSTAT will be used to do the calculations, we will let POLYSTAT check this for us.

Hypothesis Test for Difference Between Proportions

As was covered in the last section, with the exception of the two minor adjustments dealing with the pooled standard error and minimum sample size, to test the hypothesis of a difference between two proportions, we follow the six-step process we have been using all along. Similar to what was seen for single proportion hypothesis tests, since we have to conduct this test for large samples, it stands to reason that the z-score will always be used as our test statistic.

Example 13.1

A public health official wants to know whether two high schools, one in the suburbs and the other in the inner city, differ in the percentage of students who smoke. A random survey of students

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 316 of 370. PageID #: 19143

yields the following results. For a sample of 100 students in the suburbs, 34 admitted they smoked and, for a random sample of 90 in the inner city, 43 indicated that they smoked. Using a 0.05 level of significance, is there sufficient evidence to suggest that a difference exists?

Solution.

1. Null hypothesis: Ho: $\pi_1 = \pi_2$.
   Research hypothesis: Ha: $\pi_1 \neq \pi_2$.
   Since we are only testing for a difference, this is a two-tail test.
2. Level of significance: $\alpha = 0.05$.
3. Both samples are sufficiently large, so we can use the z-distribution.
4. Test criterion at 0.05 α: $z = \pm 1.96$.
5. Calculate the z-score:

$$\pi_p = 0.405 = \frac{0.34 * 100 + 0.477 * 90}{100 + 90} \tag{13.3}$$

$$z = \frac{\frac{34}{100} - \frac{43}{90}}{\sqrt{\frac{0.405(1-0.405)}{100} + \frac{0.405(1-0.405)}{90}}} = \frac{0.340 - 0.477}{\sqrt{0.0024 + 0.0027}} = -1.92. \tag{13.4}$$

6. Decision: Since the calculated $z = -1.92$ is greater than the critical z-score of -1.96, it is in the acceptance region, and the null hypothesis cannot be rejected. With a 95 percent level of confidence, there is evidence to suggest that no difference exists in the percent of students who smoke in the suburbs compared to the inner city.

Example 13.2

Castaneda v. Partida (430 U.S. 482, 1977) was an important Supreme Court case in which statistical methods were used as part of the legal argument. In *Castaneda v. Partida,* the plaintiffs alleged that the (discriminatory) method for selecting juries in Hidalgo County, Texas, was biased against Mexican Americans. For the period at issue, 181,535 persons were eligible for jury duty, of which 143,611 were Mexican Americans. Of the 870 people actually selected to serve on a jury, 339 were Mexican American. Using a 0.05 level of significance, is there sufficient evidence of discrimination? (Is the proportion of Mexican American's eligible for jury duty greater than the proportion of Mexican American's selected for jury duty?)

Solution.

1. Null hypothesis: Ho: $\pi_1 \leq \pi_2$.
   Research hypothesis: Ha: $\pi_1 > \pi_2$.
   Since we are testing if one proportion is greater than the other, this is a one-tail test.
2. Level of significance: $\alpha = 0.05$.

3. Both samples are sufficiently large, so we can use the z-distribution.
4. Test criterion at 0.05 α: $z = +1.65$.
5. Calculate the z-score:

$$\pi_p = 0.789 = \frac{0.791 * 181,535 + 0.400 * 870}{181,535 + 870} \tag{13.5}$$

$$z = 28.2 = \frac{0.791 - 0.400}{\sqrt{\frac{0.789(1-0.789)}{181,535} + \frac{0.789(1-0.789)}{870}}}. \tag{13.6}$$

6. Decision: This is an easy decision. Since the calculated $z = 28.2$ is (much) greater than the critical z-score of +1.65, it is in the rejection region, and the null hypothesis is rejected. With a 95 percent level of confidence, there is evidence to suggest that a bias exists toward Mexican Americans with regard to jury selection.

In retrospect, the Supreme Court review of this case has served as a precedent for many subsequent applications using statistical methods as part of the proceedings. The Court recognized that, as a general rule, if the difference between the expected value and the observed number is greater than two or three standard deviations, the hypothesis would be suspect to a social scientist.

Example 13.3

Since it is generally believed that the use of seat belts reduces serious traffic accident injuries, the state legislature is considering a mandatory seat belt law. To test the hypothesis that seat belt use reduces serious injuries, an analyst collects some data from a neighboring state that recently introduced such a law. Prior to the introduction of the mandatory seat belt law, a sample of 1,000 accident records on a stretch of the highway during a given month revealed 152 serious injuries. After the mandatory seat belt law was introduced, a sample of 1,400 accident records on the same stretch of highway revealed 165 serious accident injuries. Using a 0.01 level of significance, is there sufficient evidence to suggest that the proportion of serious traffic accident injuries was less after the mandatory seat belt law was introduced?

Solution.

1. Null hypothesis: Ho: $\pi_1 \geq \pi_2$.
   Research hypothesis: Ha: $\pi_1 < \pi_2$.
   Since we are testing if one proportion is less than the other, this is a one-tail test.
2. Level of significance: $\alpha = 0.05$.
3. Both samples are sufficiently large, so we can use the z-distribution.
4. Test criterion at 0.01 α: $z = -2.33$.
5. Calculate the z-score:

$$\pi_p = 0.132 = \frac{0.118 * 1,400 + 0.152 * 1,000}{1,400 + 1,000} \tag{13.7}$$

Case: 4:08-cv-00160-BYP Doc #: 388-2 Filed: 03/01/17 318 of 370. PageID #: 19143

$$z = -2.43 = \frac{0.118 - 0.152}{\sqrt{\frac{0.132(1-0.132)}{1,400} + \frac{0.132(1-0.132)}{1,000}}}. \tag{13.8}$$

6. Decision: Since the calculated $z = -2.43$ is less than the critical z-score of -2.33, it is in the rejection region, and the null hypothesis is rejected. With a 95 percent level of confidence, there is evidence to suggest that the proportion of serious accidents was lower after the mandatory seat belt law was introduced.

Using POLYSTAT to Test a Hypothesis Comparing Population Proportions

Although Microsoft Excel does not directly have a canned procedure to perform a hypothesis test between two proportions, this does not mean that Excel cannot be used to conduct this analysis. In this section, we present a straightforward method to easily analyze a hypothesis that tests for differences between population proportions using POLYSTAT.

To test if two population proportions are statistically different, open POLYSTAT and click on the Two Proportion Hypothesis Test (sheet 8). The contents of the spreadsheet will look like that shown in Exhibit 13.1. The population proportion test is designed to run based on the results of two samples.

Exhibit 13.1

	A	B	C
1	**Hypothesis Test for Two Population Proportions**		
2		**Sample 1**	**Sample 2**
3			
4	**Sample Proportion**	0.000	0.000
5	**Sample Size**	0	0
6	**Level of Significance**		0.05
7			
8	n * p =		#DIV/0!
9	n * (1 - p) =		#DIV/0!
10	Average Proportion		#DIV/0!
11	Standard Deviation of p		#DIV/0!
12			
13	Calculated z-score		#DIV/0!
14			
15	two-tail upper critical z-score		1.96
16	two-tail lower critical z-score		-1.96
17	P(value) two-tail		#DIV/0!
18	Decision:		#DIV/0!
19			
20			
21	Upper one-tail critical z-score		1.64
22	P(value) one-tail		#DIV/0!
23	Decision:		#DIV/0!
24			
25	Lower one-tail critical z-score		-1.64
26	P(value) one-tail		#DIV/0!
27	Decision:		#DIV/0!

This feature is designed to calculate the z-score statistic for a hypothesis test between two proportions from two samples. You enter the proportions from the samples in cells B4 and C4. You also enter the sample sizes in cells B5 and C5, as well as the level of significance in cell C69. The default is the 0.05 level.

The report runs automatically as each number is entered. Both n * p and n * (1 - p) have to be greater than five. The calculated z-score statistic and the critical z-scores are shown in the shadow box. To print the report, click FILE, PRINT, and OK.

Exhibit IV

Pages from statistics textbook cited by Dr. Feinstein in his report:

Probability and Statistics for Engineering and the Sciences

PROBABILITY AND STATISTICS

FOR ENGINEERING AND THE SCIENCES

NINTH EDITION

JAY L. DEVORE

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 321 of 370. PageID #: 19148

NINTH EDITION

Probability and Statistics for Engineering and the Sciences

JAY DEVORE

California Polytechnic State University, San Luis Obispo

CENGAGE Learning

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Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 322 of 370. PageID #: 19149

CENGAGE Learning

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Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 323 of 370. PageID #: 19150

Inferences Based on Two Samples

9

INTRODUCTION

Chapters 7 and 8 presented confidence intervals (CI's) and hypothesis-testing procedures for a single mean μ, single proportion p, and a single variance σ^2. Here we extend these methods to situations involving the means, proportions, and variances of two different population distributions. For example, let μ_1 denote true average Rockwell hardness for heat-treated steel specimens and μ_2 denote true average hardness for cold-rolled specimens. Then an investigator might wish to use samples of hardness observations from each type of steel as a basis for calculating an interval estimate of $\mu_1 - \mu_2$, the difference between the two true average hardnesses. As another example, let p_1 denote the true proportion of nickel-cadmium cells produced under current operating conditions that are defective because of internal shorts, and let p_2 represent the true proportion of cells with internal shorts produced under modified operating conditions. If the rationale for the modified conditions is to reduce the proportion of defective cells, a quality engineer would want to use sample information to test the null hypothesis H_0: $p_1 - p_2 = 0$ (i.e., $p_1 = p_2$) versus the alternative hypothesis, H_a: $p_1 - p_2 > 0$ (i.e., $p_1 > p_2$).

Section 9.1 presents z intervals and tests for making inferences about a difference between two population means (i.e., procedures developed by starting with a standardized variable that has at least approximately a standard normal distribution). Two-sample t procedures for making inferences about $\mu_1 - \mu_2$ are the focus of Section 9.2. The validity of methods described in the first two sections depends on selecting samples from the two populations independently of one another. Often in practice data is gathered in pairs. For example, a sample of individuals might be selected, a measurement of some sort made before a treatment is applied, and then another measurement subsequent to application of the treatment. The analysis of

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 324 of 370. PageID #: 19151

such paired data is described in Section 9.3. Section 9.4 considers inferences about a difference between two population proportions, and Section 9.5 does the same thing for a ratio of population variances or standard deviations.

9.1 *z* Tests and Confidence Intervals for a Difference Between Two Population Means

The inferences discussed in this section concern a difference $\mu_1 - \mu_2$ between the means of two different population distributions. An investigator might, for example, wish to test hypotheses about the difference between true average breaking strengths of two different types of corrugated fiberboard. One such hypothesis would state that $\mu_1 - \mu_2 = 0$ that is, that $\mu_1 = \mu_2$. Alternatively, it may be appropriate to estimate $\mu_1 - \mu_2$ by computing a 95% CI. Such inferences necessitate obtaining a sample of strength observations for each type of fiberboard.

Basic Assumptions

1. $X_1, X_2, \ldots, X_m$ is a random sample from a distribution with mean μ_1 and variance σ_1^2.
2. $Y_1, Y_2, \ldots, Y_n$ is a random sample from a distribution with mean μ_2 and variance σ_2^2.
3. The X and Y samples are independent of one another.

The use of m for the number of observations in the first sample and n for the number of observations in the second sample allows for the two sample sizes to be different. Sometimes this is because it is more difficult or expensive to sample one population than another. In other situations, equal sample sizes may initially be specified, but for reasons beyond the scope of the experiment, the actual sample sizes may differ. For example, the abstract of the article **"A Randomized Controlled Trial Assessing the Effectiveness of Professional Oral Care by Dental Hygienists" (*Intl. J. of Dental Hygiene*, 2008: 63–67)** states that "Forty patients were randomly assigned to either the POC group ($m = 20$) or the control group ($n = 20$). One patient in the POC group and three in the control group dropped out because of exacerbation of underlying disease or death." The data analysis was then based on $m = 19$ and $n = 16$.

The natural estimator of $\mu_1 - \mu_2$ is $\overline{X} - \overline{Y}$, the difference between the corresponding sample means. Inferential procedures are based on standardizing this estimator, so we need expressions for the expected value and standard deviation of $\overline{X} - \overline{Y}$.

PROPOSITION

The expected value of $\overline{X} - \overline{Y}$ is $\mu_1 - \mu_2$, so $\overline{X} - \overline{Y}$ is an unbiased estimator of $\mu_1 - \mu_2$. The standard deviation of $\overline{X} - \overline{Y}$ is

$$\sigma_{\overline{X}-\overline{Y}} = \sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}}$$

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 325 of 370. PageID #: 19152

Proof Both these results depend on the rules of expected value and variance presented in Chapter 5. Since the expected value of a difference is the difference of expected values,

$$E(\overline{X} - \overline{Y}) = E(\overline{X}) - E(\overline{Y}) = \mu_1 - \mu_2$$

Because the X and Y samples are independent, $\overline{X}$ and $\overline{Y}$ are independent quantities. Then the variance of the difference is the *sum* of $V(\overline{X})$ and $V(\overline{Y})$:

$$V(\overline{X} - \overline{Y}) = V(\overline{X}) + V(\overline{Y}) = \frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}$$

The standard deviation of $\overline{X} - \overline{Y}$ is the square root of this expression. ■

If we regard $\mu_1 - \mu_2$ as a parameter θ, then its estimator is $\hat{\theta} = \overline{X} - \overline{Y}$ with standard deviation $\sigma_{\hat{\theta}}$ given by the proposition. When σ_1^2 and σ_2^2 both have known values, the value of this standard deviation can be calculated. The sample variances must be used to estimate $\sigma_{\hat{\theta}}$ when σ_1^2 and σ_2^2 are unknown.

Test Procedures for Normal Populations with Known Variances

In Chapters 7 and 8, the first CI and test procedure for a population mean μ were based on the assumption that the population distribution was normal with the value of the population variance σ^2 known to the investigator. Similarly, we first assume here that *both* population distributions are normal and that the values of *both* σ_1^2 and σ_2^2 are known. Situations in which one or both of these assumptions can be dispensed with will be presented shortly.

Because the population distributions are normal, both $\overline{X}$ and $\overline{Y}$ have normal distributions. Furthermore, independence of the two samples implies that the two sample means are independent of one another. Thus the difference $\overline{X} - \overline{Y}$ is normally distributed, with expected value $\mu_1 - \mu_2$ and standard deviation $\sigma_{\overline{X}-\overline{Y}}$ given in the foregoing proposition. Standardizing $\overline{X} - \overline{Y}$ gives the standard normal variable

$$Z = \frac{\overline{X} - \overline{Y} - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}}} \qquad (9.1)$$

In a hypothesis-testing problem, the null hypothesis will state that $\mu_1 - \mu_2$ has a specified value. Denoting this null value by Δ_0, we have H_0: $\mu_1 - \mu_2 = \Delta_0$. Often $\Delta_0 = 0$, in which case H_0 says that $\mu_1 = \mu_2$. If μ_1 represents the true average fuel efficiency (mpg) for automobiles of a certain type equipped with a six-cylinder engine and μ_2 denotes true average efficiency for automobiles of the same type equipped with a four-cylinder engine, a sensible null hypothesis of interest might be H_0: $\mu_1 - \mu_2 = -3$. This is a fancy way of saying that on average the fuel efficiency for four-cylinder engines is 3 mpg higher than it is for six-cylinder engines.

Consider the alternative hypothesis H_a: $\mu_1 - \mu_2 > \Delta_0$. A value $\bar{x} - \bar{y}$ that considerably exceeds Δ_0 (the expected value of $\overline{X} - \overline{Y}$ when H_0 is true) provides evidence against H_0 and for H_a. Such a value of $\bar{x} - \bar{y}$ corresponds to a positive and large value of the test statistic. This implies that if the calculated sample means and

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 326 of 370. PageID #: 19153

sample sizes are substituted into the formula for Z and the resulting value is z, then values more contradictory to H_0 than z itself are those larger than z. Thus

$$\begin{aligned}P\text{-value} &= P(\text{obtaining a test statistic value at least}\\ &\qquad \text{as contradictory to } z \text{ when } H_0 \text{ is true})\\ &= P(\text{a standard normal rv is } \geq z)\\ &= \text{the area under the standard normal}\\ &\quad\ \text{curve to the right of } z\\ &= 1 - \Phi(z)\end{aligned}$$

The test procedure in this case is *upper-tailed* because the P-value is an upper-tail z curve area.

When the alternative hypothesis contains the inequality $<$, test statistic values more contradictory to H_0 than z itself are those smaller than z. The P-value is then the area under the standard normal curve to the left of z; the test is *lower-tailed.* Lastly, if the inequality $\neq$ appears in H_a, then values either larger than $|z|$ or smaller than $-|z|$ are more contradictory to H_0 than z itself (the absolute value around z takes care of both the z positive case and the z negative case). The implication is that the P-value is the sum of the area under the standard normal curve to the left of $-|z|$ and the area to the right of $|z|$—that is, a *two-tailed* test. This sum of two tail areas is the same as doubling the captured tail area.

Null hypothesis: H_0: $\mu_1 - \mu_2 = \Delta_0$

Test statistic value: $z = \dfrac{\bar{x} - \bar{y} - \Delta_0}{\sqrt{\dfrac{\sigma_1^2}{m} + \dfrac{\sigma_2^2}{n}}}$

Alternative Hypothesis	P-Value Determination
H_a: $\mu_1 - \mu_2 > \Delta_0$	Area under the standard normal curve to the right of z
H_a: $\mu_1 - \mu_2 < \Delta_0$	Area under the standard normal curve to the left of z
H_a: $\mu_1 - \mu_2 \neq \Delta_0$	$2 \cdot$ (Area under the standard normal curve to the right of $\lvert z \rvert$)

Assumptions: Two normal population distributions with known values of σ_1 and σ_2, two independent random samples.

EXAMPLE 9.1 Analysis of a random sample consisting of $m = 20$ specimens of cold-rolled steel to determine yield strengths resulted in a sample average strength of $\bar{x} = 29.8$ ksi. A second random sample of $n = 25$ two-sided galvanized steel specimens gave a sample average strength of $\bar{y} = 34.7$ ksi. Assuming that the two yield-strength distributions are normal with $\sigma_1 = 4.0$ and $\sigma_2 = 5.0$ (suggested by a graph in the article **"Zinc-Coated Sheet Steel: An Overview,"** ***Automotive Engr.*****, Dec. 1984: 39–43**), does the data indicate that the corresponding true average yield strengths μ_1 and μ_2 are different? Let's carry out a test at significance level $\alpha = .01$.

1. The parameter of interest is $\mu_1 - \mu_2$, the difference between the true average strengths for the two types of steel.
2. The null hypothesis is H_0: $\mu_1 - \mu_2 = 0$.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 327 of 370. PageID #: 191154

3. The alternative hypothesis is H_a: $\mu_1 - \mu_2 \neq 0$; if H_a is true, then μ_1 and μ_2 are different.
4. With $\Delta_0 = 0$, the test statistic value is

$$z = \frac{\bar{x} - \bar{y}}{\sqrt{\dfrac{\sigma_1^2}{m} + \dfrac{\sigma_2^2}{n}}}$$

5. Substituting $m = 20$, $\bar{x} = 29.8$, $\sigma_1^2 = 16.0$, $n = 25$, $\bar{y} = 34.7$, and $\sigma_2^2 = 25.0$ into the formula for z yields

$$z = \frac{29.8 - 34.7}{\sqrt{\dfrac{16.0}{20} + \dfrac{25.0}{25}}} = \frac{-4.90}{1.34} = -3.66$$

That is, the observed value of $\bar{x} - \bar{y}$ is more than 3 standard deviations below what would be expected were H_0 true.

6. The $\neq$ inequality in H_a implies that a two-tailed test is appropriate. The P-value is

$$2[1 - \Phi(3.66)] \approx 2(0) = 0 \quad \text{(software gives .00025).}$$

7. Since P-value $\approx 0 \leq .01 = \alpha$, H_0 is therefore rejected at level .01 in favor of the conclusion that $\mu_1 \neq \mu_2$. In fact, with a P-value this small, the null hypothesis would be rejected at *any* sensible significance level. The sample data strongly suggests that the true average yield strength for cold-rolled steel differs from that for galvanized steel. ■

Using a Comparison to Identify Causality

Investigators are often interested in comparing either the effects of two different treatments on a response or the response after treatment with the response after no treatment (treatment vs. control). If the individuals or objects to be used in the comparison are not assigned by the investigators to the two different conditions, the study is said to be **observational**. The difficulty with drawing conclusions based on an observational study is that although statistical analysis may indicate a significant difference in response between the two groups, the difference may be due to some underlying factors that had not been controlled rather than to any difference in treatments.

EXAMPLE 9.2 A letter in the ***Journal of the American Medical Association* (May 19, 1978)** reported that of 215 male physicians who were Harvard graduates and died between November 1974 and October 1977, the 125 in full-time practice lived an average of 48.9 years beyond graduation, whereas the 90 with academic affiliations lived an average of 43.2 years beyond graduation. Does the data suggest that the mean lifetime after graduation for doctors in full-time practice exceeds the mean lifetime for those who have an academic affiliation? (If so, those medical students who say that they are "dying to obtain an academic affiliation" may be closer to the truth than they realize; in other words, is "publish or perish" really "publish and perish"?)

Let μ_1 denote the true average number of years lived beyond graduation for physicians in full-time practice, and let μ_2 denote the same quantity for physicians with academic affiliations. Assume the 125 and 90 physicians to be random samples from populations 1 and 2, respectively (which may not be sensible if there is reason to believe that Harvard graduates have special characteristics that differentiate them from all other physicians—in this case inferences would be restricted just to the

"Harvard populations"). The letter from which the data was taken gave no information about variances, so for illustration assume that $\sigma_1 = 14.6$ and $\sigma_2 = 14.4$. The hypotheses are H_0: $\mu_1 - \mu_2 = 0$ versus H_a: $\mu_1 - \mu_2 > 0$, so Δ_0 is zero. The computed value of the test statistic is

$$z = \frac{48.9 - 43.2}{\sqrt{\dfrac{(14.6)^2}{125} + \dfrac{(14.4)^2}{90}}} = \frac{5.70}{\sqrt{1.70 + 2.30}} = 2.85$$

The P-value for an upper-tailed test is $1 - \Phi(2.85) = .0022$. At significance level .01, H_0 is rejected (because $\alpha > P$-value) in favor of the conclusion that $\mu_1 - \mu_2 > 0$ ($\mu_1 > \mu_2$). This is consistent with the information reported in the letter.

This data resulted from a **retrospective** observational study; the investigator did not start out by selecting a sample of doctors and assigning some to the "academic affiliation" treatment and the others to the "full-time practice" treatment, but instead identified members of the two groups by looking backward in time (through obituaries!) to past records. Can the statistically significant result here really be attributed to a difference in the type of medical practice after graduation, or is there some other underlying factor (e.g., age at graduation, exercise regimens, etc.) that might also furnish a plausible explanation for the difference? Observational studies have been used to argue for a causal link between smoking and lung cancer. There are many studies that show that the incidence of lung cancer is significantly higher among smokers than among nonsmokers. However, individuals had decided whether to become smokers long before investigators arrived on the scene, and factors in making this decision may have played a causal role in the contraction of lung cancer. ■

A **randomized controlled experiment** results when investigators assign subjects to the two treatments in a random fashion. When statistical significance is observed in such an experiment, the investigator and other interested parties will have more confidence in the conclusion that the difference in response has been caused by a difference in treatments. A very famous example of this type of experiment and conclusion is the Salk polio vaccine experiment described in Section 9.4. Various aspects of experimental and sampling design are discussed at greater length in the (nonmathematical) books by Moore and by Freedman et al., listed in the Chapter 1 references.

β and the Choice of Sample Size

The probability of a type II error is easily calculated when both population distributions are normal with known values of σ_1 and σ_2. Consider the case in which the alternative hypothesis is H_a: $\mu_1 - \mu_2 > \Delta_0$. Let Δ' denote a value of $\mu_1 - \mu_2$ that exceeds Δ_0 (a value for which H_0 is false). As with the upper-tailed z tests of Chapter 8, the inequality P-value $\le \alpha$ is equivalent to $z \ge z_\alpha$ (the area captured in the upper tail of the z curve will be at most α if and only if the calculated z is on or to the right of the z critical value that captures area α). This in turn is equivalent to $\bar{x} - \bar{y} \ge \Delta_0 + z_\alpha \sigma_{\bar{X}-\bar{Y}}$. Thus

$$\begin{aligned}\beta(\Delta') &= P(\text{not rejecting } H_0 \text{ when } \mu_1 - \mu_2 = \Delta') \\ &= P(\bar{X} - \bar{Y} < \Delta_0 + z_\alpha \sigma_{\bar{X}-\bar{Y}} \text{ when } \mu_1 - \mu_2 = \Delta')\end{aligned}$$

When $\mu_1 - \mu_2 = \Delta'$, $\bar{X} - \bar{Y}$ is normally distributed with mean value Δ' and standard deviation $\sigma_{\bar{X}-\bar{Y}}$ (the same standard deviation as when H_0 is true); using these values to standardize the inequality in parentheses gives the desired probability.

Alternative Hypothesis	$\beta(\Delta') = P(\text{type II error when } \mu_1 - \mu_2 = \Delta')$
$H_a\colon \mu_1 - \mu_2 > \Delta_0$	$\Phi\left(z_\alpha - \dfrac{\Delta' - \Delta_0}{\sigma}\right)$
$H_a\colon \mu_1 - \mu_2 < \Delta_0$	$1 - \Phi\left(-z_\alpha - \dfrac{\Delta' - \Delta_0}{\sigma}\right)$
$H_a\colon \mu_1 - \mu_2 \neq \Delta_0$	$\Phi\left(z_{\alpha/2} - \dfrac{\Delta' - \Delta_0}{\sigma}\right) - \Phi\left(-z_{\alpha/2} - \dfrac{\Delta' - \Delta_0}{\sigma}\right)$

where $\sigma = \sigma_{\overline{X}-\overline{Y}} = \sqrt{(\sigma_1^2/m) + (\sigma_2^2/n)}$

EXAMPLE 9.3 (Example 9.1 continued)

Suppose that when μ_1 and μ_2 (the true average yield strengths for the two types of steel) differ by as much as 5, the probability of detecting such a departure from H_0 (the power of the test) should be .90. Does a level .01 test with sample sizes $m = 20$ and $n = 25$ satisfy this condition? The value of σ for these sample sizes (the denominator of z) was previously calculated as 1.34. The probability of a type II error for the two-tailed level .01 test when $\mu_1 - \mu_2 = \Delta' = 5$ is

$$\begin{aligned}\beta(5) &= \Phi\left(2.58 - \frac{5-0}{1.34}\right) - \Phi\left(-2.58 - \frac{5-0}{1.34}\right)\\ &= \Phi(-1.15) - \Phi(-6.31) = .1251\end{aligned}$$

It is easy to verify that $\beta(-5) = .1251$ also. Thus the power is $1 - \beta(5) = .8749$. Because this is somewhat less than .9, slightly larger sample sizes should be used. ■

As in Chapter 8, sample sizes m and n can be determined that will satisfy both $P(\text{type I error}) =$ a specified α and $P(\text{type II error when } \mu_1 - \mu_2 = \Delta') =$ a specified β. For an upper-tailed test, equating the previous expression for $\beta(\Delta')$ to the specified value of β gives

$$\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n} = \frac{(\Delta' - \Delta_0)^2}{(z_\alpha + z_\beta)^2}$$

When the two sample sizes are equal, this equation yields

$$m = n = \frac{(\sigma_1^2 + \sigma_2^2)(z_\alpha + z_\beta)^2}{(\Delta' - \Delta_0)^2}$$

These expressions are also correct for a lower-tailed test, whereas α is replaced by $\alpha/2$ for a two-tailed test.

Large-Sample Tests

The assumptions of normal population distributions and known values of σ_1 and σ_2 are fortunately unnecessary when both sample sizes are sufficiently large. In this case, the Central Limit Theorem guarantees that $\overline{X} - \overline{Y}$ has approximately a normal distribution regardless of the underlying population distributions. Furthermore,

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 330 of 370. PageID #: 19157

using S_1^2 and S_2^2 in place of σ_1^2 and σ_2^2 in Expression (9.1) gives a variable whose distribution is approximately standard normal:

$$Z = \frac{\overline{X} - \overline{Y} - (\mu_1 - \mu_2)}{\sqrt{\dfrac{S_1^2}{m} + \dfrac{S_2^2}{n}}}$$

A large-sample test statistic results from replacing $\mu_1 - \mu_2$ by Δ_0, the expected value of $\overline{X} - \overline{Y}$ when H_0 is true. This statistic Z then has approximately a standard normal distribution when H_0 is true, which allows for straightforward determination of a P-value as a z curve area.

Use of the test statistic value

$$z = \frac{\bar{x} - \bar{y} - \Delta_0}{\sqrt{\dfrac{s_1^2}{m} + \dfrac{s_2^2}{n}}}$$

along with the previously stated prescriptions for P-value determination gives large-sample tests whose significance levels are approximately α. These tests are usually appropriate if both $m > 40$ and $n > 40$.

EXAMPLE 9.4 What impact does fast-food consumption have on various dietary and health characteristics? The article **"Effects of Fast-Food Consumption on Energy Intake and Diet Quality Among Children in a National Household Study" (*Pediatrics*, 2004: 112–118)** reported the accompanying summary data on daily calorie intake both for a sample of teens who said they did not typically eat fast food and another sample of teens who said they did usually eat fast food.

Eat Fast Food	Sample Size	Sample Mean	Sample SD
No	663	2258	1519
Yes	413	2637	1138

Does this data provide strong evidence for concluding that true average calorie intake for teens who typically eat fast food exceeds by more than 200 calories per day the true average intake for those who don't typically eat fast food? Let's investigate by carrying out a test of hypotheses at a significance level of approximately .05.

The parameter of interest is $\mu_1 - \mu_2$, where μ_1 is the true average calorie intake for teens who don't typically eat fast food and μ_2 is true average intake for teens who do typically eat fast food. The hypotheses of interest are

$$H_0\colon \mu_1 - \mu_2 = -200 \quad \text{versus} \quad H_a\colon \mu_1 - \mu_2 < -200$$

The alternative hypothesis asserts that true average daily intake for those who typically eat fast food exceeds that for those who don't by more than 200 calories. The test statistic value is

$$z = \frac{\bar{x} - \bar{y} - (-200)}{\sqrt{\dfrac{s_1^2}{m} + \dfrac{s_2^2}{n}}}$$

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 331 of 370. PageID #: 19158

The calculated test statistic value is

$$z = \frac{2258 - 2637 + 200}{\sqrt{\dfrac{(1519)^2}{663} + \dfrac{(1138)^2}{413}}} = \frac{-179}{81.34} = -2.20$$

The inequality in H_a implies that P-value $= \Phi(-2.20) = .0139$ (a lower-tailed test). Since $.0139 \le .05$, the null hypothesis is rejected. At a significance level of .05, it does appear that true average daily calorie intake for teens who typically eat fast food exceeds by more than 200 the true average intake for those who don't typically eat such food. However, the P-value is not small enough to justify rejecting H_0 at significance level .01.

Notice that if the label 1 had instead been used for the fast-food condition and 2 had been used for the no-fast-food condition, then 200 would have replaced -200 in both hypotheses and H_a would have contained the inequality $>$, implying an upper-tailed test. The resulting test statistic value would have been 2.20, giving the same P-value as before. ■

Confidence Intervals for $\mu_1 - \mu_2$

When both population distributions are normal, standardizing $\overline{X} - \overline{Y}$ gives a random variable Z with a standard normal distribution. Since the area under the z curve between $-z_{\alpha/2}$ and $z_{\alpha/2}$ is $1 - \alpha$, it follows that

$$P\left(-z_{\alpha/2} < \frac{\overline{X} - \overline{Y} - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{m} + \dfrac{\sigma_2^2}{n}}} < z_{\alpha/2}\right) = 1 - \alpha$$

Manipulation of the inequalities inside the parentheses to isolate $\mu_1 - \mu_2$ yields the equivalent probability statement

$$P\left(\overline{X} - \overline{Y} - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}} < \mu_1 - \mu_2 < \overline{X} - \overline{Y} + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}}\right) = 1 - \alpha$$

This implies that a $100(1 - \alpha)\%$ CI for $\mu_1 - \mu_2$ has lower limit $\bar{x} - \bar{y} - z_{\alpha/2} \cdot \sigma_{\overline{X}-\overline{Y}}$ and upper limit $\bar{x} - \bar{y} + z_{\alpha/2} \cdot \sigma_{\overline{X}-\overline{Y}}$, where $\sigma_{\overline{X}-\overline{Y}}$ is the square-root expression. This interval is a special case of the general formula $\hat{\theta} \pm z_{\alpha/2} \cdot \sigma_{\hat{\theta}}$.

If both m and n are large, the CLT implies that this interval is valid even without the assumption of normal populations; in this case, the confidence level is *approximately* $100(1 - \alpha)\%$. Furthermore, use of the sample variances S_1^2 and S_2^2 in the standardized variable Z yields a valid interval in which s_1^2 and s_2^2 replace σ_1^2 and σ_2^2.

Provided that m and n are both large, a CI for $\mu_1 - \mu_2$ with a confidence level of approximately $100(1 - \alpha)\%$ is

$$\bar{x} - \bar{y} \pm z_{\alpha/2}\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}}$$

where $-$ gives the lower limit and $+$ the upper limit of the interval. An upper or a lower confidence bound can also be calculated by retaining the appropriate sign ($+$ or $-$) and replacing $z_{\alpha/2}$ by z_α.

Our standard rule of thumb for characterizing sample sizes as large is $m > 40$ and $n > 40$.

EXAMPLE 9.5 Enhanced heavy oil recovery uses steam delivered to the production zone. The annulus between rock formation and the metal casing pipe is filled with cement. The article **"Thermal Stability of the Cement Sheath in Steam Treated Oil Wells"** (***J. of the Amer. Ceramic Soc.*, 2011: 4463–4470**) reported on a study of cement sheath performance when various thermal cements were cured at 35 °C and then heated to 230 °C. Here is summary data on Vicker's hardness (MPa) for both a control cement and an experimental cement:

Type	Sample Size	Sample Mean	Sample SD
Control	50	24.3	5.2
Experimental	50	27.0	5.8

Figure 9.1 shows a comparative boxplot of data consistent with these summary quantities. The main difference between the two samples appears to be where they are centered.

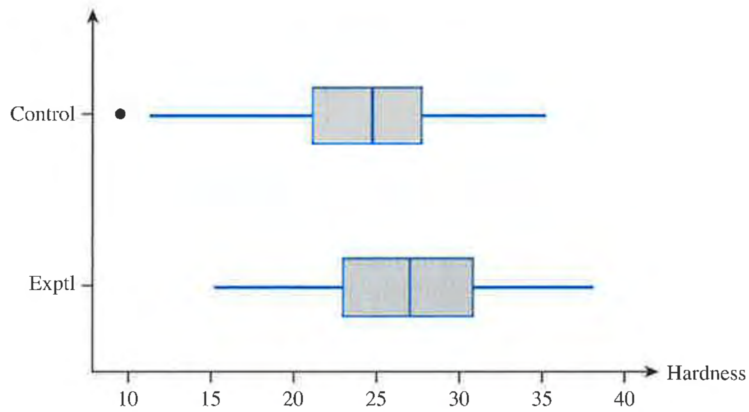


Figure 9.1 A comparative boxplot of the hardness data

Let's now calculate a confidence interval for the difference between true average hardness for the control cement (μ_1) and true average hardness for the experimental cement (μ_2) using a confidence level of 95%:

$$24.3 - 27.0 \pm (1.96)\sqrt{\frac{(5.2)^2}{50} + \frac{(5.8)^2}{50}} = -2.7 \pm (1.96)(1.1016)$$

$$= -2.7 \pm 2.2 = (-4.9, -.5)$$

That is, with 95% confidence, $-4.9 < \mu_1 - \mu_2 < -.5$. We can therefore be highly confident that true average hardness for the experimental cement exceeds that for the control cement by between .5 and 4.9 MPa. This CI does not include 0, so at the chosen confidence level, 0 is not a plausible value of $\mu_1 - \mu_2$. According to the relationship between CI's and HT's discussed in Section 8.5, the null hypothesis H_0: $\mu_1 - \mu_2 = 0$ should be rejected in favor of H_a: $\mu_1 - \mu_2 \neq 0$ at significance level .05 (the P-value for this test given in the cited article is not in agreement with other summary data).

Notice that if we relabel so that μ_1 refers to the experimental cement and μ_2 to the control cement, the CI becomes (.5, 4.9). The interpretation of the interval is exactly the same as was that of the first interval. ■

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 333 of 370. PageID #: 19160

If the variances σ_1^2 and σ_2^2 are at least approximately known and the investigator uses equal sample sizes, then the common sample size n that yields a $100(1-\alpha)\%$ interval of width w is

$$n = \frac{4z_{\alpha/2}^2(\sigma_1^2 + \sigma_2^2)}{w^2}$$

which will generally have to be rounded up to an integer.

EXERCISES Section 9.1 (1–16)

1. An article in the November 1983 ***Consumer Reports*** compared various types of batteries. The average lifetimes of Duracell Alkaline AA batteries and Eveready Energizer Alkaline AA batteries were given as 4.1 hours and 4.5 hours, respectively. Suppose these are the population average lifetimes.
   - **a.** Let $\overline{X}$ be the sample average lifetime of 100 Duracell batteries and $\overline{Y}$ be the sample average lifetime of 100 Eveready batteries. What is the mean value of $\overline{X} - \overline{Y}$ (i.e., where is the distribution of $\overline{X} - \overline{Y}$ centered)? How does your answer depend on the specified sample sizes?
   - **b.** Suppose the population standard deviations of lifetime are 1.8 hours for Duracell batteries and 2.0 hours for Eveready batteries. With the sample sizes given in part (a), what is the variance of the statistic $\overline{X} - \overline{Y}$, and what is its standard deviation?
   - **c.** For the sample sizes given in part (a), draw a picture of the approximate distribution curve of $\overline{X} - \overline{Y}$ (include a measurement scale on the horizontal axis). Would the shape of the curve necessarily be the same for sample sizes of 10 batteries of each type? Explain.

2. The **National Health Statistics Reports** dated Oct. 22, 2008, included the following information on the heights (in.) for non-Hispanic white females:

Age	Sample Size	Sample Mean	Std. Error Mean
20–39	866	64.9	.09
60 and older	934	63.1	.11

   - **a.** Calculate and interpret a confidence interval at confidence level approximately 95% for the difference between population mean height for the younger women and that for the older women.
   - **b.** Let μ_1 denote the population mean height for those aged 20–39 and μ_2 denote the population mean height for those aged 60 and older. Interpret the hypotheses H_0: $\mu_1 - \mu_2 = 1$ and H_a: $\mu_1 - \mu_2 > 1$, and then carry out a test of these hypotheses at significance level .001.
   - **c.** Based on the *P*-value calculated in (b) would you reject the null hypothesis at any reasonable significance level? Explain your reasoning.
   - **d.** What hypotheses would be appropriate if μ_1 referred to the older age group, μ_2 to the younger age group, and you wanted to see if there was compelling evidence for concluding that the population mean height for younger women exceeded that for older women by more than 1 in.?

3. Pilates is a popular set of exercises for the treatment of individuals with lower back pain. The method has six basic principles: centering, concentration, control, precision, flow, and breathing. The article **"Efficacy of the Addition of Modified Pilates Exercises to a Minimal Intervention in Patients with Chronic Low Back Pain: A Randomized Controlled Trial" (*Physical Therapy*, 2013: 309–321)** reported on an experiment involving 86 subjects with nonspecific low back pain. The participants were randomly divided into two groups of equal size. The first group received just educational materials, whereas the second group participated in 6 weeks of Pilates exercises. The sample mean level of pain (on a scale from 0 to 10) for the control group at a 6-week follow-up was 5.2 and the sample mean for the treatment group was 3.1; both sample standard deviations were 2.3.
   - **a.** Does it appear that true average pain level for the control condition exceeds that for the treatment condition? Carry out a test of hypotheses using a significance level of .01 (the cited article reported statistical significance at this α, and a sample mean difference of 2.1 also suggests practical significance).
   - **b.** Does it appear that true average pain level for the control condition exceeds that for the treatment condition by more than 1? Carry out a test of appropriate hypotheses.

4. Reliance on solid biomass fuel for cooking and heating exposes many children from developing countries to high levels of indoor air pollution. The article **"Domestic Fuels, Indoor Air Pollution, and Children's Health" (*Annals of the N.Y. Academy of Sciences*, 2008: 209–217)** presented information on various pulmonary characteristics in

samples of children whose households in India used either biomass fuel or liquefied petroleum gas (LPG). For the 755 children in biomass households, the sample mean peak expiratory flow (a person's maximum speed of expiration) was 3.30 L/s, and the sample standard deviation was 1.20. For the 750 children whose households used liquefied petroleum gas, the sample mean PEF was 4.25 and the sample standard deviation was 1.75.

a. Calculate a confidence interval at the 95% confidence level for the population mean PEF for children in biomass households and then do likewise for children in LPG households. What is the simultaneous confidence level for the two intervals?

b. Carry out a test of hypotheses at significance level .01 to decide whether true average PEF is lower for children in biomass households than it is for children in LPG households (the cited article included a *P*-value for this test).

c. FEV_1, the forced expiratory volume in 1 second, is another measure of pulmonary function. The cited article reported that for the biomass households the sample mean FEV_1 was 2.3 L/s and the sample standard deviation was .5 L/s. If this information is used to compute a 95% CI for population mean FEV_1, would the simultaneous confidence level for this interval and the first interval calculated in (a) be the same as the simultaneous confidence level determined there? Explain.

5. Persons having Reynaud's syndrome are apt to suffer a sudden impairment of blood circulation in fingers and toes. In an experiment to study the extent of this impairment, each subject immersed a forefinger in water and the resulting heat output (cal/cm^2/min) was measured. For $m = 10$ subjects with the syndrome, the average heat output was $\bar{x} = .64$, and for $n = 10$ nonsufferers, the average output was 2.05. Let μ_1 and μ_2 denote the true average heat outputs for the two types of subjects. Assume that the two distributions of heat output are normal with $\sigma_1 = .2$ and $\sigma_2 = .4$.

a. Consider testing H_0: $\mu_1 - \mu_2 = -1.0$ versus H_a: $\mu_1 - \mu_2 < -1.0$ at level .01. Describe in words what H_a says, and then carry out the test.

b. What is the probability of a type II error when the actual difference between μ_1 and μ_2 is $\mu_1 - \mu_2 = -1.2$?

c. Assuming that $m = n$, what sample sizes are required to ensure that $\beta = .1$ when $\mu_1 - \mu_2 = -1.2$?

6. An experiment to compare the tension bond strength of polymer latex modified mortar (Portland cement mortar to which polymer latex emulsions have been added during mixing) to that of unmodified mortar resulted in $\bar{x} = 18.12$ kgf/cm^2 for the modified mortar ($m = 40$) and $\bar{y} = 16.87$ kgf/cm^2 for the unmodified mortar ($n = 32$). Let μ_1 and μ_2 be the true average tension bond strengths for the modified and unmodified mortars, respectively. Assume that the bond strength distributions are both normal.

a. Assuming that $\sigma_1 = 1.6$ and $\sigma_2 = 1.4$, test H_0: $\mu_1 - \mu_2 = 0$ versus H_a: $\mu_1 - \mu_2 > 0$ at level .01.

b. Compute the probability of a type II error for the test of part (a) when $\mu_1 - \mu_2 = 1$.

c. Suppose the investigator decided to use a level .05 test and wished $\beta = .10$ when $\mu_1 - \mu_2 = 1$. If $m = 40$, what value of n is necessary?

d. How would the analysis and conclusion of part (a) change if σ_1 and σ_2 were unknown but $s_1 = 1.6$ and $s_2 = 1.4$?

7. Is there any systematic tendency for part-time college faculty to hold their students to different standards than do full-time faculty? The article **"Are There Instructional Differences Between Full-Time and Part-Time Faculty?" (*College Teaching*, 2009: 23–26)** reported that for a sample of 125 courses taught by full-time faculty, the mean course GPA was 2.7186 and the standard deviation was .63342, whereas for a sample of 88 courses taught by part-timers, the mean and standard deviation were 2.8639 and .49241, respectively. Does it appear that true average course GPA for part-time faculty differs from that for faculty teaching full-time? Test the appropriate hypotheses at significance level .01.

8. Tensile-strength tests were carried out on two different grades of wire rod **("Fluidized Bed Patenting of Wire Rods," *Wire J.*, June 1977: 56–61)**, resulting in the accompanying data.

Grade	Sample Size	Sample Mean (kg/mm^2)	Sample SD
AISI 1064	$m = 129$	$\bar{x} = 107.6$	$s_1 = 1.3$
AISI 1078	$n = 129$	$\bar{y} = 123.6$	$s_2 = 2.0$

a. Does the data provide compelling evidence for concluding that true average strength for the 1078 grade exceeds that for the 1064 grade by more than 10 kg/mm^2? Test the appropriate hypotheses using a significance level of .01.

b. Estimate the difference between true average strengths for the two grades in a way that provides information about precision and reliability.

9. The article **"Evaluation of a Ventilation Strategy to Prevent Barotrauma in Patients at High Risk for Acute Respiratory Distress Syndrome" (*New Engl. J. of Med.*, 1998: 355–358)** reported on an experiment in which 120 patients with similar clinical features were randomly divided into a control group and a treatment group, each consisting of 60 patients. The sample mean ICU stay (days) and sample standard deviation for the treatment group were 19.9 and 39.1, respectively, whereas these values for the control group were 13.7 and 15.8.

a. Calculate a point estimate for the difference between true average ICU stay for the treatment and control groups. Does this estimate suggest that

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 335 of 370. PageID #: 19162

there is a significant difference between true average stays under the two conditions?

b. Answer the question posed in part (a) by carrying out a formal test of hypotheses. Is the result different from what you conjectured in part (a)?

c. Does it appear that ICU stay for patients given the ventilation treatment is normally distributed? Explain your reasoning.

d. Estimate true average length of stay for patients given the ventilation treatment in a way that conveys information about precision and reliability.

10. An experiment was performed to compare the fracture toughness of high-purity 18 Ni maraging steel with commercial-purity steel of the same type (***Corrosion Science*, 1971: 723–736**). For $m = 32$ specimens, the sample average toughness was $\bar{x} = 65.6$ for the high-purity steel, whereas for $n = 38$ specimens of commercial steel $\bar{y} = 59.8$. Because the high-purity steel is more expensive, its use for a certain application can be justified only if its fracture toughness exceeds that of commercial-purity steel by more than 5. Suppose that both toughness distributions are normal.

a. Assuming that $\sigma_1 = 1.2$ and $\sigma_2 = 1.1$, test the relevant hypotheses using $\alpha = .001$.

b. Compute β for the test conducted in part (a) when $\mu_1 - \mu_2 = 6$.

11. The level of lead in the blood was determined for a sample of 152 male hazardous-waste workers ages 20–30 and also for a sample of 86 female workers, resulting in a mean $\pm$ standard error of 5.5 ± 0.3 for the men and 3.8 ± 0.2 for the women (**"Temporal Changes in Blood Lead Levels of Hazardous Waste Workers in New Jersey, 1984–1987,"** ***Environ. Monitoring and Assessment*, 1993: 99–107**). Calculate an estimate of the difference between true average blood lead levels for male and female workers in a way that provides information about reliability and precision.

12. The accompanying summary data on total cholesterol level (mmol/l) was obtained from a sample of Asian postmenopausal women who were vegans and another sample of such women who were omnivores (**"Vegetarianism, Bone Loss, and Vitamin D: A Longitudinal Study in Asian Vegans and Non-Vegans,"** ***European J. of Clinical Nutr.*, 2012: 75–82**).

Diet	Sample Size	Sample Mean	Sample SD
Vegan	88	5.10	1.07
Omnivore	93	5.55	1.10

Calculate and interpret a 99% CI for the difference between population mean total cholesterol level for vegans and population mean total cholesterol level for omnivores (the cited article included a 95% CI). [*Note:* The article described a more sophisticated statistical analysis for investigating bone density loss taking into account other characteristics ("covariates") such as age, body weight, and various nutritional factors; the resulting CI included 0, suggesting no diet effect.]

13. A mechanical engineer wishes to compare strength properties of steel beams with similar beams made with a particular alloy. The same number of beams, n, of each type will be tested. Each beam will be set in a horizontal position with a support on each end, a force of 2500 lb will be applied at the center, and the deflection will be measured. From past experience with such beams, the engineer is willing to assume that the true standard deviation of deflection for both types of beam is .05 in. Because the alloy is more expensive, the engineer wishes to test at level .01 whether it has smaller average deflection than the steel beam. What value of n is appropriate if the desired type II error probability is .05 when the difference in true average deflection favors the alloy by .04 in.?

14. The level of monoamine oxidase (MAO) activity in blood platelets (nm/mg protein/h) was determined for each individual in a sample of 43 chronic schizophrenics, resulting in $\bar{x} = 2.69$ and $s_1 = 2.30$, as well as for 45 normal subjects, resulting in $\bar{y} = 6.35$ and $s_2 = 4.03$. Does this data strongly suggest that true average MAO activity for normal subjects is more than twice the activity level for schizophrenics? Derive a test procedure and carry out the test using $\alpha = .01$. [*Hint*: H_0 and H_a here have a different form from the three standard cases. Let μ_1 and μ_2 refer to true average MAO activity for schizophrenics and normal subjects, respectively, and consider the parameter $\theta = 2\mu_1 - \mu_2$. Write H_0 and H_a in terms of θ, estimate θ, and derive $\hat{\sigma}_{\hat{\theta}}$ (**"Reduced Monoamine Oxidase Activity in Blood Platelets from Schizophrenic Patients,"** ***Nature*, July 28, 1972: 225–226**).]

15. **a.** Show for the upper-tailed test with σ_1 and σ_2 known that as either m or n increases, β decreases when $\mu_1 - \mu_2 > \Delta_0$.

b. For the case of equal sample sizes ($m = n$) and fixed α, what happens to the necessary sample size n as β is decreased, where β is the desired type II error probability at a fixed alternative?

16. To decide whether two different types of steel have the same true average fracture toughness values, n specimens of each type are tested, yielding the following results:

Type	Sample Average	Sample SD
1	60.1	1.0
2	59.9	1.0

Calculate the P-value for the appropriate two-sample z test, assuming that the data was based on $n = 100$. Then repeat the calculation for $n = 400$. Is the small P-value for $n = 400$ indicative of a difference that has practical significance? Would you have been satisfied with just a report of the P-value? Comment briefly.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 338 of 370. PageID #: 19163

9.2 The Two-Sample *t* Test and Confidence Interval

Values of the population variances will usually not be known to an investigator. In the previous section, we illustrated for large sample sizes the use of a z test and CI in which the sample variances were used in place of the population variances. In fact, for large samples, the CLT allows us to use these methods even when the two populations of interest are not normal.

In practice, though, it will often happen that at least one sample size is small and the population variances have unknown values. Without the CLT at our disposal, we proceed by making specific assumptions about the underlying population distributions. The use of inferential procedures that follow from these assumptions is then restricted to situations in which the assumptions are at least approximately satisfied. We could, for example, assume that both population distributions are members of the Weibull family or that they are both Poisson distributions. It shouldn't surprise you to learn that normality is often the most reasonable assumption.

ASSUMPTIONS

Both population distributions are normal, so that $X_1, X_2, \ldots, X_m$ is a random sample from a normal distribution and so is $Y_1, \ldots, Y_n$ (with the X's and Y's independent of one another). The plausibility of these assumptions can be judged by constructing a normal probability plot of the x_i's and another of the y_i's.

The test statistic and confidence interval formula are based on the same standardized variable developed in Section 9.1, but the relevant distribution is now t rather than z.

THEOREM

When the population distributions are both normal, the standardized variable

$$T = \frac{\overline{X} - \overline{Y} - (\mu_1 - \mu_2)}{\sqrt{\dfrac{S_1^2}{m} + \dfrac{S_2^2}{n}}} \tag{9.2}$$

has approximately a t distribution with df ν estimated from the data by

$$\nu = \frac{\left(\dfrac{s_1^2}{m} + \dfrac{s_2^2}{n}\right)^2}{\dfrac{(s_1^2/m)^2}{m-1} + \dfrac{(s_2^2/n)^2}{n-1}} = \frac{[(se_1)^2 + (se_2)^2]^2}{\dfrac{(se_1)^4}{m-1} + \dfrac{(se_2)^4}{n-1}}$$

where

$$se_1 = \frac{s_1}{\sqrt{m}}, \quad se_2 = \frac{s_2}{\sqrt{n}}$$

(round ν down to the nearest integer).

Manipulating T in a probability statement to isolate $\mu_1 - \mu_2$ gives a CI, whereas a test statistic results from replacing $\mu_1 - \mu_2$ by the null value Δ_0.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 337 of 370. PageID #: 19164

The **two-sample t confidence interval for $\mu_1 - \mu_2$** with confidence level $100(1-\alpha)\%$ is then

$$\bar{x} - \bar{y} \pm t_{\alpha/2,\nu}\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}}$$

A one-sided confidence bound can be calculated as described earlier.

The **two-sample t test** for testing H_0: $\mu_1 - \mu_2 = \Delta_0$ is as follows:

$$\text{Test statistic value: } t = \frac{\bar{x} - \bar{y} - \Delta_0}{\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}}}$$

Alternative Hypothesis	P-Value Determination
H_a: $\mu_1 - \mu_2 > \Delta_0$	Area under the t_ν curve to the right of t
H_a: $\mu_1 - \mu_2 < \Delta_0$	Area under the t_ν curve to the left of t
H_a: $\mu_1 - \mu_2 \neq \Delta_0$	$2 \cdot$ (Area under the t_ν curve to the right of $\lvert t \rvert$)

Assumptions: Both population distributions are normal, and the two random samples are selected independently of one another.

EXAMPLE 9.6 The void volume within a textile fabric affects comfort, flammability, and insulation properties. Permeability of a fabric refers to the accessibility of void space to the flow of a gas or liquid. The article **"The Relationship Between Porosity and Air Permeability of Woven Textile Fabrics" (*J. of Testing and Eval.*, 1997: 108–114)** gave summary information on air permeability ($cm^3/cm^2/sec$) for a number of different fabric types. Consider the following data on two different types of plain-weave fabric:

Fabric Type	Sample Size	Sample Mean	Sample Standard Deviation
Cotton	10	51.71	.79
Triacetate	10	136.14	3.59

Assuming that the porosity distributions for both types of fabric are normal, let's calculate a confidence interval for the difference between true average porosity for the cotton fabric and that for the acetate fabric, using a 95% confidence level. Before the appropriate t critical value can be selected, df must be determined:

$$\text{df} = \frac{\left(\frac{.6241}{10} + \frac{12.8881}{10}\right)^2}{\frac{(.6241/10)^2}{9} + \frac{(12.8881/10)^2}{9}} = \frac{1.8258}{.1850} = 9.87$$

Thus we use $\nu = 9$; Appendix Table A.5 gives $t_{.025,9} = 2.262$. The resulting interval is

$$51.71 - 136.14 \pm (2.262)\sqrt{\frac{.6241}{10} + \frac{12.8881}{10}} = -84.43 \pm 2.63$$
$$= (-87.06, -81.80)$$

With a high degree of confidence, we can say that true average porosity for triacetate fabric specimens exceeds that for cotton specimens by between 81.80 and 87.06 $cm^3/cm^2/sec$. ■

EXAMPLE 9.7 The deterioration of many municipal pipeline networks across the country is a growing concern. One technology proposed for pipeline rehabilitation uses a flexible liner threaded through existing pipe. The article **"Effect of Welding on a High-Density Polyethylene Liner" (*J. of Materials in Civil Engr.*, 1996: 94–100)** reported the following data on tensile strength (psi) of liner specimens both when a certain fusion process was used and when this process was not used.

No fusion	2748	2700	2655	2822	2511			
	3149	3257	3213	3220	2753			
	$m = 10$	$\bar{x} = 2902.8$		$s_1 = 277.3$				
Fused	3027	3356	3359	3297	3125	2910	2889	2902
	$n = 8$	$\bar{y} = 3108.1$		$s_2 = 205.9$				

Figure 9.2 shows normal probability plots from Minitab. The linear pattern in each plot supports the assumption that the tensile strength distributions under the two conditions are both normal.

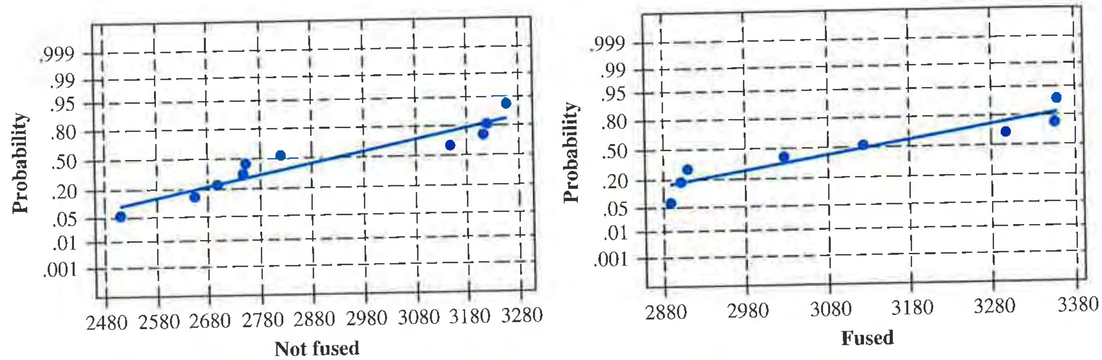


Figure 9.2 Normal probability plots from Minitab for the tensile strength data

The authors of the article stated that the fusion process increased the average tensile strength. The message from the comparative boxplot of Figure 9.3 is not all that clear. Let's carry out a test of hypotheses to see whether the data supports this conclusion.

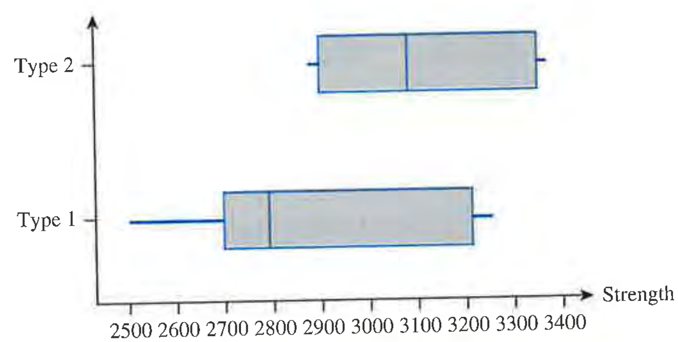


Figure 9.3 A comparative boxplot of the tensile-strength data

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 339 of 370. PageID #: 19166

1. Let μ_1 be the true average tensile strength of specimens when the no-fusion treatment is used and μ_2 denote the true average tensile strength when the fusion treatment is used.
2. H_0: $\mu_1 - \mu_2 = 0$ (no difference in the true average tensile strengths for the two treatments)
3. H_a: $\mu_1 - \mu_2 < 0$ (true average tensile strength for the no-fusion treatment is less than that for the fusion treatment, so that the investigators' conclusion is correct)
4. The null value is $\Delta_0 = 0$, so the test statistic value is

$$t = \frac{\bar{x} - \bar{y}}{\sqrt{\dfrac{s_1^2}{m} + \dfrac{s_2^2}{n}}}$$

5. We now compute both the test statistic value and df for the test:

$$t = \frac{2902.8 - 3108.1}{\sqrt{\dfrac{(277.3)^2}{10} + \dfrac{(205.9)^2}{8}}} = \frac{-205.3}{113.97} = -1.8$$

Using $s_1^2/m = 7689.529$ and $s_2^2/n = 5299.351$,

$$\nu = \frac{(7689.529 + 5299.351)^2}{(7689.529)^2/9 + (5299.351)^2/7} = \frac{168{,}711{,}003.7}{10{,}581{,}747.35} = 15.94$$

so the test will be based on 15 df.

6. Appendix Table A.8 shows that the area under the 15 df *t* curve to the right of 1.8 is .046, so the *P*-value for a lower-tailed test is also .046. The following Minitab output summarizes all the computations:

```
Two-sample T for nofusion vs fused

            N     Mean    StDev    SE Mean
not fused  10     2903      277         88
fused       8     3108      206         73

95% C.I. for mu nofusion-mu fused: (-488, 38)
t-Test mu not fused = mu fused (vs <): T = -1.80 P = 0.046 DF = 15
```

7. Using a significance level of .05, we can barely reject the null hypothesis in favor of the alternative hypothesis, confirming the conclusion stated in the article. However, someone demanding more compelling evidence might select $\alpha = .01$, a level for which H_0 cannot be rejected.

If the question posed had been whether fusing increased true average strength by more than 100 psi, then the relevant hypotheses would have been H_0: $\mu_1 - \mu_2 = -100$ versus H_a: $\mu_1 - \mu_2 < -100$; that is, the null value would have been $\Delta_0 = -100$. ■

Pooled *t* Procedures

Alternatives to the two-sample *t* procedures just described result from assuming not only that the two population distributions are normal but also that they have equal variances ($\sigma_1^2 = \sigma_2^2$). That is, the two population distribution curves are assumed normal with equal spreads, the only possible difference between them being where they are centered.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 340 of 370. PageID #: 19167

Let σ^2 denote the common population variance. Then standardizing $\overline{X} - \overline{Y}$ gives

$$Z = \frac{\overline{X} - \overline{Y} - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma^2}{m} + \dfrac{\sigma^2}{n}}} = \frac{\overline{X} - \overline{Y} - (\mu_1 - \mu_2)}{\sqrt{\sigma^2\left(\dfrac{1}{m} + \dfrac{1}{n}\right)}}$$

which has a standard normal distribution. Before this variable can be used as a basis for making inferences about $\mu_1 - \mu_2$, the common variance must be estimated from sample data. One estimator of σ^2 is S_1^2, the variance of the m observations in the first sample, and another is S_2^2, the variance of the second sample. Intuitively, a better estimator than either individual sample variance results from combining the two sample variances. A first thought might be to use $(S_1^2 + S_2^2)/2$. However, if $m > n$, then the first sample contains more information about σ^2 than does the second sample, and an analogous comment applies if $m < n$. The following *weighted* average of the two sample variances, called the **pooled** (i.e., combined) **estimator of σ^2**, adjusts for any difference between the two sample sizes:

$$S_p^2 = \frac{m-1}{m+n-2} \cdot S_1^2 + \frac{n-1}{m+n-2} \cdot S_2^2$$

The first sample contributes $m - 1$ degrees of freedom to the estimate of σ^2, and the second sample contributes $n - 1$ df, for a total of $m + n - 2$ df. Statistical theory says that if S_p^2 replaces σ^2 in the expression for Z, the resulting standardized variable has a t distribution based on $m + n - 2$ df. In the same way that earlier standardized variables were used as a basis for deriving confidence intervals and test procedures, this t variable immediately leads to the pooled t CI for estimating $\mu_1 - \mu_2$ and the pooled t test for testing hypotheses about a difference between means.

In the past, many statisticians recommended these pooled t procedures over the two-sample t procedures. The pooled t test, for example, can be derived from the likelihood ratio principle, whereas the two-sample t test is not a likelihood ratio test. Furthermore, the significance level for the pooled t test is exact, whereas it is only approximate for the two-sample t test. However, recent research has shown that although the pooled t test does outperform the two-sample t test by a bit (smaller β's for the same α) when $\sigma_1^2 = \sigma_2^2$, the former test can easily lead to erroneous conclusions if applied when the variances are different. Analogous comments apply to the behavior of the two confidence intervals. That is, the pooled t procedures are not robust to violations of the equal variance assumption.

It has been suggested that one could carry out a preliminary test of H_0: $\sigma_1^2 = \sigma_2^2$ and use a pooled t procedure if this null hypothesis is not rejected. Unfortunately, the usual "F test" of equal variances (Section 9.5) is quite sensitive to the assumption of normal population distributions—much more so than t procedures. We therefore recommend the conservative approach of using two-sample t procedures unless there is really compelling evidence for doing otherwise, particularly when the two sample sizes are different.

Type II Error Probabilities

Determining type II error probabilities (or equivalently, power $= 1 - \beta$) for the two-sample t test is complicated. There does not appear to be any simple way to use the β curves of Appendix Table A.17. The most recent version of Minitab (Version 16) will calculate power for the pooled t test but not for the two-sample t test. However, the UCLA Statistics Department homepage (http://www.stat.ucla.edu)

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 341 of 370. PageID #: 19168

permits access to a power calculator that will do this. For example, we specified $m = 10$, $n = 8$, $\sigma_1 = 300$, $\sigma_2 = 225$ (these are the sample sizes for Example 9.7, whose sample standard deviations are somewhat smaller than these values of σ_1 and σ_2) and asked for the power of a two-tailed level .05 test of H_0: $\mu_1 - \mu_2 = 0$ when $\mu_1 - \mu_2 = 100$, 250, and 500. The resulting values of the power were .1089, .4609, and .9635 (corresponding to $\beta = .89$, .54, and .04), respectively. In general, β will decrease as the sample sizes increase, as α increases, and as $\mu_1 - \mu_2$ moves farther from 0. The software will also calculate sample sizes necessary to obtain a specified value of power for a particular value of $\mu_1 - \mu_2$.

EXERCISES **Section 9.2 (17–35)**

17. Determine the number of degrees of freedom for the two-sample *t* test or CI in each of the following situations:

a. $m = 10, n = 10, s_1 = 5.0, s_2 = 6.0$

b. $m = 10, n = 15, s_1 = 5.0, s_2 = 6.0$

c. $m = 10, n = 15, s_1 = 2.0, s_2 = 6.0$

d. $m = 12, n = 24, s_1 = 5.0, s_2 = 6.0$

18. Which way of dispensing champagne, the traditional vertical method or a tilted beer-like pour, preserves more of the tiny gas bubbles that improve flavor and aroma? The following data was reported in the article **"On the Losses of Dissolved CO_2 during Champagne Serving" (*J. Agr. Food Chem.*, 2010: 8768–8775)**.

Temp (°C)	Type of Pour	n	Mean (g/L)	SD
18	Traditional	4	4.0	.5
18	Slanted	4	3.7	.3
12	Traditional	4	3.3	.2
12	Slanted	4	2.0	.3

Assume that the sampled distributions are normal.

a. Carry out a test at significance level .01 to decide whether true average CO_2 loss at 18 °C for the traditional pour differs from that for the slanted pour.

b. Repeat the test of hypotheses suggested in (a) for the 12° temperature. Is the conclusion different from that for the 18° temperature? *Note:* The 12° result was reported in the popular media.

19. Suppose μ_1 and μ_2 are true mean stopping distances at 50 mph for cars of a certain type equipped with two different types of braking systems. Use the two-sample *t* test at significance level .01 to test H_0: $\mu_1 - \mu_2 = -10$ versus H_a: $\mu_1 - \mu_2 < -10$ for the following data: $m = 6$, $\bar{x} = 115.7$, $s_1 = 5.03$, $n = 6$, $\bar{y} = 129.3$, and $s_2 = 5.38$.

20. Use the data of Exercise 19 to calculate a 95% CI for the difference between true average stopping distance for cars equipped with system 1 and cars equipped with system 2. Does the interval suggest that precise information about the value of this difference is available?

21. Quantitative noninvasive techniques are needed for routinely assessing symptoms of peripheral neuropathies, such as carpal tunnel syndrome (CTS). The article **"A Gap Detection Tactility Test for Sensory Deficits Associated with Carpal Tunnel Syndrome" (*Ergonomics*, 1995: 2588–2601)** reported on a test that involved sensing a tiny gap in an otherwise smooth surface by probing with a finger; this functionally resembles many work-related tactile activities, such as detecting scratches or surface defects. When finger probing was not allowed, the sample average gap detection threshold for $m = 8$ normal subjects was 1.71 mm, and the sample standard deviation was .53; for $n = 10$ CTS subjects, the sample mean and sample standard deviation were 2.53 and .87, respectively. Does this data suggest that the true average gap detection threshold for CTS subjects exceeds that for normal subjects? State and test the relevant hypotheses using a significance level of .01.

22. According to the article **"Modeling and Predicting the Effects of Submerged Arc Weldment Process Parameters on Weldment Characteristics and Shape Profiles" (*J. of Engr. Manuf.*, 2012: 1230–1240)**, the submerged arc welding (SAW) process is commonly used for joining thick plates and pipes. The heat affected zone (HAZ), a band created within the base metal during welding, was of particular interest to the investigators. Here are observations on depth (mm) of the HAZ both when the current setting was high and when it was lower.

Non-high	1.04	1.15	1.23	1.69	1.92
	1.98	2.36	2.49	2.72	
	1.37	1.43	1.57	1.71	1.94
	2.06	2.55	2.64	2.82	
High	1.55	2.02	2.02	2.05	2.35
	2.57	2.93	2.94	2.97	

a. Construct a comparative boxplot and comment on interesting features.

b. Is it reasonable to use the two-sample *t* test to test hypotheses about the difference between true average HAZ depths for the two conditions?

c. Does it appear that true average HAZ depth is larger for the higher current condition than for the lower condition? Carry out a test of appropriate hypotheses using a significance level of .01.

23. Fusible interlinings are being used with increasing frequency to support outer fabrics and improve the shape and drape of various pieces of clothing. The article **"Compatibility of Outer and Fusible Interlining Fabrics in Tailored Garments" (*Textile Res. J.*, 1997: 137–142)** gave the accompanying data on extensibility (%) at 100 gm/cm for both high-quality (H) fabric and poor-quality (P) fabric specimens.

H	1.2	.9	.7	1.0	1.7	1.7	1.1	.9	1.7
	1.9	1.3	2.1	1.6	1.8	1.4	1.3	1.9	1.6
	.8	2.0	1.7	1.6	2.3	2.0			
P	1.6	1.5	1.1	2.1	1.5	1.3	1.0	2.6	

a. Construct normal probability plots to verify the plausibility of both samples having been selected from normal population distributions.

b. Construct a comparative boxplot. Does it suggest that there is a difference between true average extensibility for high-quality fabric specimens and that for poor-quality specimens?

c. The sample mean and standard deviation for the high-quality sample are 1.508 and .444, respectively, and those for the poor-quality sample are 1.588 and .530. Use the two-sample t test to decide whether true average extensibility differs for the two types of fabric.

24. Damage to grapes from bird predation is a serious problem for grape growers. The article **"Experimental Method to Investigate and Monitor Bird Behavior and Damage to Vineyards" (*Amer. J. of Enology and Viticulture*, 2004: 288–291)** reported on an experiment involving a bird-feeder table, time-lapse video, and artificial foods. Information was collected for two different bird species at both the experimental location and at a natural vineyard setting. Consider the following data on time (sec) spent on a single visit to the location.

Species	Location	n	$\bar{x}$	SE mean
Blackbirds	Exptl	65	13.4	2.05
Blackbirds	Natural	50	9.7	1.76
Silvereyes	Exptl	34	49.4	4.78
Silvereyes	Natural	46	38.4	5.06

a. Calculate an upper confidence bound for the true average time that blackbirds spend on a single visit at the experimental location.

b. Does it appear that true average time spent by blackbirds at the experimental location exceeds the true average time birds of this type spend at the natural location? Carry out a test of appropriate hypotheses.

c. Estimate the difference between the true average time blackbirds spend at the natural location and true average time that silvereyes spend at the natural location, and do so in a way that conveys information about reliability and precision.

[*Note:* The sample medians reported in the article all seemed significantly smaller than the means, suggesting substantial population distribution skewness. The authors actually used the distribution-free test procedure presented in Section 2 of Chapter 15.]

25. The accompanying data consists of prices (\$) for one sample of California cabernet sauvignon wines that received ratings of 93 or higher in the May 2013 issue of ***Wine Spectator*** and another sample of California cabernets that received ratings of 89 or lower in the same issue.

≥ 93:	100	100	60	135	195	195	
	125	135	95	42	75	72	
≤ 89:	80	75	75	85	75	35	85
	65	45	100	28	38	50	28

Assume that these are both random samples of prices from the population of all wines recently reviewed that received ratings of at least 93 and at most 89, respectively.

a. Investigate the plausibility of assuming that both sampled populations are normal.

b. Construct a comparative boxplot. What does it suggest about the difference in true average prices?

c. Calculate a confidence interval at the 95% confidence level to estimate the difference between μ_1, the mean price in the higher rating population, and μ_2, the mean price in the lower rating population. Is the interval consistent with the statement "Price rarely equates to quality" made by a columnist in the cited issue of the magazine?

26. The article **"The Influence of Corrosion Inhibitor and Surface Abrasion on the Failure of Aluminum-Wired Twist-On Connections" (*IEEE Trans. on Components, Hybrids, and Manuf. Tech.*, 1984: 20–25)** reported data on potential drop measurements for one sample of connectors wired with alloy aluminum and another sample wired with EC aluminum. Does the accompanying SAS output suggest that the true average potential drop for alloy connections (type 1) is higher than that wfor EC connections (as stated in the article)? Carry out the appropriate test using a significance level of .01. In reaching your conclusion, what type of error might you have committed? [*Note:* SAS reports the P-value for a two-tailed test.]

```
Type    N          Mean        Std Dev      Std Error
1      20   17.49900000     0.55012821     0.12301241
2      20   16.90000000     0.48998389     0.10956373

     Variances          T          DF        Prob>|T|
       Unequal     3.6362        37.5          0.0008
         Equal     3.6362        38.0          0.0008
```

27. Anorexia Nervosa (AN) is a psychiatric condition leading to substantial weight loss among women who are fearful of becoming fat. The article **"Adipose Tissue Distribution After Weight Restoration and Weight Maintenance in Women with Anorexia Nervosa" (*Amer. J. of Clinical***

Nutr., 2009: 1132–1137) used whole-body magnetic resonance imagery to determine various tissue characteristics for both an AN sample of individuals who had undergone acute weight restoration and maintained their weight for a year and a comparable (at the outset of the study) control sample. Here is summary data on intermuscular adipose tissue (IAT; kg).

Condition	Sample Size	Sample Mean	Sample SD
AN	16	.52	.26
Control	8	.35	.15

Assume that both samples were selected from normal distributions.

a. Calculate an estimate for true average IAT under the described AN protocol, and do so in a way that conveys information about the reliability and precision of the estimation.

b. Calculate an estimate for the difference between true average AN IAT and true average control IAT, and do so in a way that conveys information about the reliability and precision of the estimation. What does your estimate suggest about true average AN IAT relative to true average control IAT?

28. As the population ages, there is increasing concern about accident-related injuries to the elderly. The article **"Age and Gender Differences in Single-Step Recovery from a Forward Fall" (*J. of Gerontology*, 1999: M44–M50)** reported on an experiment in which the maximum lean angle—the farthest a subject is able to lean and still recover in one step—was determined for both a sample of younger females (21–29 years) and a sample of older females (67–81 years). The following observations are consistent with summary data given in the article:

YF: 29, 34, 33, 27, 28, 32, 31, 34, 32, 27
OF: 18, 15, 23, 13, 12

Does the data suggest that true average maximum lean angle for older females is more than 10 degrees smaller than it is for younger females? State and test the relevant hypotheses at significance level .10.

29. The article **"Effect of Internal Gas Pressure on the Compression Strength of Beverage Cans and Plastic Bottles" (*J. of Testing and Evaluation*, 1993: 129–131)** includes the accompanying data on compression strength (lb) for a sample of 12-oz aluminum cans filled with strawberry drink and another sample filled with cola. Does the data suggest that the extra carbonation of cola results in a higher average compression strength? Base your answer on a P-value. What assumptions are necessary for your analysis?

Beverage	Sample Size	Sample Mean	Sample SD
Strawberry drink	15	540	21
Cola	15	554	15

30. The article **"Flexure of Concrete Beams Reinforced with Advanced Composite Orthogrids" (*J. of Aerospace Engr.*, 1997: 7–15)** gave the accompanying data on ultimate load (kN) for two different types of beams.

Type	Sample Size	Sample Mean	Sample SD
Fiberglass grid	26	33.4	2.2
Commercial carbon grid	26	42.8	4.3

a. Assuming that the underlying distributions are normal, calculate and interpret a 99% CI for the difference between true average load for the fiberglass beams and that for the carbon beams.

b. Does the upper limit of the interval you calculated in part (a) give a 99% upper confidence bound for the difference between the two μ's? If not, calculate such a bound. Does it strongly suggest that true average load for the carbon beams is more than that for the fiberglass beams? Explain.

31. Refer to Exercise 33 in Section 7.3. The cited article also gave the following observations on degree of polymerization for specimens having viscosity times concentration in a higher range:

429	430	430	431	436	437
440	441	445	446	447	

a. Construct a comparative boxplot for the two samples, and comment on any interesting features.

b. Calculate a 95% confidence interval for the difference between true average degree of polymerization for the middle range and that for the high range. Does the interval suggest that μ_1 and μ_2 may in fact be different? Explain your reasoning.

32. The degenerative disease osteoarthritis most frequently affects weight-bearing joints such as the knee. The article **"Evidence of Mechanical Load Redistribution at the Knee Joint in the Elderly When Ascending Stairs and Ramps" (*Annals of Biomed. Engr.*, 2008: 467–476)** presented the following summary data on stance duration (ms) for samples of both older and younger adults.

Age	Sample Size	Sample Mean	Sample SD
Older	28	801	117
Younger	16	780	72

Assume that both stance duration distributions are normal.

a. Calculate and interpret a 99% CI for true average stance duration among elderly individuals.

b. Carry out a test of hypotheses at significance level .05 to decide whether true average stance duration is larger among elderly individuals than among younger individuals.

33. The article **"The Effects of a Low-Fat, Plant-Based Dietary Intervention on Body Weight, Metabolism,**

and Insulin Sensitivity in Postmenopausal Women" (***Amer. J. of Med.*, 2005: 991–997**) reported on the results of an experiment in which half of the individuals in a group of 64 postmenopausal overweight women were randomly assigned to a particular vegan diet, and the other half received a diet based on National Cholesterol Education Program guidelines. The sample mean decrease in body weight for those on the vegan diet was 5.8 kg, and the sample SD was 3.2, whereas for those on the control diet, the sample mean weight loss and standard deviation were 3.8 and 2.8, respectively. Does it appear the true average weight loss for the vegan diet exceeds that for the control diet by more than 1 kg? Carry out an appropriate test of hypotheses at significance level .05.

34. Consider the pooled t variable

$$T = \frac{(\overline{X} - \overline{Y}) - (\mu_1 - \mu_2)}{S_p\sqrt{\dfrac{1}{m} + \dfrac{1}{n}}}$$

which has a t distribution with $m + n - 2$ df when both population distributions are normal with $\sigma_1 = \sigma_2$ (see the Pooled t Procedures subsection for a description of S_p).

a. Use this t variable to obtain a pooled t confidence interval formula for $\mu_1 - \mu_2$.

b. A sample of ultrasonic humidifiers of one particular brand was selected for which the observations on maximum output of moisture (oz) in a controlled chamber were 14.0, 14.3, 12.2, and 15.1. A sample of the second brand gave output values 12.1, 13.6, 11.9, and 11.2 (**"Multiple Comparisons of Means Using Simultaneous Confidence Intervals,"** ***J. of Quality Technology*, 1989: 232–241**). Use the pooled t formula from part (a) to estimate the difference between true average outputs for the two brands with a 95% confidence interval.

c. Estimate the difference between the two μ's using the two-sample t interval discussed in this section, and compare it to the interval of part (b).

35. Refer to Exercise 34. Describe the pooled t test for testing H_0: $\mu_1 - \mu_2 = \Delta_0$ when both population distributions are normal with $\sigma_1 = \sigma_2$. Then use this test procedure to test the hypotheses suggested in Exercise 33.

9.3 Analysis of Paired Data

In Sections 9.1 and 9.2, we considered making an inference about a difference between two means μ_1 and μ_2. This was done by utilizing the results of a random sample $X_1, X_2, \ldots X_m$ from the distribution with mean μ_1 and a completely independent (of the X's) sample $Y_1, \ldots, Y_n$ from the distribution with mean μ_2. That is, either m individuals were selected from population 1 and n different individuals from population 2, or m individuals (or experimental objects) were given one treatment and another set of n individuals were given the other treatment. In contrast, there are a number of experimental situations in which there is only one set of n individuals or experimental objects; making two observations on each one results in a natural pairing of values.

EXAMPLE 9.8 Trace metals in drinking water affect the flavor, and unusually high concentrations can pose a health hazard. The article **"Trace Metals of South Indian River"** (***Envir. Studies*, 1982: 62–66**) reported on a study in which six river locations were selected (six experimental objects) and the zinc concentration (mg/L) determined for both surface water and bottom water at each location. The six pairs of observations are displayed in the accompanying table. Does the data suggest that true average concentration in bottom water exceeds that of surface water?

	Location					
	1	2	3	4	5	6
Zinc concentration in bottom water (x)	.430	.266	.567	.531	.707	.716
Zinc concentration in surface water (y)	.415	.238	.390	.410	.605	.609
Difference	.015	.028	.177	.121	.102	.107

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 345 of 370. PageID #: 19172

Figure 9.4(a) displays a plot of this data. At first glance, there appears to be little difference between the x and y samples. From location to location, there is a great deal of variability in each sample, and it looks as though any differences between the samples can be attributed to this variability. However, when the observations are identified by location, as in Figure 9.4(b), a different view emerges. At each location, bottom concentration exceeds surface concentration. This is confirmed by the fact that all $x - y$ differences displayed in the bottom row of the data table are positive. A correct analysis of this data focuses on these differences.

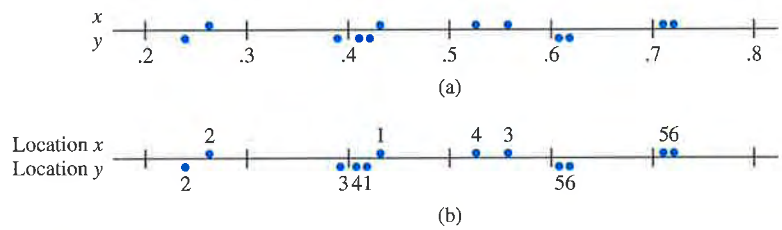


Figure 9.4 Plot of paired data from Example 9.8: (a) observations not identified by location; (b) observations identified by location

ASSUMPTIONS

> The data consists of n independently selected pairs $(X_1, Y_1), (X_2, Y_2), \ldots (X_n, Y_n)$, with $E(X_i) = \mu_1$ and $E(Y_i) = \mu_2$. Let $D_1 = X_1 - Y_1, D_2 = X_2 - Y_2, \ldots, D_n = X_n - Y_n$ so the D_i's are the differences within pairs. The D_i's are assumed to be normally distributed with mean value μ_D and variance σ_D^2 (this is usually a consequence of the X_i's and Y_i's themselves being normally distributed).

We are again interested in making an inference about the difference $\mu_1 - \mu_2$. The two-sample t confidence interval and test statistic were obtained by assuming independent samples and applying the rule $V(\overline{X} - \overline{Y}) = V(\overline{X}) + V(\overline{Y})$. However, with paired data, the X and Y observations within each pair are often not independent. Then $\overline{X}$ and $\overline{Y}$ are not independent of one another. We must therefore abandon the two-sample t procedures and look for an alternative method of analysis.

The Paired *t* Test

Because different pairs are independent, the D_i's are independent of one another. Let $D = X - Y$, where X and Y are the first and second observations, respectively, within an arbitrary pair. Then the expected difference is

$$\mu_D = E(X - Y) = E(X) - E(Y) = \mu_1 - \mu_2$$

(the rule of expected values used here is valid even when X and Y are dependent). Thus any hypothesis about $\mu_1 - \mu_2$ can be phrased as a hypothesis about the mean difference μ_D. But since the D_i's constitute a normal random sample (of differences) with mean μ_D, hypotheses about μ_D can be tested using a one-sample t test. That is, *to test hypotheses about $\mu_1 - \mu_2$ when data is paired, form the differences $D_1, D_2, \ldots, D_n$ and carry out a one-sample t test (based on $n - 1$ df) on these differences.*

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 346 of 370. PageID #: 19173

The Paired t Test

Null hypothesis: H_0: $\mu_D = \Delta_0$ (where $D = X - Y$ is the difference between the first and second observations within a pair, and $\mu_D = \mu_1 - \mu_2$)

Test statistic value: $t = \dfrac{\bar{d} - \Delta_0}{s_D/\sqrt{n}}$ (where $\bar{d}$ and s_D are the sample mean and standard deviation, respectively, of the d_i's)

Alternative Hypothesis	P-Value Determination
H_a: $\mu_D > \Delta_0$	Area under the t_{n-1} curve to the right of t
H_a: $\mu_D < \Delta_0$	Area under the t_{n-1} curve to the left of t
H_a: $\mu_D \neq \Delta_0$	$2 \cdot$ (Area under the t_{n-1} curve to the right of $\lvert t \rvert$)

Assumptions: The D_is constitute a random sample from a normal "difference" population.

EXAMPLE 9.9 Musculoskeletal neck-and-shoulder disorders are all too common among office staff who perform repetitive tasks using visual display units. The article **"Upper-Arm Elevation During Office Work" (*Ergonomics*, 1996: 1221–1230)** reported on a study to determine whether more varied work conditions would have any impact on arm movement. The accompanying data was obtained from a sample of $n = 16$ subjects. Each observation is the amount of time, expressed as a proportion of total time observed, during which arm elevation was below 30°. The two measurements from each subject were obtained 18 months apart. During this period, work conditions were changed, and subjects were allowed to engage in a wider variety of work tasks. Does the data suggest that true average time during which elevation is below 30° differs after the change from what it was before the change?

Subject	1	2	3	4	5	6	7	8
Before	81	87	86	82	90	86	96	73
After	78	91	78	78	84	67	92	70
Difference	3	−4	8	4	6	19	4	3
Subject	9	10	11	12	13	14	15	16
Before	74	75	72	80	66	72	56	82
After	58	62	70	58	66	60	65	73
Difference	16	13	2	22	0	12	−9	9

Figure 9.5 shows a normal probability plot of the 16 differences; the pattern in the plot is quite straight, supporting the normality assumption. A boxplot of these differences appears in Figure 9.6; the boxplot is located considerably to the right of zero, suggesting that perhaps $\mu_D > 0$ (note also that 13 of the 16 differences are positive and only two are negative).

Let's now test the appropriate hypotheses.

1. Let μ_D denote the true average difference between elevation time before the change in work conditions and time after the change.
2. H_0: $\mu_D = 0$ (there is no difference between true average time before the change and true average time after the change)
3. H_0: $\mu_D \neq 0$

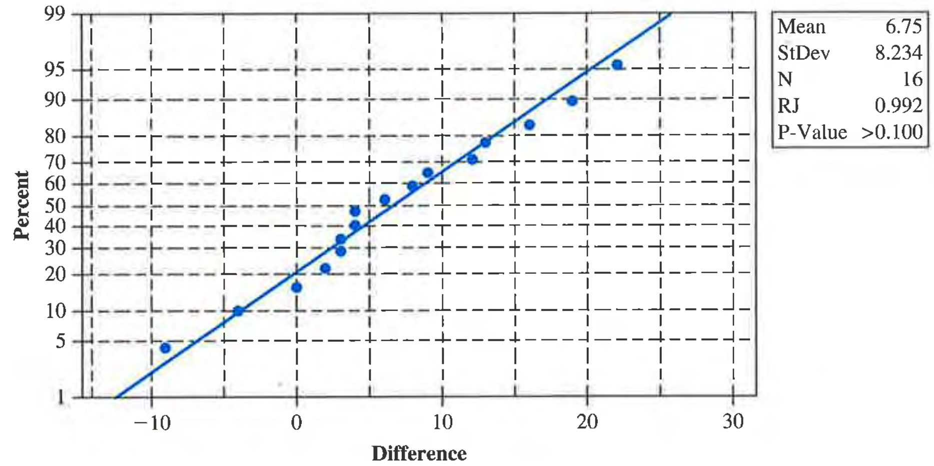


Figure 9.5 A normal probability plot from Minitab of the differences in Example 9.9

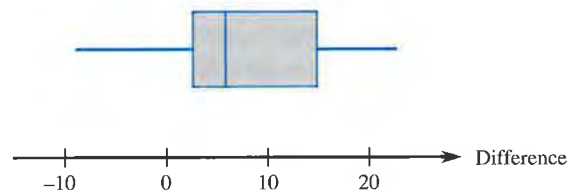


Figure 9.6 A boxplot of the differences in Example 9.9

4. $t = \dfrac{\bar{d} - 0}{s_D/\sqrt{n}} = \dfrac{\bar{d}}{s_D/\sqrt{n}}$

5. $n = 16$, $\Sigma d_i = 108$, and $\Sigma d_i^2 = 1746$, from which $\bar{d} = 6.75$, $s_D = 8.234$, and

$$t = \frac{6.75}{8.234/\sqrt{16}} = 3.28 \approx 3.3$$

6. Appendix Table A.8 shows that the area to the right of 3.3 under the t curve with 15 df is .002. The inequality in H_a implies that a two-tailed test is appropriate, so the P-value is approximately 2(.002) = .004 (Minitab gives .0051).

7. Since .004 < .01, the null hypothesis can be rejected at either significance level .05 or .01. It does appear that the true average difference between times is something other than zero; that is, true average time after the change is different from that before the change. ■

When the number of pairs is large, the assumption of a normal difference distribution is not necessary. The CLT validates the resulting z test.

The Paired t Confidence Interval

In the same way that the t CI for a single population mean μ is based on the t variable $T = (\bar{X} - \mu)/(S/\sqrt{n})$, a t confidence interval for $\mu_D (= \mu_1 - \mu_2)$ is based on the fact that

$$T = \frac{\bar{D} - \mu_D}{S_D/\sqrt{n}}$$

has a t distribution with $n - 1$ df. Manipulation of this t variable, as in previous derivations of CI's, yields the following $100(1 - \alpha)\%$ CI:

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 348 of 370. PageID #: 19175

The **paired t CI for μ_D** is

$$\bar{d} \pm t_{\alpha/2,n-1} \cdot s_D/\sqrt{n}$$

A one-sided confidence bound results from retaining the relevant sign and replacing $t_{\alpha/2}$ by t_α.

When n is small, the validity of this interval requires that the distribution of differences be at least approximately normal. For large n, the CLT ensures that the resulting z interval is valid without any restrictions on the distribution of differences.

EXAMPLE 9.10 Magnetic resonance imaging is a commonly used noninvasive technique for assessing the extent of cartilage damage. However, there is concern that the MRI sizing of articular cartilage defects may not be accurate. The article **"Preoperative MRI Underestimates Articular Cartilage Defect Size Compared with Findings at Arthroscopic Knee Surgery" (*Amer. J. of Sports Med.*, 2013: 590–595)** reported on a study involving a sample of 92 cartilage defects. For each one, the size of the lesion area was determined by an MRI analysis and also during arthroscopic surgery. Each MRI value was then subtracted from the corresponding arthroscopic value to obtain a difference value. The sample mean difference was calculated to be 1.04 cm^2, with a sample standard deviation of 1.67. Let's now calculate a confidence interval using a confidence level of (at least approximately) 95% for μ_D, the mean difference for the population of all such defects (as did the authors of the cited article). Because n is quite large here, we use the z critical value $z_{.025} = 1.96$ (an entry at the very bottom of our t table). The resulting CI is

$$1.04 \pm (1.96) \cdot \frac{1.67}{\sqrt{92}} = 1.04 \pm .34 = (.70, 1.38)$$

At the 95% confidence level, we believe that $.70 < \mu_D < 1.38$. Perhaps the most interesting aspect of this interval is that 0 is not included; only certain positive values of μ_D are plausible. It is this fact that led the investigators to conclude that MRIs tend to underestimate defect size. ■

Paired Data and Two-Sample t Procedures

Consider using the two-sample t test on paired data. The numerators of the two test statistics are identical, since $\bar{d} = \Sigma d_i/n = [\Sigma(x_i - y_i)]/n = (\Sigma x_i)/n - (\Sigma y_i)/n = \bar{x} - \bar{y}$. The difference between the statistics is due entirely to the denominators. Each test statistic is obtained by standardizing $\bar{X} - \bar{Y}\ (= \bar{D})$. But in the presence of dependence the two-sample t standardization is incorrect. To see this, recall from Section 5.5 that

$$V(X \pm Y) = V(X) + V(Y) \pm 2\,\mathrm{Cov}(X, Y)$$

The correlation between X and Y is

$$\rho = \mathrm{Corr}(X, Y) = \mathrm{Cov}(X, Y)/[\sqrt{V(X)} \cdot \sqrt{V(Y)}]$$

It follows that

$$V(X - Y) = \sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2$$

Applying this to $\bar{X} - \bar{Y}$ yields

$$V(\bar{X} - \bar{Y}) = V(\bar{D}) = V\left(\frac{1}{n}\Sigma D_i\right) = \frac{V(D_i)}{n} = \frac{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}{n}$$

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 349 of 370. PageID #: 19176

The two-sample t test is based on the assumption of independence, in which case $\rho = 0$. But in many paired experiments, there will be a strong *positive* dependence between X and Y (large X associated with large Y), so that ρ will be positive and the variance of $\overline{X} - \overline{Y}$ will be smaller than $\sigma_1^2/n + \sigma_2^2/n$. Thus *whenever there is positive dependence within pairs, the denominator for the paired t statistic should be smaller than for t of the independent-samples test.* Often two-sample t will be much closer to zero than paired t, considerably understating the significance of the data.

Similarly, when data is paired, the paired t CI will usually be narrower than the (incorrect) two-sample t CI. This is because there is typically much less variability in the differences than in the x and y values.

Paired Versus Unpaired Experiments

In our examples, paired data resulted from two observations on the same subject (Example 9.9) or experimental object (location in Example 9.8). Even when this cannot be done, paired data with dependence within pairs can be obtained by matching individuals or objects on one or more characteristics thought to influence responses. For example, in a medical experiment to compare the efficacy of two drugs for lowering blood pressure, the experimenter's budget might allow for the treatment of 20 patients. If 10 patients are randomly selected for treatment with the first drug and another 10 independently selected for treatment with the second drug, an independent-samples experiment results.

However, the experimenter, knowing that blood pressure is influenced by age and weight, might decide to create pairs of patients so that within each of the resulting 10 pairs, age and weight were approximately equal (though there might be sizable differences between pairs). Then each drug would be given to a different patient within each pair for a total of 10 observations on each drug.

Without this matching (or "blocking"), one drug might appear to outperform the other just because patients in one sample were lighter and younger and thus more susceptible to a decrease in blood pressure than the heavier and older patients in the second sample. However, there is a price to be paid for pairing—a smaller number of degrees of freedom for the paired analysis—so we must ask when one type of experiment should be preferred to the other.

There is no straightforward and precise answer to this question, but there are some useful guidelines. If we have a choice between two t tests that are both valid (and carried out at the same level of significance α), we should prefer the test that has the larger number of degrees of freedom. The reason for this is that a larger number of degrees of freedom means smaller β for any fixed alternative value of the parameter or parameters. That is, for a fixed type I error probability, the probability of a type II error is decreased by increasing degrees of freedom.

However, if the experimental units are quite heterogeneous in their responses, it will be difficult to detect small but significant differences between two treatments. This is essentially what happened in the data set in Example 9.8; for both "treatments" (bottom water and surface water), there is great between-location variability, which tends to mask differences in treatments within locations. If there is a high positive correlation within experimental units or subjects, the variance of $\overline{D} = \overline{X} - \overline{Y}$ will be much smaller than the unpaired variance. Because of this reduced variance, it will be easier to detect a difference with paired samples than with independent samples. The pros and cons of pairing can now be summarized as follows.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 350 of 370. PageID #: 19147

1. If there is great heterogeneity between experimental units and a large correlation within experimental units (large positive ρ), then the loss in degrees of freedom will be compensated for by the increased precision associated with pairing, so a paired experiment is preferable to an independent-samples experiment.
2. If the experimental units are relatively homogeneous and the correlation within pairs is not large, the gain in precision due to pairing will be outweighed by the decrease in degrees of freedom, so an independent-samples experiment should be used.

Of course, values of σ_1^2, σ_2^2, and ρ will not usually be known very precisely, so an investigator will be required to make an educated guess as to whether Situation 1 or 2 obtains. In general, if the number of observations that can be obtained is large, then a loss in degrees of freedom (e.g., from 40 to 20) will not be serious; but if the number is small, then the loss (say, from 16 to 8) because of pairing may be serious if not compensated for by increased precision. Similar considerations apply when choosing between the two types of experiments to estimate $\mu_1 - \mu_2$ with a confidence interval.

EXERCISES Section 9.3 (36–48)

36. Consider the accompanying data on breaking load (kg/25 mm width) for various fabrics in both an unabraded condition and an abraded condition (**"The Effect of Wet Abrasive Wear on the Tensile Properties of Cotton and Polyester-Cotton Fabrics,"** ***J. Testing and Evaluation*, 1993: 84–93**). Use the paired t test, as did the authors of the cited article, to test H_0: $\mu_D = 0$ versus H_a: $\mu_D > 0$ at significance level .01.

	Fabric							
	1	2	3	4	5	6	7	8
U	36.4	55.0	51.5	38.7	43.2	48.8	25.6	49.8
A	28.5	20.0	46.0	34.5	36.5	52.5	26.5	46.5

37. Hexavalent chromium has been identified as an inhalation carcinogen and an air toxin of concern in a number of different locales. The article **"Airborne Hexavalent Chromium in Southwestern Ontario"** (***J. of Air and Waste Mgmnt. Assoc.*, 1997: 905–910**) gave the accompanying data on both indoor and outdoor concentration (nanograms/m^3) for a sample of houses selected from a certain region.

	House								
	1	2	3	4	5	6	7	8	9
Indoor	.07	.08	.09	.12	.12	.12	.13	.14	.15
Outdoor	.29	.68	.47	.54	.97	.35	.49	.84	.86

	House							
	10	11	12	13	14	15	16	17
Indoor	.15	.17	.17	.18	.18	.18	.18	.19
Outdoor	.28	.32	.32	1.55	.66	.29	.21	1.02

	House							
	18	19	20	21	22	23	24	25
Indoor	.20	.22	.22	.23	.23	.25	.26	.28
Outdoor	1.59	.90	.52	.12	.54	.88	.49	1.24

	House							
	26	27	28	29	30	31	32	33
Indoor	.28	.29	.34	.39	.40	.45	.54	.62
Outdoor	.48	.27	.37	1.26	.70	.76	.99	.36

a. Calculate a confidence interval for the population mean difference between indoor and outdoor concentrations using a confidence level of 95%, and interpret the resulting interval.

b. If a 34th house were to be randomly selected from the population, between what values would you predict the difference in concentrations to lie?

38. Adding computerized medical images to a database promises to provide great resources for physicians.

However, there are other methods of obtaining such information, so the issue of efficiency of access needs to be investigated. The article **"The Comparative Effectiveness of Conventional and Digital Image Libraries"(*J. of Audiovisual Media in Medicine*, 2001: 8–15)** reported on an experiment in which 13 computer-proficient medical professionals were timed both while retrieving an image from a library of slides and while retrieving the same image from a computer database with a Web front end.

Subject	1	2	3	4	5	6	7
Slide	30	35	40	25	20	30	35
Digital	25	16	15	15	10	20	7
Difference	5	19	25	10	10	10	28

Subject	8	9	10	11	12	13
Slide	62	40	51	25	42	33
Digital	16	15	13	11	19	19
Difference	46	25	38	14	23	14

a. Construct a comparative boxplot of times for the two types of retrieval, and comment on any interesting features.

b. Estimate the difference between true average times for the two types of retrieval in a way that conveys information about precision and reliability. Be sure to check the plausibility of any assumptions needed in your analysis. Does it appear plausible that the true average times for the two types of retrieval are identical? Why or why not?

39. Scientists and engineers frequently wish to compare two different techniques for measuring or determining the value of a variable. In such situations, interest centers on testing whether the mean difference in measurements is zero. The article **"Evaluation of the Deuterium Dilution Technique Against the Test Weighing Procedure for the Determination of Breast Milk Intake" (*Amer. J. of Clinical Nutr.*, 1983: 996–1003)** reports the accompanying data on amount of milk ingested by each of 14 randomly selected infants.

Infant	1	2	3	4	5	6	7	8	9	10	11	12	13	14
DD method	1509	1418	1561	1556	2169	1760	1098	1198	1479	1281	1414	1954	2174	2058
TW method	1498	1254	1336	1565	2000	1318	1410	1129	1342	1124	1468	1604	1722	1518
Difference	11	164	225	−9	169	442	−312	69	137	157	−54	350	452	540

a. Is it plausible that the population distribution of differences is normal?

b. Does it appear that the true average difference between intake values measured by the two methods is something other than zero? Determine the *P*-value of the test, and use it to reach a conclusion at significance level .05.

40. Lactation promotes a temporary loss of bone mass to provide adequate amounts of calcium for milk production. The paper **"Bone Mass Is Recovered from Lactation to Postweaning in Adolescent Mothers with Low Calcium Intakes" (*Amer. J. of Clinical Nutr.*, 2004: 1322–1326)** gave the following data on total body bone mineral content (TBBMC) (g) for a sample both during lactation (L) and in the postweaning period (P).

Subject	1	2	3	4	5	6	7	8	9	10
L	1928	2549	2825	1924	1628	2175	2114	2621	1843	2541
P	2126	2885	2895	1942	1750	2184	2164	2626	2006	2627

a. Does the data suggest that true average total body bone mineral content during postweaning exceeds that during lactation by more than 25 g? State and test the appropriate hypotheses using a significance level of .05. [*Note:* The appropriate normal probability plot shows some curvature but not enough to cast substantial doubt on a normality assumption.]

b. Calculate an upper confidence bound using a 95% confidence level for the true average difference between TBBMC during postweaning and during lactation.

c. Does the (incorrect) use of the two-sample t test to test the hypotheses suggested in (a) lead to the same conclusion that you obtained there? Explain.

41. Antipsychotic drugs are widely prescribed for conditions such as schizophrenia and bipolar disease. The article **"Cardiometabolic Risk of Second-Generation Antipsychotic Medications During First-Time Use in Children and Adolescents" (*J. of the Amer. Med. Assoc.*, 2009)** reported on body composition and metabolic changes for individuals who had taken various antipsychotic drugs for short periods of time.

a. The sample of 41 individuals who had taken aripiprazole had a mean change in total cholesterol (mg/dL) of 3.75, and the estimated standard error $s_D/\sqrt{n}$ was

3.878. Calculate a confidence interval with confidence level approximately 95% for the true average increase in total cholesterol under these circumstances (the cited article included this CI).

b. The article also reported that for a sample of 36 individuals who had taken quetiapine, the sample mean cholesterol level change and estimated standard error were 9.05 and 4.256, respectively. Making any necessary assumptions about the distribution of change in cholesterol level, does the choice of significance level impact your conclusion as to whether true average cholesterol level increases? Explain. [*Note:* The article included a *P*-value.]

c. For the sample of 45 individuals who had taken olanzapine, the article reported (7.38, 9.69) as a 95% CI for true average weight gain (kg). What is a 99% CI?

42. Many freeways have service (or logo) signs that give information on attractions, camping, lodging, food, and gas services prior to off-ramps. These signs typically do not provide information on distances. The article **"Evaluation of Adding Distance Information to Freeway-Specific Service (Logo) Signs" (*J. of Transp. Engr.*, 2011: 782–788)** reported that in one investigation, six sites along Virginia interstate highways where service signs are posted were selected. For each site, crash data was obtained for a three-year period before distance information was added to the service signs and for a one-year period afterward. The number of crashes per year before and after the sign changes were as follows:

Before:	15	26	66	115	62	64
After:	16	24	42	80	78	73

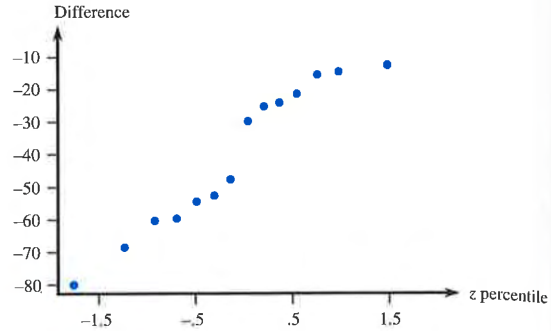
a. The cited article included the statement "A paired *t* test was performed to determine whether there was any change in the mean number of crashes before and after the addition of distance information on the signs." Carry out such a test. [*Note:* The relevant normal probability plot shows a substantial linear pattern.]

b. If a seventh site were to be randomly selected among locations bearing service signs, between what values would you predict the difference in number of crashes to lie?

43. Cushing's disease is characterized by muscular weakness due to adrenal or pituitary dysfunction. To provide effective treatment, it is important to detect childhood Cushing's disease as early as possible. Age at onset of symptoms and age at diagnosis (months) for 15 children suffering from the disease were given in the article **"Treatment of Cushing's Disease in Childhood and Adolescence by Transphenoidal Microadenomectomy" (*New Engl. J. of Med.*, 1984: 889)**. Here are the values of the differences between age at onset of symptoms and age at diagnosis:

−24	−12	−55	−15	−30	−60	−14	−21
−48	−12	−25	−53	−61	−69	−80	

a. Does the accompanying normal probability plot cast strong doubt on the approximate normality of the population distribution of differences?



b. Calculate a lower 95% confidence bound for the population mean difference, and interpret the resulting bound.

c. Suppose the (age at diagnosis) – (age at onset) differences had been calculated. What would be a 95% upper confidence bound for the corresponding population mean difference?

44. Refer back to the previous exercise.

a. By far the most frequently tested null hypothesis when data is paired is H_0: $\mu_D = 0$. Is that a sensible hypothesis in this context? Explain.

b. Carry out a test of hypotheses to decide whether there is compelling evidence for concluding that on average diagnosis occurs more than 25 months after the onset of symptoms.

45. Torsion during hip external rotation (ER) and extension may be responsible for certain kinds of injuries in golfers and other athletes. The article **"Hip Rotational Velocities During the Full Golf Swing" (*J. of Sports Science and Medicine*, 2009: 296–299)** reported on a study in which peak ER velocity and peak IR (internal rotation) velocity (both in deg.sec^{-1}) were determined for a sample of 15 female collegiate golfers during their swings. The following data was supplied by the article's authors.

Golfer	ER	IR	diff	*z* perc
1	−130.6	−98.9	−31.7	−1.28
2	−125.1	−115.9	−9.2	−0.97
3	−51.7	−161.6	109.9	0.34
4	−179.7	−196.9	17.2	−0.73
5	−130.5	−170.7	40.2	−0.34
6	−101.0	−274.9	173.9	0.97
7	−24.4	−275.0	250.6	1.83
8	−231.1	−275.7	44.6	−0.17
9	−186.8	−214.6	27.8	−0.52
10	−58.5	−117.8	59.3	0.00
11	−219.3	−326.7	107.4	0.17
12	−113.1	−272.9	159.8	0.73
13	−244.3	−429.1	184.8	1.28
14	−184.4	−140.6	−43.8	−1.83
15	−199.2	−345.6	146.4	0.52

46.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 353 of 370. PageID #: 19180

a. Is it plausible that the differences came from a normally distributed population?

b. The article reported that mean (± SD) = −145.3(68.0) for ER velocity and = −227.8(96.6) for IR velocity. Based just on this information, could a test of hypotheses about the difference between true average IR velocity and true average ER velocity be carried out? Explain.

c. The article stated that "The lead hip peak IR velocity was significantly greater than the trail hip ER velocity ($p = 0.003$, t value $= 3.65$)." (The phrasing suggests that an upper-tailed test was used.) Is that in fact the case? [*Note*: "$p = .033$" in Table 2 of the article is erroneous.]

46. Example 7.11 gave data on the modulus of elasticity obtained 1 minute after loading in a certain configuration. The cited article also gave the values of modulus of elasticity obtained 4 weeks after loading for the same lumber specimens. The data is presented here.

Observation	1 min	4 weeks	Difference
1	10,490	9,110	1380
2	16,620	13,250	3370
3	17,300	14,720	2580
4	15,480	12,740	2740
5	12,970	10,120	2850
6	17,260	14,570	2690
7	13,400	11,220	2180
8	13,900	11,100	2800
9	13,630	11,420	2210
10	13,260	10,910	2350
11	14,370	12,110	2260
12	11,700	8,620	3080
13	15,470	12,590	2880
14	17,840	15,090	2750
15	14,070	10,550	3520
16	14,760	12,230	2530

Calculate and interpret an upper confidence bound for the true average difference between 1-minute modulus and 4-week modulus; first check the plausibility of any necessary assumptions.

47. The article **"Slender High-Strength RC Columns Under Eccentric Compression"** (***Magazine of Concrete Res.*, 2005: 361–370**) gave the accompanying data on cylinder strength (MPa) for various types of columns cured under both moist conditions and laboratory drying conditions.

	Type					
	1	**2**	**3**	**4**	**5**	**6**
M:	82.6	87.1	89.5	88.8	94.3	80.0
LD:	86.9	87.3	92.0	89.3	91.4	85.9
	7	**8**	**9**	**10**	**11**	**12**
M:	86.7	92.5	97.8	90.4	94.6	91.6
LD:	89.4	91.8	94.3	92.0	93.1	91.3

a. Estimate the difference in true average strength under the two drying conditions in a way that conveys information about reliability and precision, and interpret the estimate. What does the estimate suggest about how true average strength under moist drying conditions compares to that under laboratory drying conditions?

b. Check the plausibility of any assumptions that underlie your analysis of (a).

48. Construct a paired data set for which $t = \infty$, so that the data is highly significant when the correct analysis is used, yet t for the two-sample t test is quite near zero, so the incorrect analysis yields an insignificant result.

9.4 Inferences Concerning a Difference Between Population Proportions

Having presented methods for comparing the means of two different populations, we now turn attention to the comparison of two population proportions. Regard an individual or object as a success S if he/she/it possesses some characteristic of interest (someone who graduated from college, a refrigerator with an icemaker, etc.). Let

$$p_1 = \text{the proportion of } S\text{'s in population } \# \ 1$$
$$p_2 = \text{the proportion of } S\text{'s in population } \# \ 2$$

Alternatively, $p_1(p_2)$ can be regarded as the probability that a randomly selected individual or object from the first (second) population is a success.

Suppose that a sample of size m is selected from the first population and independently a sample of size n is selected from the second one. Let X denote the number

of S's in the first sample and Y be the number of S's in the second. Independence of the two samples implies that X and Y are independent. Provided that the two sample sizes are much smaller than the corresponding population sizes, X and Y can be regarded as having binomial distributions. The natural estimator for $p_1 - p_2$, the difference in population proportions, is the corresponding difference in sample proportions $X/m - Y/n$.

PROPOSITION

Let $\hat{p}_1 = X/m$ and $\hat{p}_2 = Y/n$, where $X \sim \text{Bin}(m, p_1)$ and $Y \sim \text{Bin}(n, p_2)$ with X and Y independent variables. Then

$$E(\hat{p}_1 - \hat{p}_2) = p_1 - p_2$$

so $\hat{p}_1 - \hat{p}_2$ is an unbiased estimator of $p_1 - p_2$, and

$$V(\hat{p}_1 - \hat{p}_2) = \frac{p_1q_1}{m} + \frac{p_2q_2}{n} \quad (\text{where } q_i = 1 - p_i) \tag{9.3}$$

Proof Since $E(X) = mp_1$ and $E(Y) = np_2$,

$$E\left(\frac{X}{m} - \frac{Y}{n}\right) = \frac{1}{m}E(X) - \frac{1}{n}E(Y) = \frac{1}{m}mp_1 - \frac{1}{n}np_2 = p_1 - p_2$$

Since $V(X) = mp_1q_1$, $V(Y) = np_2q_2$, and X and Y are independent,

$$V\left(\frac{X}{m} - \frac{Y}{n}\right) = V\left(\frac{X}{m}\right) + V\left(\frac{Y}{n}\right) = \frac{1}{m^2}V(X) + \frac{1}{n^2}V(Y) = \frac{p_1q_1}{m} + \frac{p_2q_2}{n} \qquad \blacksquare$$

We will focus first on situations in which both m and n are large. Then because $\hat{p}_1$ and $\hat{p}_2$ individually have approximately normal distributions, the estimator $\hat{p}_1 - \hat{p}_2$ also has approximately a normal distribution. Standardizing $\hat{p}_1 - \hat{p}_2$ yields a variable Z whose distribution is approximately standard normal:

$$Z = \frac{\hat{p}_1 - \hat{p}_2 - (p_1 - p_2)}{\sqrt{\dfrac{p_1q_1}{m} + \dfrac{p_2q_2}{n}}}$$

A Large-Sample Test Procedure

The most general null hypothesis an investigator might consider would be of the form H_0: $p_1 - p_2 = \Delta_0$. Although for population means the case $\Delta_0 \neq 0$ presented no difficulties, for population proportions $\Delta_0 = 0$ and $\Delta_0 \neq 0$ must be considered separately. Since the vast majority of actual problems of this sort involve $\Delta_0 = 0$ (i.e., the null hypothesis $p_1 = p_2$), we'll concentrate on this case. When H_0: $p_1 - p_2 = 0$ is true, let p denote the common value of p_1 and p_2 (and similarly for q). Then the standardized variable

$$Z = \frac{\hat{p}_1 - \hat{p}_2 - 0}{\sqrt{pq\left(\dfrac{1}{m} + \dfrac{1}{n}\right)}} \tag{9.4}$$

has approximately a standard normal distribution when H_0 is true. However, this Z cannot serve as a test statistic because the value of p is unknown—H_0 asserts only that there is a common value of p, but does not say what that value is. A test statistic results from replacing p and q in (9.4) by appropriate estimators.

Assuming that $p_1 = p_2 = p$, instead of separate samples of size m and n from two different populations (two different binomial distributions), we really have a single sample of size $m + n$ from one population with proportion p. The total number of individuals in this combined sample having the characteristic of interest is $X + Y$. The natural estimator of p is then

$$\hat{p} = \frac{X + Y}{m + n} = \frac{m}{m + n} \cdot \hat{p}_1 + \frac{n}{m + n} \cdot \hat{p}_2 \qquad (9.5)$$

The second expression for $\hat{p}$ shows that it is actually a weighted average of estimators $\hat{p}_1$ and $\hat{p}_2$ obtained from the two samples. Using $\hat{p}$ and $\hat{q} = 1 - \hat{p}$ in place of p and q in (9.4) gives a test statistic having approximately a standard normal distribution when H_0 is true.

Null hypothesis: $H_0\colon p_1 - p_2 = 0$

Test statistic value (large samples): $z = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\dfrac{1}{m} + \dfrac{1}{n}\right)}}$

Alternative Hypothesis	P-Value Determination
$H_a\colon p_1 - p_2 > 0$	Area under the standard normal curve to the right of z
$H_a\colon p_1 - p_2 < 0$	Area under the standard normal curve to the left of z
$H_a\colon p_1 - p_2 \neq 0$	$2 \cdot$ (Area under the standard normal curve to the right of $\lvert z\rvert$)

The test can safely be used as long as $m\hat{p}_1$, $m\hat{q}_1$, $n\hat{p}_2$, and $n\hat{q}_2$ are all at least 10.

EXAMPLE 9.11 The article **"Aspirin Use and Survival After Diagnosis of Colorectal Cancer" (*J. of the Amer. Med. Assoc.*, 2009: 649–658)** reported that of 549 study participants who regularly used aspirin after being diagnosed with colorectal cancer, there were 81 colorectal cancer-specific deaths, whereas among 730 similarly diagnosed individuals who did not subsequently use aspirin, there were 141 colorectal cancer-specific deaths. Does this data suggest that the regular use of aspirin after diagnosis will decrease the incidence rate of colorectal cancer-specific deaths? Let's test the appropriate hypotheses using a significance level of .05.

The parameter of interest is the difference $p_1 - p_2$, where p_1 is the true proportion of deaths for those who regularly used aspirin and p_2 is the true proportion of deaths for those who did not use aspirin. The use of aspirin is beneficial if $p_1 < p_2$, which corresponds to a negative difference between the two proportions. The relevant hypotheses are therefore

$$H_0\colon p_1 - p_2 = 0 \qquad \text{versus} \qquad H_a\colon p_1 - p_2 < 0$$

Parameter estimates are $\hat{p}_1 = 81/549 = .1475$, $\hat{p}_2 = 141/730 = .1932$, and $\hat{p} = (81 + 141)/(549 + 730) = .1736$. A z test is appropriate here because all of $m\hat{p}_1$, $m\hat{q}_1$, $n\hat{p}_2$, and $n\hat{q}_2$ are at least 10. The resulting test statistic value is

$$z = \frac{.1475 - .1932}{\sqrt{(.1736)(.8264)\left(\dfrac{1}{549} + \dfrac{1}{730}\right)}} = \frac{-.0457}{.021397} = -2.14$$

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 358 of 370. PageID #: 19183

The corresponding P-value for a lower-tailed z test is $\Phi(-2.14) = .0162$. Because $.0162 \le .05$, the null hypothesis can be rejected at significance level .05. So anyone adopting this significance level would be convinced that the use of aspirin in these circumstances is beneficial. However, someone looking for more compelling evidence might select a significance level .01 and then not be persuaded. ■

Type II Error Probabilities and Sample Sizes

Here the determination of β is a bit more cumbersome than it was for other large-sample tests. The reason is that the denominator of Z is an estimate of the standard deviation of $\hat{p} - \hat{p}_2$, assuming that $p_1 = p_2 = p$. When H_0 is false, $\hat{p}_1 - \hat{p}_2$ must be restandardized using

$$\sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{p_1 q_1}{m} + \frac{p_2 q_2}{n}} \tag{9.6}$$

The form of σ implies that β is not a function of just $p_1 - p_2$, so we denote it by $\beta(p_1, p_2)$.

Alternative Hypothesis	$\beta(p_1, p_2)$
H_a: $p_1 - p_2 > 0$	$\Phi\left[\dfrac{z_\alpha\sqrt{\bar{p}\,\bar{q}\left(\frac{1}{m}+\frac{1}{n}\right)} - (p_1 - p_2)}{\sigma}\right]$
H_a: $p_1 - p_2 < 0$	$1 - \Phi\left[\dfrac{-z_\alpha\sqrt{\bar{p}\,\bar{q}\left(\frac{1}{m}+\frac{1}{n}\right)} - (p_1 - p_2)}{\sigma}\right]$
H_a: $p_1 - p_2 \ne 0$	$\Phi\left[\dfrac{z_{\alpha/2}\sqrt{\bar{p}\,\bar{q}\left(\frac{1}{m}+\frac{1}{n}\right)} - (p_1 - p_2)}{\sigma}\right]$ $-\Phi\left[\dfrac{-z_{\alpha/2}\sqrt{\bar{p}\,\bar{q}\left(\frac{1}{m}+\frac{1}{n}\right)} - (p_1 - p_2)}{\sigma}\right]$

where $\bar{p} = (mp_1 + np_2)/(m+n)$, $\bar{q} = (mq_1 + nq_2)/(m+n)$, and σ is given by (9.6).

Proof For the upper-tailed test (H_a: $p_1 - p_2 > 0$),

$$\beta(p_1, p_2) = P\left[\hat{p}_1 - \hat{p}_2 < z_\alpha\sqrt{\hat{p}\hat{q}\left(\frac{1}{m}+\frac{1}{n}\right)}\right]$$

$$= P\left[\frac{(\hat{p}_1 - \hat{p}_2 - (p_1 - p_2))}{\sigma} < \frac{z_\alpha\sqrt{\hat{p}\hat{q}\left(\frac{1}{m}+\frac{1}{n}\right)} - (p_1 - p_2)}{\sigma}\right]$$

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 357 of 370. PageID #: 19184

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When m and n are both large,

$$\hat{p} = (m\hat{p}_1 + n\hat{p}_2)/(m+n) \approx (mp_1 + np_2)/(m+n) = \bar{p}$$

and $\hat{q} \approx \bar{q}$, which yields the previous (approximate) expression for $\beta(p_1, p_2)$. ■

Alternatively, for specified p_1, p_2 with $p_1 - p_2 = d$, the sample sizes necessary to achieve $\beta(p_1, p_2) = \beta$ can be determined. For example, for the upper-tailed test, we equate $-z_\beta$ to the argument of $\Phi(\cdot)$ (i.e., what's inside the parentheses) in the foregoing box. If $m = n$, there is a simple expression for the common value.

> For the case $m = n$, the level α test has type II error probability β at the alternative values p_1, p_2 with $p_1 - p_2 = d$ when
>
> $$n = \frac{\left[z_\alpha\sqrt{(p_1 + p_2)(q_1 + q_2)/2} + z_\beta\sqrt{p_1q_1 + p_2q_2}\right]^2}{d^2} \qquad (9.7)$$
>
> for an upper- or lower-tailed test, with $\alpha/2$ replacing α for a two-tailed test.

EXAMPLE 9.12 One of the truly impressive applications of statistics occurred in connection with the design of the 1954 Salk polio-vaccine experiment and analysis of the resulting data. Part of the experiment focused on the efficacy of the vaccine in combating paralytic polio. Because it was thought that without a control group of children, there would be no sound basis for assessment of the vaccine, it was decided to administer the vaccine to one group and a placebo injection (visually indistinguishable from the vaccine but known to have no effect) to a control group. For ethical reasons and also because it was thought that the knowledge of vaccine administration might have an effect on treatment and diagnosis, the experiment was conducted in a **double-blind** manner. That is, neither the individuals receiving injections nor those administering them actually knew who was receiving vaccine and who was receiving the placebo (samples were numerically coded). (Remember: at that point it was not at all clear whether the vaccine was beneficial.)

Let p_1 and p_2 be the probabilities of a child getting paralytic polio for the control and treatment conditions, respectively. The objective was to test H_0: $p_1 - p_2 = 0$ versus H_a: $p_1 - p_2 > 0$ (the alternative states that a vaccinated child is less likely to contract polio than an unvaccinated child). Supposing the true value of p_1 is .0003 (an incidence rate of 30 per 100,000), the vaccine would be a significant improvement if the incidence rate was halved—that is, $p_2 = .00015$. Using a level $\alpha = .05$ test, it would then be reasonable to ask for sample sizes for which $\beta = .1$ when $p_1 = .0003$ and $p_2 = .00015$. Assuming equal sample sizes, the required n is obtained from (9.7) as

$$n = \frac{\left[1.645\sqrt{(.5)(.00045)(1.99955)} + 1.28\sqrt{(.00015)(.99985) + (.0003)(.9997)}\right]^2}{(.0003 - .00015)^2}$$

$$= [(.0349 + .0271)/.00015]^2 \approx 171{,}000$$

The actual data for this experiment follows. Sample sizes of approximately 200,000 were used. The reader can easily verify that $z = 6.43$—a highly significant value. The vaccine was judged a resounding success!

Placebo: $m = 201{,}229$, x = number of cases of paralytic polio = 110
Vaccine: $n = 200{,}745$, $y = 33$ ■

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 358 of 370. PageID #: 19185

A Large-Sample Confidence Interval

As with means, many two-sample problems involve the objective of comparison through hypothesis testing, but sometimes an interval estimate for $p_1 - p_2$ is appropriate. Both $\hat{p}_1 = X/m$ and $\hat{p}_2 = Y/n$ have approximate normal distributions when m and n are both large. If we identify θ with $p_1 - p_2$, then $\hat{\theta} = \hat{p}_1 - \hat{p}_2$ satisfies the conditions necessary for obtaining a large-sample CI. In particular, the estimated standard deviation of $\hat{\theta}$ is $\sqrt{(\hat{p}_1\hat{q}_1/m) + (\hat{p}_2\hat{q}_2/n)}$. The general $100(1 - \alpha)\%$ interval $\hat{\theta} \pm z_{\alpha/2} \cdot \hat{\sigma}_{\hat{\theta}}$ then takes the following form.

> A CI for $p_1 - p_2$ with confidence level approximately $100(1 - \alpha)\%$ is
>
> $$\hat{p}_1 - \hat{p}_2 \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{m} + \frac{\hat{p}_2\hat{q}_2}{n}}$$
>
> This interval can safely be used as long as $m\hat{p}_1$, $m\hat{q}_1$, $n\hat{p}_2$, and $n\hat{q}_2$ are all at least 10.

Notice that the estimated standard deviation of $\hat{p}_1 - \hat{p}_2$ (the square-root expression) is different here from what it was for hypothesis testing when $\Delta_0 = 0$.

Recent research has shown that the actual confidence level for the traditional CI just given can sometimes deviate substantially from the nominal level (the level you think you are getting when you use a particular z critical value—e.g., 95% when $z_{\alpha/2} = 1.96$). The suggested improvement is to add one success and one failure to each of the two samples and then replace the $\hat{p}$'s and $\hat{q}$'s in the foregoing formula by $\tilde{p}$'s and $\tilde{q}$'s where $\tilde{p}_1 = (x + 1)/(m + 2)$, etc. This modified interval can also be used when sample sizes are quite small.

EXAMPLE 9.13 Do people who work long hours have more trouble sleeping? An investigation into this issue was described in the article **"Long Working Hours and Sleep Disturbances: The Whitehall II Prospective Cohort Study" (*Sleep*, 2009: 737–745)**. In one sample of 1501 British civil servants who worked more than 40 hours a week, 750 said they usually get less than 7 hours of sleep per night. In another sample of 958 British civil servants who worked between 35 and 40 hours per week, 407 said they usually get less than 7 hours of sleep per night. The investigators believed that these samples were representative of the populations to which they belong.

Let p_1 denote the proportion of British civil servants working more than 40 hours per week who usually get less than 7 hours of sleep per night, and let p_2 be the corresponding proportion for the 35–40 hours population. The point estimates of p_1 and p_2 are

$$\hat{p}_1 = \frac{750}{1501} = .500,\ \hat{p}_2 = \frac{407}{958} = .425$$

from which $\hat{q}_1 = .500$, $\hat{q}_2 = .575$. All quantities $m\hat{p}_1$, $m\hat{q}_1$, $n\hat{p}_2$, $n\hat{q}_2$ are much larger than 10, so the large-sample CI for $p_1 - p_2$ can be used. The 99% interval is

$$.500 - .425 \pm 2.58\sqrt{\frac{(.500)(.500)}{1501} + \frac{(.425)(.575)}{958}} = .075 \pm (2.58)(.020534)$$
$$= 0.75 \pm 0.53 = (.022, .128)$$

At the 99% confidence level, we estimate that the proportion of those working longer hours who usually get less than 7 hours of sleep per night exceeds the corresponding

proportion for those who work fewer hours by between .022 and .128. The fact that this interval includes only positive values suggests that those who work longer hours tend to get less sleep. But the study is observational rather than randomized controlled, so it would be dangerous to infer a causal relationship between work hours and amount of sleep. Because of the large sample sizes, the modified interval that uses $\tilde{p}_1$, $\tilde{q}_1$, $\tilde{p}_2$, and $\tilde{q}_2$ is identical to the one we calculated. ■

Small-Sample Inferences

On occasion an inference concerning $p_1 - p_2$ may have to be based on samples for which at least one sample size is small. Appropriate methods for such situations are not as straightforward as those for large samples, and there is more controversy among statisticians as to recommended procedures. One frequently used test, called the Fisher–Irwin test, is based on the hypergeometric distribution. Your friendly neighborhood statistician can be consulted for more information.

EXERCISES Section 9.4 (49–58)

49. Consider the following two questions designed to assess quantitative literacy:

a. What is 15% of 1000?

b. A store is offering a 15% off sale on all TVs. The most popular television is normally priced at $1000. How much money would a customer save on the television during this sale?

Suppose the first question is asked of 200 randomly selected college students, with 164 answering correctly; the second one is asked of a different random sample of 200 college students, resulting in 140 correct responses (the sample percentages agree with those given in the article **"Using the Right Yardstick: Assessing Financial Literacy Measures by Way of Financial Well-Being,"** ***J. of Consumer Affairs*, 2013: 243–262**; the investigators found that those who answered such questions correctly, particularly questions with context, were significantly more successful in their investment decisions than those who did not answer correctly). Carry out a test of hypotheses at significance level .05 to decide if the true proportion of correct responses to the question without context exceeds that for the one with context.

50. Recent incidents of food contamination have caused great concern among consumers. The article **"How Safe Is That Chicken?" (*Consumer Reports*, Jan. 2010: 19–23)** reported that 35 of 80 randomly selected Perdue brand broilers tested positively for either campylobacter or salmonella (or both), the leading bacterial causes of food-borne disease, whereas 66 of 80 Tyson brand broilers tested positive.

a. Does it appear that the true proportion of non-contaminated Perdue broilers differs from that for the Tyson brand? Carry out a test of hypotheses using a significance level .01.

b. If the true proportions of non-contaminated chickens for the Perdue and Tyson brands are .50 and .25, respectively, how likely is it that the null hypothesis of equal proportions will be rejected when a .01 significance level is used and the sample sizes are both 80?

51. It is well known that a placebo, a fake medication or treatment, can sometimes have a positive effect just because patients often expect the medication or treatment to be helpful. The article **"Beware the Nocebo Effect" (*New York Times*, Aug. 12, 2012)** gave examples of a less familiar phenomenon, the tendency for patients informed of possible side effects to actually experience those side effects. The article cited a study reported in ***The Journal of Sexual Medicine*** in which a group of patients diagnosed with benign prostatic hyperplasia was randomly divided into two subgroups. One subgroup of size 55 received a compound of proven efficacy along with counseling that a potential side effect of the treatment was erectile dysfunction. The other subgroup of size 52 was given the same treatment without counseling. The percentage of the no-counseling subgroup that reported one or more sexual side effects was 15.3%, whereas 43.6% of the counseling subgroup reported at least one sexual side effect. State and test the appropriate hypotheses at significance level .05 to decide whether the nocebo effect is operating here. [*Note:* The estimated expected number of "successes" in the no-counseling sample is a bit shy of 10, but not by enough to be of great concern (some sources use a less conservative cutoff of 5 rather than 10).]

52. Do teachers find their work rewarding and satisfying? The article **"Work-Related Attitudes" (*Psychological Reports*, 1991: 443–450)** reports the results of a survey

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 360 of 370. PageID #: 19187

of 395 elementary school teachers and 266 high school teachers. Of the elementary school teachers, 224 said they were very satisfied with their jobs, whereas 126 of the high school teachers were very satisfied with their work. Estimate the difference between the proportion of all elementary school teachers who are very satisfied and all high school teachers who are very satisfied by calculating and interpreting a CI.

53. Olestra is a fat substitute approved by the FDA for use in snack foods. Because there have been anecdotal reports of gastrointestinal problems associated with olestra consumption, a randomized, double-blind, placebo-controlled experiment was carried out to compare olestra potato chips to regular potato chips with respect to GI symptoms (**"Gastrointestinal Symptoms Following Consumption of Olestra or Regular Triglyceride Potato Chips,"** ***J. of the Amer. Med. Assoc.*, 1998: 150–152**). Among 529 individuals in the TG control group, 17.6% experienced an adverse GI event, whereas among the 563 individuals in the olestra treatment group, 15.8% experienced such an event.

a. Carry out a test of hypotheses at the 5% significance level to decide whether the incidence rate of GI problems for those who consume olestra chips according to the experimental regimen differs from the incidence rate for the TG control treatment.

b. If the true percentages for the two treatments were 15% and 20%, respectively, what sample sizes ($m = n$) would be necessary to detect such a difference with probability .90?

54. Teen Court is a juvenile diversion program designed to circumvent the formal processing of first-time juvenile offenders within the juvenile justice system. The article **"An Experimental Evaluation of Teen Courts"** (***J. of Experimental Criminology*, 2008: 137–163**) reported on a study in which offenders were randomly assigned either to Teen Court or to the traditional Department of Juvenile Services method of processing. Of the 56 TC individuals, 18 subsequently recidivated (look it up!) during the 18-month follow-up period, whereas 12 of the 51 DJS individuals did so. Does the data suggest that the true proportion of TC individuals who recidivate during the specified follow-up period differs from the proportion of DJS individuals who do so? State and test the relevant hypotheses using a significance level of .10.

55. In medical investigations, the ratio $\theta = p_1/p_2$ is often of more interest than the difference $p_1 - p_2$ (e.g., individuals given treatment 1 are how many times as likely to recover as those given treatment 2?). Let $\hat{\theta} = \hat{p}_1/\hat{p}_2$. When m and n are both large, the statistic $\ln(\hat{\theta})$ has approximately a normal distribution with approximate mean value $\ln(\theta)$ and approximate standard deviation $[(m - x)/(mx) + (n - y)/(ny)]^{1/2}$.

a. Use these facts to obtain a large-sample 95% CI formula for estimating $\ln(\theta)$, and then a CI for θ itself.

b. Return to the heart-attack data of Example 1.3, and calculate an interval of plausible values for θ at the 95% confidence level. What does this interval suggest about the efficacy of the aspirin treatment?

56. Sometimes experiments involving success or failure responses are run in a paired or before/after manner. Suppose that before a major policy speech by a political candidate, n individuals are selected and asked whether (S) or not (F) they favor the candidate. Then after the speech the same n people are asked the same question. The responses can be entered in a table as follows:

		After S	After F
Before	S	x_1	x_2
	F	x_3	x_4

where $x_1 + x_2 + x_3 + x_4 = n$. Let p_1, p_2, p_3, and p_4 denote the four cell probabilities, so that $p_1 = P(S$ before and S after), and so on. We wish to test the hypothesis that the true proportion of supporters (S) after the speech has not increased against the alternative that it has increased.

a. State the two hypotheses of interest in terms of p_1, p_2, p_3, and p_4.

b. Construct an estimator for the after/before difference in success probabilities.

c. When n is large, it can be shown that the rv $(X_i - X_j)/n$ has approximately a normal distribution with variance given by $[p_i + p_j - (p_i - p_j)^2]/n$. Use this to construct a test statistic with approximately a standard normal distribution when H_0 is true (the result is called McNemar's test).

d. If $x_1 = 350$, $x_2 = 150$, $x_3 = 200$, and $x_4 = 300$, what do you conclude?

57. Two different types of alloy, A and B, have been used to manufacture experimental specimens of a small tension link to be used in a certain engineering application. The ultimate strength (ksi) of each specimen was determined, and the results are summarized in the accompanying frequency distribution.

	A	B
26 – < 30	6	4
30 – < 34	12	9
34 – < 38	15	19
38 – < 42	7	10
	$m = 40$	$m = 42$

Compute a 95% CI for the difference between the true proportions of all specimens of alloys A and B that have an ultimate strength of at least 34 ksi.

58. Using the traditional formula, a 95% CI for $p_1 - p_2$ is to be constructed based on equal sample sizes from the two populations. For what value of n (= m) will the resulting interval have a width at most of .1, irrespective of the results of the sampling?

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 361 of 370. PageID #: 19188

9.5 Inferences Concerning Two Population Variances

Methods for comparing two population variances (or standard deviations) are occasionally needed, though such problems arise much less frequently than those involving means or proportions. For the case in which the populations under investigation are normal, the procedures are based on a new family of probability distributions.

The *F* Distribution

The F probability distribution has two parameters, denoted by ν_1 and ν_2. The parameter ν_1 is called the *number of numerator degrees of freedom*, and ν_2 is the *number of denominator degrees of freedom;* here ν_1 and ν_2 are positive integers. A random variable that has an F distribution cannot assume a negative value. Since the density function is complicated and will not be used explicitly, we omit the formula. There is an important connection between an F variable and chi-squared variables. If X_1 and X_2 are independent chi-squared rv's with ν_1 and ν_2 df, respectively, then the rv

$$F = \frac{X_1/\nu_1}{X_2/\nu_2} \tag{9.8}$$

(the ratio of the two chi-squared variables divided by their respective degrees of freedom), can be shown to have an F distribution.

Figure 9.7 illustrates the graph of a typical F density function. Analogous to the notation $t_{\alpha,\nu}$ and $\chi^2_{\alpha,\nu}$, we use F_{α,ν_1,ν_2} for the value on the horizontal axis that captures α of the area under the F density curve with ν_1 and ν_2 df in the upper tail. The density curve is not symmetric, so it would seem that both upper- and lower-tail critical values must be tabulated. This is not necessary, though, because of the fact that $F_{1-\alpha,\nu_1,\nu_2} = 1/F_{\alpha,\nu_2,\nu_1}$.

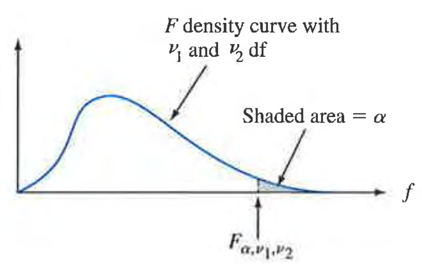

Figure 9.7 An *F* density curve and critical value

Appendix Table A.9 gives F_{α,ν_1,ν_2} for $\alpha = .10$, $.05$, $.01$, and $.001$, and various values of ν_1 (in different columns of the table) and ν_2 (in different groups of rows of the table). For example, $F_{.05,6,10} = 3.22$ and $F_{.05,10,6} = 4.06$. The critical value $F_{.95,6,10}$, which captures .95 of the area to its right (and thus .05 to the left) under the F curve with $\nu_1 = 6$ and $\nu_2 = 10$, is $F_{.95,6,10} = 1/F_{.05,10,6} = 1/4.06 = .246$.

The *F* Test for Equality of Variances

A test procedure for hypotheses concerning the ratio σ_1^2/σ_2^2 is based on the following result.

THEOREM

Let $X_1,\ldots, X_m$ be a random sample from a normal distribution with variance σ_1^2, let $Y_1,\ldots, Y_n$ be another random sample (independent of the X_i's) from a normal distribution with variance σ_2^2, and let S_1^2 and S_2^2 denote the two sample variances. Then the rv

$$F = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \tag{9.9}$$

has an F distribution with $\nu_1 = m - 1$ and $\nu_2 = n - 1$.

This theorem results from combining (9.8) with the fact that the variables $(m - 1)S_1^2/\sigma_1^2$ and $(n - 1)S_2^2/\sigma_2^2$ each have a chi-squared distribution with $m - 1$ and $n - 1$ df, respectively (see Section 7.4). Because F involves a ratio rather than a difference, the test statistic is the ratio of sample variances. The claim that $\sigma_1^2 = \sigma_2^2$ is implausible if the ratio differs by too much from 1.

Recall that the P-value for an upper-tailed t test is the area under an appropriate t curve to the right of the calculated t, whereas for a lower-tailed test the P-value is the area under the curve to the left of t. Analogously, the P-value for an upper-tailed F test is the area under an appropriate F curve (the one with specified numerator and denominator dfs) to the right of f, and the P-value for a lower-tailed test is the area under an F curve to the left of f. Because t curves are symmetric, the P-value for a two-tailed test is double the captured lower tail area if t is negative and double the captured upper tail area if t is positive. Although F curves are not symmetric, by analogy the P-value for a two-tailed F test is twice the captured lower tail area if f is below the median and twice the captured upper tail area if it is above the median. Figure 9.8 illustrates this for an upper-tailed test based on $\nu_1 = 4$ and $\nu_2 = 6$.

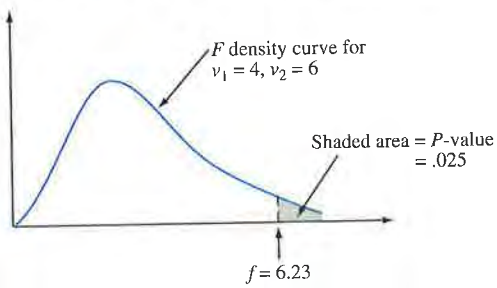

Figure 9.8 A P-value for an upper-tailed F test

Null hypothesis: $H_0\colon \sigma_1^2 = \sigma_2^2$

Test statistic value: $f = s_1^2/s_2^2$

Alternative Hypothesis	P-Value Determination
$H_a\colon \sigma_1^2 > \sigma_2^2$	A_R = Area under the $F_{m-1,\,n-1}$ curve to the right of f
$H_a\colon \sigma_1^2 < \sigma_2^2$	A_L = Area under the $F_{m-1,\,n-1}$ curve to the left of f
$H_a\colon \sigma_1^2 \neq \sigma_2^2$	$2 \cdot \min(A_R, A_L)$

Assumption: The population distributions are both normal, and the two random samples are independent of one another.

Case: 4:08-cv-00160-BYP Doc #: 383-2 Filed: 09/01/17 363 of 370. PageID #: 19190

Tabulation of F-curve upper-tail areas is much more cumbersome than for t curves because two df's are involved. For each combination of ν_1 and ν_2, our F table gives only the four critical values that capture areas .10, .05, .01, and .001. Because of this, the table will generally provide only an upper or lower bound (or both) on the P-value. For example, suppose the test is upper-tailed and based on 4 numerator df and 6 denominator df. If $f = 5.82$, then the P-value is the area under the $F_{4,6}$ curve to the right of 5.82. Because $F_{.05,4,6} = 4.53$, the area to the right of 4.53 is by definition .05. Similarly, $F_{.01,4,6} = 9.15$ implies that the area under the curve to the right of this value is .01. Since 5.82 lies in between 4.53 and 9.15, the area to the right of 5.82 must be between .01 and .05. That is, $.01 < P\text{-value} < .05$. Figure 9.9 shows what can be said about the P-value depending on where f falls relative to the four relevant tabulated critical values.

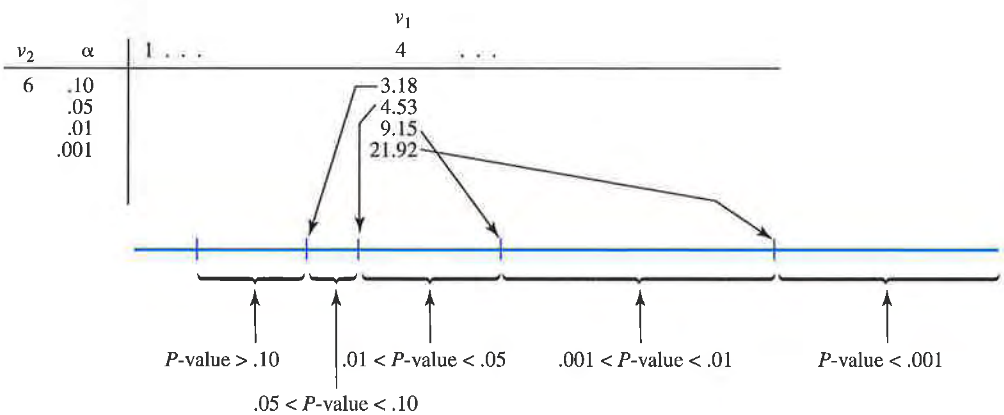

Figure 9.9 Obtaining P-value information from the F table for an upper-tailed F test

Again considering a test with $\nu_1 = 4$ and $\nu_2 = 6$,

$$f = 5.82 \Rightarrow .01 < P\text{-value} < .05$$
$$f = 2.16 \Rightarrow P\text{-value} > .10$$
$$f = 25.03 \Rightarrow P\text{-value} < .001$$

Only if f equals a tabulated value do we obtain an exact P-value (e.g., if $f = 4.53$, then $P\text{-value} = .05$). Once we know that $.01 < P\text{-value} < .05$, H_0 would be rejected at a significance level of .05 but not at a level of .01. When $P\text{-value} < .001$, H_0 should be rejected at any reasonable significance level.

The F tests discussed in succeeding chapters will all be upper-tailed. If, however, a lower-tailed F test is appropriate, then lower-tailed critical values should be obtained as described earlier so that a bound or bounds on the P-value can be established. In the case of a two-tailed test, the bound or bounds from a one-tailed test should be multiplied by 2. For example, if $f = 5.82$ when $\nu_1 = 4$ and $\nu_2 = 6$, then since 5.82 falls between the .05 and .01 critical values, $2(.01) < P\text{-value} < 2(.05)$, giving $.02 < P\text{-value} < .10$. H_0 would then be rejected if $\alpha = .10$ but not if $\alpha = .01$. In this case, we cannot say from our table what conclusion is appropriate when $\alpha = .05$ (since we don't know whether the P-value is smaller or larger than this). However, statistical software shows that the area to the right of 5.82 under this F curve is .029, so the P-value is .058 and the null hypothesis should therefore not be rejected at level .05. Various statistical software packages will, of course, provide an exact P-value for any F test.

Case: 4:08-cv-00160-BYP Doc #: 383-12 Filed: 09/01/17 364 of 370. PageID #: 191991

EXAMPLE 9.14 A random sample of 200 vehicles traveling on gravel roads in a county with a posted speed limit of 35 mph on such roads resulted in a sample mean speed of 37.5 mph and a sample standard deviation of 8.6 mph, whereas another random sample of 200 vehicles in a county with a posted speed limit of 55 mph resulted in a sample mean and sample standard deviation of 35.8 mph and 9.2 mph, respectively (these means and standard deviations were reported in the article **"Evaluation of Criteria for Setting Speed Limits on Gravel Roads" (*J. of Transp. Engr.*, 2011: 57–63)**; the actual sample sizes result in dfs that exceed the largest of those in our F table). Let's carry out a test at significance level .10 to decide whether the two population distribution variances are identical.

1. σ_1^2 is the variance of the speed distribution on the 35 mph roads, and σ_2^2 is the variance of the speed distribution on 55 mph roads.
2. H_0: $\sigma_1^2 = \sigma_2^2$
3. H_a: $\sigma_1^2 \neq \sigma_2^2$
4. Test statistic value: $f = s_1^2/s_2^2$
5. Calculation: $f = (8.6)^2/(9.2)^2 = .87$
6. P-value determination: .87 lies in the lower tail of the F curve with 199 numerator df and 199 denominator df. A glance at the F table shows that $F_{.10,199,199} \approx F_{.10,200,200} \approx 1.20$ (consult the $\nu_1 = 120$ and $\nu_1 = 1000$ columns), implying $F_{.90,199,199} \approx 1/1.20 = .83$ (these values are confirmed by software). That is, the area under the relevant F curve to the left of .83 is .10. Thus the area under the curve to the left of .87 exceeds .10, and so P-value $> 2(.10) = .2$ (software gives .342).
7. The P-value clearly exceeds the mandated significance level. The null hypothesis therefore cannot be rejected; it is plausible that the two speed distribution variances are identical.

The sample sizes in the cited article were 2665 and 1868, respectively, and the P-value reported there was .0008. So for the actual data, the hypothesis of equal variances would be rejected not only at significance level .10—in contrast to our conclusion—but also at level .05, .01, and even .001. This illustrates again how quite large sample sizes can magnify a small difference in estimated values. Note also that the sample mean speed for the county with the lower posted speed limit was higher than for the county with the lower limit, a counterintuitive result that surprised the investigators; and because of the very large sample sizes, this difference in means is highly statistically significant. ■

A Confidence Interval for σ_1/σ_2

The CI for σ_1^2/σ_2^2 is based on replacing F in the probability statement

$$P(F_{1-\alpha/2,\nu_1,\nu_2} < F < F_{\alpha/2,\nu_1,\nu_2}) = 1 - \alpha$$

by the F variable (9.9) and manipulating the inequalities to isolate σ_1^2/σ_2^2. An interval for σ_1/σ_2 results from taking the square root of each limit. The details are left for an exercise.

EXERCISES Section 9.5 (59–66)

59. Obtain or compute the following quantities:
   a. $F_{.05,5,8}$ b. $F_{.05,8,5}$ c. $F_{.95,5,8}$ d. $F_{.95,8,5}$
   e. The 99th percentile of the F distribution with $\nu_1 = 10$, $\nu_2 = 12$
   f. The 1st percentile of the F distribution with $\nu_1 = 10$, $\nu_2 = 12$
   g. $P(F \leq 6.16)$ for $\nu_1 = 6$, $\nu_2 = 4$
   h. $P(.177 \leq F \leq 4.74)$ for $\nu_1 = 10$, $\nu_2 = 5$

60. Give as much information as you can about the *P*-value of the *F* test in each of the following situations:

a. $\nu_1 = 5, \nu_2 = 10$, upper-tailed test, $f = 4.75$
b. $\nu_1 = 5, \nu_2 = 10$, upper-tailed test, $f = 2.00$
c. $\nu_1 = 5, \nu_2 = 10$, two-tailed test, $f = 5.64$
d. $\nu_1 = 5, \nu_2 = 10$, lower-tailed test, $f = .200$
e. $\nu_1 = 35, \nu_2 = 20$, upper-tailed test, $f = 3.24$

61. Return to the data on maximum lean angle given in Exercise 28 of this chapter. Carry out a test at significance level .10 to see whether the population standard deviations for the two age groups are different (normal probability plots support the necessary normality assumption).

62. Refer to Example 9.7. Does the data suggest that the standard deviation of the strength distribution for fused specimens is smaller than that for not-fused specimens? Carry out a test at significance level .01.

63. Toxaphene is an insecticide that has been identified as a pollutant in the Great Lakes ecosystem. To investigate the effect of toxaphene exposure on animals, groups of rats were given toxaphene in their diet. The article **"Reproduction Study of Toxaphene in the Rat" (*J. of Environ. Sci. Health*, 1988: 101–126)** reports weight gains (in grams) for rats given a low dose (4 ppm) and for control rats whose diet did not include the insecticide. The sample standard deviation for 23 female control rats was 32 g and for 20 female low-dose rats was 54 g. Does this data suggest that there is more variability in low-dose weight gains than in control weight gains? Assuming normality, carry out a test of hypotheses at significance level .05.

64. The following observations are on time (h) for a AA 1.5-volt alkaline battery to reach a 0.8 voltage (**"Comparing the Lifetimes of Two Brands of Batteries," *J. of Statistical Educ.*, 2013, online**):

Energizer:	8.65	8.74	8.91	8.72	8.85
Ultracell:	8.76	8.81	8.81	8.70	8.73
Energizer:	8.52	8.62	8.68	8.86	
Ultracell:	8.76	8.68	8.64	8.79	

Normal probability plots support the assumption that the population distributions are normal. Does the data suggest that the variance of the Energizer population distribution differs from that of the Ultracell population distribution? Test the relevant hypotheses using a significance level of .05. [*Note:* The two-sample *t* test for equality of population means gives a *P*-value of .763.] The Energizer batteries are much more expensive than the Ultracell batteries. Would you pay the extra money?

65. The article **"Enhancement of Compressive Properties of Failed Concrete Cylinders with Polymer Impregnation" (*J. of Testing and Evaluation*, 1977: 333–337)** reports the following data on impregnated compressive modulus (psi $\times 10^6$) when two different polymers were used to repair cracks in failed concrete.

Epoxy	1.75	2.12	2.05	1.97
MMA prepolymer	1.77	1.59	1.70	1.69

Obtain a 90% CI for the ratio of variances by first using the method suggested in the text to obtain a general confidence interval formula.

66. Reconsider the data of Example 9.6, and calculate a 95% upper confidence bound for the ratio of the standard deviation of the triacetate porosity distribution to that of the cotton porosity distribution.

SUPPLEMENTARY EXERCISES (67–95)

67. The accompanying summary data on compression strength (lb) for $12 \times 10 \times 8$ in. boxes appeared in the article **"Compression of Single-Wall Corrugated Shipping Containers Using Fixed and Floating Test Platens" (*J. Testing and Evaluation*, 1992: 318–320)**. The authors stated that "the difference between the compression strength using fixed and floating platen method was found to be small compared to normal variation in compression strength between identical boxes." Do you agree? Is your analysis predicated on any assumptions?

Method	Sample Size	Sample Mean	Sample SD
Fixed	10	807	27
Floating	10	757	41

68. The article **"Supervised Exercise Versus Non-Supervised Exercise for Reducing Weight in Obese Adults" (*The J. of Sports Med. and Physical Fitness*, 2009: 85–90)** reported on an investigation in which participants were randomly assigned to either a supervised exercise program or a control group. Those in the control group were told only that they should take measures to lose weight. After 4 months, the sample mean decrease in body fat for the 17 individuals in the experimental group was 6.2 kg with a sample standard deviation of 4.5 kg, whereas the sample mean and sample standard deviation for the 17 people in the control group were 1.7 kg and 3.1 kg, respectively. Assume normality of the two weight-loss distributions (as did the investigators).

a. Calculate a 99% lower prediction bound for the weight loss of a single randomly selected individual subjected to the supervised exercise program. Can

you be highly confident that such an individual will actually lose weight?

b. Does it appear that true average decrease in body fat is more than two kg larger for the experimental condition than for the control condition? Use the accompanying Minitab output to reach a conclusion at significance level of .01. [*Note:* Minitab accepts such summary data as well as individual observations. Also, because the test is upper-tailed, the software provides a lower confidence bound rather than a conventional CI.]

```
Sample      N    Mean    StDev    SE Mean
Exptl.     17    6.20     4.50        1.1
Control    17    1.70     3.10       0.75
Difference = mu (1) − mu (2)
Estimate for difference:  4.50
95% lower bound for difference: 2.25
T-Test of difference = 2 (vs >):
T-Value = 1.89
P-Value = 0.035 DF = 28
```

69. Is the response rate for questionnaires affected by including some sort of incentive to respond along with the questionnaire? In one experiment, 110 questionnaires with no incentive resulted in 75 being returned, whereas 98 questionnaires that included a chance to win a lottery yielded 66 responses (**"Charities, No; Lotteries, No; Cash, Yes,"** ***Public Opinion Quarterly*, 1996: 542–562**). Does this data suggest that including an incentive increases the likelihood of a response? State and test the relevant hypotheses at significance level .10.

70. Shoveling is not exactly a high-tech activity, but it will continue to be a required task even in our information age. The article **"A Shovel with a Perforated Blade Reduces Energy Expenditure Required for Digging Wet Clay"** (***Human Factors*, 2010: 492–502**) reported on an experiment in which 13 workers were each provided with both a conventional shovel and a shovel whose blade was perforated with small holes. The authors of the cited article provided the following data on stable energy expenditure [(kcal/kg(subject)/lb(clay)]:

Worker:	1	2	3	4
Conventional:	.0011	.0014	.0018	.0022
Perforated:	.0011	.0010	.0019	.0013

Worker:	5	6	7
Conventional	.0010	.0016	.0028
Perforated:	.0011	.0017	.0024

Worker:	8	9	10
Conventional:	.0020	.0015	.0014
Perforated:	.0020	.0013	.0013

Worker:	11	12	13
Conventional:	.0023	.0017	.0020
Perforated:	.0017	.0015	.0013

a. Calculate a confidence interval at the 95% confidence level for the true average difference between energy expenditure for the conventional shovel and the perforated shovel (the relevant normal probability plot shows a reasonably linear pattern). Based on this interval, does it appear that the shovels differ with respect to true average energy expenditure? Explain.

b. Carry out a test of hypotheses at significance level .05 to see if true average energy expenditure using the conventional shovel exceeds that using the perforated shovel.

71. The article **"Quantitative MRI and Electrophysiology of Preoperative Carpal Tunnel Syndrome in a Female Population"** (***Ergonomics*, 1997: 642–649**) reported that (−473.13, 1691.9) was a large-sample 95% confidence interval for the difference between true average thenar muscle volume (mm^3) for sufferers of carpal tunnel syndrome and true average volume for nonsufferers. Calculate and interpret a 90% confidence interval for this difference.

72. The following summary data on bending strength (lb-in/in) of joints is taken from the article **"Bending Strength of Corner Joints Constructed with Injection Molded Splines"** (***Forest Products J.*, April, 1997: 89–92**).

Type	Sample Size	Sample Mean	Sample SD
Without side coating	10	80.95	9.59
With side coating	10	63.23	5.96

a. Calculate a 95% lower confidence bound for true average strength of joints with a side coating.

b. Calculate a 95% lower prediction bound for the strength of a single joint with a side coating.

c. Calculate an interval that, with 95% confidence, includes the strength values for at least 95% of the population of all joints with side coatings.

d. Calculate a 95% confidence interval for the difference between true average strengths for the two types of joints.

73. The article **"Urban Battery Litter"** cited in Example 8.14 gave the following summary data on zinc mass (g) for two different brands of size D batteries:

Brand	Sample Size	Sample Mean	Sample SD
Duracell	15	138.52	7.76
Energizer	20	149.07	1.52

Assuming that both zinc mass distributions are at least approximately normal, carry out a test at significance level .05 to decide whether true average zinc mass is different for the two types of batteries.

74. The derailment of a freight train due to the catastrophic failure of a traction motor armature bearing provided the impetus for a study reported in the article **"Locomotive Traction Motor Armature Bearing Life Study"** (***Lubrication Engr.*, Aug. 1997: 12–19**). A sample of 17

high-mileage traction motors was selected, and the amount of cone penetration (mm/10) was determined both for the pinion bearing and for the commutator armature bearing, resulting in the following data:

	Motor 1	2	3	4	5	6
Commutator	211	273	305	258	270	209
Pinion	226	278	259	244	273	236

	Motor 7	8	9	10	11	12
Commutator	223	288	296	233	262	291
Pinion	290	287	315	242	288	242

	Motor 13	14	15	16	17
Commutator	278	275	210	272	264
Pinion	278	208	281	274	268

Calculate an estimate of the population mean difference between penetration for the commutator armature bearing and penetration for the pinion bearing, and do so in a way that conveys information about the reliability and precision of the estimate. [*Note:* A normal probability plot validates the necessary normality assumption.] Would you say that the population mean difference has been precisely estimated? Does it look as though population mean penetration differs for the two types of bearings? Explain.

75. *Headability* is the ability of a cylindrical piece of material to be shaped into the head of a bolt, screw, or other cold-formed part without cracking. The article **"New Methods for Assessing Cold Heading Quality" (*Wire J. Intl.*, Oct. 1996: 66–72)** described the result of a headability impact test applied to 30 specimens of aluminum killed steel and 30 specimens of silicon killed steel. The sample mean headability rating number for the steel specimens was 6.43, and the sample mean for aluminum specimens was 7.09. Suppose that the sample standard deviations were 1.08 and 1.19, respectively. Do you agree with the article's authors that the difference in headability ratings is significant at the 5% level (assuming that the two headability distributions are normal)?

76. The article **"Fatigue Testing of Condoms"** cited in Exercise 7.32 reported that for a sample of 20 natural latex condoms of a certain type, the sample mean and sample standard deviation of the number of cycles to break were 4358 and 2218, respectively, whereas a sample of 20 polyisoprene condoms gave a sample mean and sample standard deviation of 5805 and 3990, respectively. Is there strong evidence for concluding that true average number of cycles to break for the polyisoprene condom exceeds that for the natural latex condom by more than 1000 cycles? Carry out a test using a significance level of .01. [*Note:* The cited paper reported P-values of t tests for comparing means of the various types considered.]

77. Information about hand posture and forces generated by the fingers during manipulation of various daily objects is needed for designing high-tech hand prosthetic devices. The article **"Grip Posture and Forces During Holding Cylindrical Objects with Circular Grips" (*Ergonomics*, 1996: 1163–1176)** reported that for a sample of 11 females, the sample mean four-finger pinch strength (N) was 98.1 and the sample standard deviation was 14.2. For a sample of 15 males, the sample mean and sample standard deviation were 129.2 and 39.1, respectively.

a. A test carried out to see whether true average strengths for the two genders were different resulted in $t = 2.51$ and P-value $= .019$. Does the appropriate test procedure described in this chapter yield this value of t and the stated P-value?

b. Is there substantial evidence for concluding that true average strength for males exceeds that for females by more than 25 N? State and test the relevant hypotheses.

78. The article **"Pine Needles as Sensors of Atmospheric Pollution" (*Environ. Monitoring*, 1982: 273–286)** reported on the use of neutron-activity analysis to determine pollutant concentration in pine needles. According to the article's authors, "These observations strongly indicated that for those elements which are determined well by the analytical procedures, the distribution of concentration is lognormal. Accordingly, in tests of significance the logarithms of concentrations will be used." The given data refers to bromine concentration in needles taken from a site near an oil-fired steam plant and from a relatively clean site. The summary values are means and standard deviations of the log-transformed observations.

Site	Sample Size	Mean Log Concentration	SD of Log Concentration
Steam plant	8	18.0	4.9
Clean	9	11.0	4.6

Let μ_1^* be the true average *log* concentration at the first site, and define μ_2^* analogously for the second site.

a. Use the pooled t test (based on assuming normality *and* equal standard deviations) to decide at significance level .05 whether the two concentration distribution means are equal.

b. If σ_1^* and σ_2^* (the standard deviations of the two log concentration distributions) are not equal, would μ_1 and μ_2 (the means of the concentration distributions) be the same if $\mu_1^* = \mu_2^*$? Explain your reasoning.

79. The article **"The Accuracy of Stated Energy Contents of Reduced-Energy, Commercially Prepared Foods" (*J. of the Amer. Dietetic Assoc.*, 2010: 116–123)**

presented the accompanying data on vendor-stated gross energy and measured value (both in kcal) for 10 different supermarket convenience meals):

Meal:	1	2	3	4	5	6	7	8	9	10
Stated:	180	220	190	230	200	370	250	240	80	180
Measured:	212	319	231	306	211	431	288	265	145	228

Carry out a test of hypotheses to decide whether the true average % difference from that stated differs from zero. [*Note*: The article stated "Although formal statistical methods do not apply to convenience samples, standard statistical tests were employed to summarize the data for exploratory purposes and to suggest directions for future studies."]

80. Arsenic is a known carcinogen and poison. The standard laboratory procedures for measuring arsenic concentration (μg/L) in water are expensive. Consider the accompanying summary data and Minitab output for comparing a laboratory method to a new relatively quick and inexpensive field method (from the article **"Evaluation of a New Field Measurement Method for Arsenic in Drinking Water Samples,"** ***J. of Envir. Engr.*****, 2008: 382–388**).

Two-Sample T-Test and CI

```
Sample    N      Mean     StDev     SE Mean
1         3     19.70      1.10        0.64
2         3     10.90      0.60        0.35
Estimate for difference: 8.800
95% CI for difference: (6.498, 11.102)
T-Test of difference = 0 (vs not =):
T-Value = 12.16 P-Value = 0.001 DF = 3
```

What conclusion do you draw about the two methods, and why? Interpret the given confidence interval. [*Note*: One of the article's authors indicated in private communication that they were unsure why the two methods disagreed.]

81. The accompanying data on response time appeared in the article **"The Extinguishment of Fires Using Low-Flow Water Hose Streams—Part II"** (***Fire Technology*****, 1991: 291–320**).

Good visibility

.43 1.17 .37 .47 .68 .58 .50 2.75

Poor visibility

1.47 .80 1.58 1.53 4.33 4.23 3.25 3.22

The authors analyzed the data with the pooled t test. Does the use of this test appear justified? [*Hint*: Check for normality. The z percentiles for $n = 8$ are $-1.53, -.89, -.49, -.15, .15, .49, .89$, and 1.53.]

82. Acrylic bone cement is commonly used in total joint arthroplasty as a grout that allows for the smooth transfer of loads from a metal prosthesis to bone structure. The paper **"Validation of the Small-Punch Test as a Technique for Characterizing the Mechanical Properties of Acrylic Bone Cement"** (***J. of Engr. in Med.*****, 2006: 11–21**) gave the following data on breaking force (N):

Temp	Medium	n	$\bar{x}$	s
22°	Dry	6	170.60	39.08
37°	Dry	6	325.73	34.97
22°	Wet	6	366.36	34.82
37°	Wet	6	306.09	41.97

Assume that all population distributions are normal.

a. Estimate true average breaking force in a dry medium at 37° in a way that conveys information about reliability and precision, and interpret your estimate.

b. Estimate the difference between true average breaking force in a dry medium at 37° and true average force at the same temperature in a wet medium, and do so in a way that conveys information about precision and reliability. Then interpret your estimate.

c. Is there strong evidence for concluding that true average force in a dry medium at the higher temperature exceeds that at the lower temperature by more than 100 N?

83. In an experiment to compare bearing strengths of pegs inserted in two different types of mounts, a sample of 14 observations on stress limit for red oak mounts resulted in a sample mean and sample standard deviation of 8.48 MPa and .79 MPa, respectively, whereas a sample of 12 observations when Douglas fir mounts were used gave a mean of 9.36 and a standard deviation of 1.52 (**"Bearing Strength of White Oak Pegs in Red Oak and Douglas Fir Timbers,"** ***J. of Testing and Evaluation*****, 1998, 109–114**). Consider testing whether or not true average stress limits are identical for the two types of mounts. Compare df's and P-values for the unpooled and pooled t tests.

84. How does energy intake compare to energy expenditure? One aspect of this issue was considered in the article **"Measurement of Total Energy Expenditure by the Doubly Labelled Water Method in Professional Soccer Players"** (***J. of Sports Sciences*****, 2002: 391–397**), which contained the accompanying data (MJ/day).

	Player						
	1	2	3	4	5	6	7
Expenditure	14.4	12.1	14.3	14.2	15.2	15.5	17.8
Intake	14.6	9.2	11.8	11.6	12.7	15.0	16.3

Test to see whether there is a significant difference between intake and expenditure. Does the conclusion depend on whether a significance level of .05, .01, or .001 is used?

85. An experimenter wishes to obtain a CI for the difference between true average breaking strength for cables manufactured by company I and by company II. Suppose breaking strength is normally distributed for both types of cable with $\sigma_1 = 30$ psi and $\sigma_2 = 20$ psi.

a. If costs dictate that the sample size for the type I cable should be three times the sample size for the type II cable, how many observations are required if the 99% CI is to be no wider than 20 psi?

b. Suppose a total of 400 observations is to be made. How many of the observations should be made on type I cable samples if the width of the resulting interval is to be a minimum?

86. A study was carried out to compare two different methods, injection and nasal spray, for administering flu vaccine to children under the age of 5. All 8000 children in the study were given both an injection and a spray. However, the vaccine given to 4000 of the children actually contained just saltwater, and the spray given to the other 4000 children also contained just saltwater. At the end of the flu season, it was determined that 3.9% of the children who received the real vaccine via nasal spray contracted the flu, whereas 8.6% of the 4000 children receiving the real vaccine via injection contracted the flu.

a. Why do you think each child received both an injection and a spray?

b. Does one method for delivering the vaccine appear to be superior to the other? Test the appropriate hypotheses. [*Note:* The study was described in the article **"Spray Flu Vaccine May Work Better Than Injections for Tots,"** ***San Luis Obispo Tribune*, May 2, 2006.**]

87. Wait staff at restaurants have employed various strategies to increase tips. An article in the **Sept. 5, 2005, *New Yorker*** reported that "In one study a waitress received 50% more in tips when she introduced herself by name than when she didn't." Consider the following (fictitious) data on tip amount as a percentage of the bill:

Introduction:	$m = 50$	$\bar{x} = 22.63$	$s_1 = 7.82$
No introduction:	$n = 50$	$\bar{y} = 14.15$	$s_2 = 6.10$

Does this data suggest that an introduction increases tips on average by more than 50%? State and test the relevant hypotheses. [*Hint:* Consider the parameter $\theta = \mu_1 - 1.5\mu_2$.]

88. The paper **"Quantitative Assessment of Glenohumeral Translation in Baseball Players" (*The Amer. J. of Sports Med.*, 2004: 1711–1715)** considered various aspects of shoulder motion for a sample of pitchers and another sample of position players [*glenohumeral* refers to the articulation between the humerus (ball) and the glenoid (socket)]. The authors kindly supplied the following data on anteroposterior translation (mm), a measure of the extent of anterior and posterior motion, both for the dominant arm and the nondominant arm.

	Pos Dom Tr	Pos ND Tr	Pit Dom Tr	Pit ND Tr
1	30.31	32.54	27.63	24.33
2	44.86	40.95	30.57	26.36
3	22.09	23.48	32.62	30.62
4	31.26	31.11	39.79	33.74
5	28.07	28.75	28.50	29.84
6	31.93	29.32	26.70	26.71
7	34.68	34.79	30.34	26.45
8	29.10	28.87	28.69	21.49
9	25.51	27.59	31.19	20.82
10	22.49	21.01	36.00	21.75
11	28.74	30.31	31.58	28.32
12	27.89	27.92	32.55	27.22
13	28.48	27.85	29.56	28.86
14	25.60	24.95	28.64	28.58
15	20.21	21.59	28.58	27.15
16	33.77	32.48	31.99	29.46
17	32.59	32.48	27.16	21.26
18	32.60	31.61		
19	29.30	27.46		
mean	29.4463	29.2137	30.7112	26.6447
sd	5.4655	4.7013	3.3310	3.6679

a. Estimate the true average difference in translation between dominant and nondominant arms for pitchers in a way that conveys information about reliability and precision, and interpret the resulting estimate.

b. Repeat (a) for position players.

c. The authors asserted that "pitchers have greater difference in side-to-side anteroposterior translation of their shoulders compared with position players." Do you agree? Explain.

89. Suppose a level .05 test of $H_0\colon \mu_1 - \mu_2 = 0$ versus $H_a\colon \mu_1 - \mu_2 > 0$ is to be performed, assuming $\sigma_1 = \sigma_2 = 10$ and normality of both distributions, using equal sample sizes ($m = n$). Evaluate the probability of a type II error when $\mu_1 - \mu_2 = 1$ and $n = 25, 100, 2500,$ and $10{,}000$. Can you think of real problems in which the difference $\mu_1 - \mu_2 = 1$ has little practical significance? Would sample sizes of $n = 10{,}000$ be desirable in such problems?

90. The invasive diatom species *Didymosphenia geminata* has the potential to inflict substantial ecological and economic damage in rivers. The article **"Substrate Characteristics Affect Colonization by the Bloom-Forming Diatom *Didymosphenia geminata*" (*Acquatic Ecology*, 2010: 33–40)** described an investigation of colonization behavior. One aspect of particular interest was whether the roughness of stones impacted the degree of colonization. The authors of the cited article kindly provided the accompanying data on roughness ratio (dimensionless) for specimens of sandstone and shale.

Sandstone:	5.74	2.07	3.29	0.75	1.23	
	2.95	1.58	1.83	1.61	1.12	
	2.91	3.22	2.84	1.97	2.48	
	3.45	2.17	0.77	1.44	3.79	
Shale:	.56	.84	.40	.55	.36	.72
	.29	.47	.66	.48	.28	
	.72	.31	.35	.32	.37	.43
	.60	.54	.43	.51		

Normal probability plots of both samples show a reasonably linear pattern. Estimate the difference between true average roughness for sandstone and that for shale in a way that provides information about reliability and precision, and interpret your estimate. Does it appear that true average roughness differs for the two

types of rocks (a formal test of this was reported in the article)? [*Note:* The investigators concluded that more diatoms colonized the rougher surface than the smoother surface.]

91. Researchers sent 5000 resumes in response to job ads that appeared in the *Boston Globe* and *Chicago Tribune*. The resumes were identical except that 2500 of them had "white sounding" first names, such as Brett and Emily, whereas the other 2500 had "black sounding" names such as Tamika and Rasheed. The resumes of the first type elicited 250 responses and the resumes of the second type only 167 responses (these numbers are very consistent with information that appeared in a **Jan. 15, 2003**, report by the **Associated Press**). Does this data strongly suggest that a resume with a "black" name is less likely to result in a response than is a resume with a "white" name?

92. McNemar's test, developed in Exercise 56, can also be used when individuals are paired (matched) to yield n pairs and then one member of each pair is given treatment 1 and the other is given treatment 2. Then X_1 is the number of pairs in which both treatments were successful, and similarly for X_2, X_3, and X_4. The test statistic for testing equal efficacy of the two treatments is given by $(X_2 - X_3)/\sqrt{(X_2 + X_3)}$, which has approximately a standard normal distribution when H_0 is true. Use this to test whether the drug ergotamine is effective in the treatment of migraine headaches.

		Ergotamine	
		S	***F***
Placebo	***S***	44	34
	F	46	30

The data is fictitious, but the conclusion agrees with that in the article **"Controlled Clinical Trial of Ergotamine Tartrate" (*British Med. J.*, 1970: 325–327)**.

93. The article **"Evaluating Variability in Filling Operations" (*Food Tech.*, 1984: 51–55)** describes two different filling operations used in a ground-beef packing plant. Both filling operations were set to fill packages with 1400 g of ground beef. In a random sample of size 30 taken from each filling operation, the resulting means and standard deviations were 1402.24 g and 10.97 g for operation 1 and 1419.63 g and 9.96 g for operation 2.

a. Using a .05 significance level, is there sufficient evidence to indicate that the true mean weight of the packages differs for the two operations?

b. Does the data from operation 1 suggest that the true mean weight of packages produced by operation 1 is higher than 1400 g? Use a .05 significance level.

94. Let $X_1, \ldots, X_m$ be a random sample from a Poisson distribution with parameter μ_1, and let $Y_1, \ldots, Y_n$ be a random sample from another Poisson distribution with parameter μ_2. We wish to test H_0: $\mu_1 - \mu_2 = 0$ against one of the three standard alternatives. When m and n are large, the large-sample z test of Section 9.1 can be used. However, the fact that $V(\overline{X}) = \mu/n$ suggests that a different denominator should be used in standardizing $\overline{X} - \overline{Y}$. Develop a large-sample test procedure appropriate to this problem, and then apply it to the following data to test whether the plant densities for a particular species are equal in two different regions (where each observation is the number of plants found in a randomly located square sampling quadrate having area 1 m^2, so for region 1 there were 40 quadrates in which one plant was observed, etc.):

	Frequency								
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	
Region 1	28	40	28	17	8	2	1	1	$m = 125$
Region 2	14	25	30	18	49	2	1	1	$n = 140$

95. Referring to Exercise 94, develop a large-sample confidence interval formula for $\mu_1 - \mu_2$. Calculate the interval for the data given there using a confidence level of 95%.

BIBLIOGRAPHY

See the bibliography at the end of Chapter 7.